ECE 20100 – Fall 2018
Exam #2
October 16, 2018

Sections (include on scantron)

Qi (2:30) – 0002    Michelusi (8:30) – 0004    Cui (10:30) – 0005

Elliott (1:30) – 0012    Zhu (12:30) – 0013    Kildishev (12:30) – 0014

Narimanov (11:30) – 0015    Peleato-Inarrea (3:30) – 0016

Name Solution PUID

Instructions

1. DO NOT START UNTIL TOLD TO DO SO.
2. Write your name, section, professor, and student ID# on your Scantron sheet. We may check PUIDs.
3. This is a CLOSED BOOKS and CLOSED NOTES exam.
4. The use of a TI-30X IIS calculator is allowed, but not necessary.
5. If extra paper is needed, use the back of test pages.
6. All questions are worth 10 points.
7. Cheating will not be tolerated and will be dealt with according to the policy in your section. In particular, continuing to write after the exam time is up is regarded as cheating.
8. If you cannot solve a question, be sure to look at the other ones, and come back to it if time permits.
9. All of the problems on Exam #2 provide evidence for satisfaction of this ECE 20100 Learning Objective:
   
   ii) An ability to analyze 1st order linear circuits with sources and/or passive elements.

The minimum score needed to satisfy this objective will be posted on Blackboard after the exam has been graded. Remediation options will be posted in Blackboard if you fail to satisfy any of the course outcomes.

By signing the scantron sheet, you affirm you have not received or provided assistance on this exam.
Question 1

Find the Thevenin equivalent network for the circuit shown below.

Step 2: Find \( R_{th} \)

Disable 90 V source by shorting it:

\[
\frac{12 \ \Omega / 6 \ \Omega}{12 + 6} = 30 \ \text{V}
\]

\[
R_{th} = \left(\frac{12 \times 6}{12 + 6}\right) + 4 \ \Omega = 8 \ \Omega
\]

Step 1: Find \( V_{oc} \)

\[
V_{oc} = V_{bg}
\]

Since \( I_{ab} = 0 \)

\[
V_{ab} = 0 = V_{ba}
\]

So \( V_{oc} = V_{ba} + V_{ag} = V_{ag} \)

Use voltage division

\[
V_{ag} = 90 \left(\frac{6}{12 + 6}\right)
\]

\[
= 30 \ \text{V}
\]

\[
= V_{oc}
\]

(1) (2) (3) (4) (5) (6) (7) None of the above
Question 2

The linear network below contains only resistors, dependent sources and independent sources. Attached to the linear network is a variable resistor, $R_L$. The following data was measured,

$$V = 10 \text{ V}; \quad I_L = 0 \text{ A}$$

$$V = -10 \text{ V}; \quad I_L = 2 \text{ A}$$

Find the Thevenin equivalent resistance, $R_{th}$, for the linear network (in $\Omega$).

Replace the linear network with its Thevenin equivalent, we have:

$$V = V_{oc} - I_L \cdot R_{th}$$

plug in the values:

\[
\begin{align*}
10 &= V_{oc} - 0 \\
-10 &= V_{oc} - 2 \cdot R_{th}
\end{align*}
\]

So $R_{th} = 10 \Omega$

Note: the load resistor, $R_L$, should be interpreted as capable of being infinite or negative, so as to satisfy the V-I relationship in this question.
Question 3

In the circuit shown, determine the maximum power delivered to the load resistor \( R_L \) (in W).

(1) 11.25  
(2) 12  
(3) 15  
(4) 18  
(5) 22.5  
(6) 30  
(7) 33.75  
(8) 45  
(9) None of the above

Step 1: Remove the \( R_L \) and solve for \( V_{OC} \):

\[ V_{OC} = V_{BG} = V_{BA} + V_{AG} \]

\[ V_{AG} = (6A)(6\Omega) = 30V \]

So \( V_{OC} = V_{AG} = 30V \)

Step 2: Remove \( R_L \) & disable 5A source

\[ R_{TH} = 4 + 6 = 10\Omega \]

Step 3:

\[ P_{L, max} = \frac{V_{OC}^2}{4R_{TH}} = \frac{30^2}{4 \times 10} = 22.5 \text{ W} \]
Question 4

In the circuit shown, the current through the resistor at time $t = 0$ is $i(0) = 0.5$ mA. Find the energy (in mJ) dissipated through the resistor in the time interval $(0, \infty)$.

Step 1: $A t \ t = \infty, \ V_c(\infty) = 0 \ V$

Step 2: Energy dissipated between $(0, \infty)$

$$i_s = \frac{1}{2} C V_c^2(0) - \frac{1}{2} C V_c^2(\infty)$$

$$= \frac{1}{2} \left(2 \times 10^{-3}\right) \left(0.5 \text{ mA} \times 6 \text{k}\Omega\right)^2 - 0$$

$$= 10^{-3} \left(3\right)^2$$

$$= 9 \text{ mJ}$$
Question 5

Find the equivalent inductance, $L_{eq}$, as shown in the figure (in mH).

(1) 1
(2) 2
(3) 3
(4) 4
(5) 5
(6) 6
(7) 7
(8) 8
(9) None of the above
Questions 6 and 7

The circuit shown is used for Questions 6-7. The switch in the circuit opens at $t = 0$ s after being closed for a long time.

Question 6

Because of the continuity principle, $v_L(0^-) = v_L(0^+)$ is always valid.

(1) True
(2) False

\[ v_L(0^+) = v_L(0^-) \quad \textit{NOT } v_L \]

Question 7

Find the value of $i_L(0^-)$ (in A).

(1) 1
(2) 2
(3) 3
(4) 4
(5) 5
(6) 6
(7) 7
(8) 8
(9) None of the above

Since the circuit is at equilibrium at $t = 0$ we can replace the inductor with a short circuit.

We define $i_L(0^-)$ using current division:

\[ i_L(0^-) = \frac{12}{12+4} = 6 \text{ A} \]
Question 8

At $t = 0$ s, the inductor current is $i_L(0) = 5$ mA. Find the time constant for the discharging inductor when $t > 0$ (in msec).

\[ \text{Remove the inductor,} \]
\[ \text{and disable all independent sources (there is none)} \]

\[ \text{Add an auxiliary voltage source of 12 Volts:} \]

\[ \text{KCL @ node A:} \]
\[ I_{IN} + 0.25i_{12} = i_{12} \]
\[ \Rightarrow I_{IN} = 0.75i_{12} = 0.75A \]

\[ R_{th} = \frac{12V}{I_{IN}} = \frac{12}{0.75} = 16\Omega \]

\[ \Rightarrow \tau = \frac{L}{R_{th}} = \frac{40 \text{mH}}{16\Omega} = 2.5 \text{ msec} \]
Question 9

In the circuit below, find the capacitor voltage for $t \geq 0$ sec (in V).

1. $3.5 + 2e^{-4t}$
2. $3.5 - 5e^{-0.5t}$
3. $7 + 3.5e^{-0.5t}$
4. $2 - 4e^{-0.25t}$
5. $7e^{-3t}$
6. $7e^{-0.5t}$
7. $3.5e^{4t}$
8. $3.5e^{-0.25t}$
9. None of the above

Step 1: at $t = 0^-$, replace $C$ with an open circuit.

- $V_c(0^-) = 7$ V
- So $V_c(0^+) = 7$ V

Step 2: at $t = \infty$, again replace $C$ with an open circuit.

- $V_c(\infty) = 0$ V

Step 3: disable voltage source.

- $R_{Th} = 2\, k\Omega$

Step 4: $C = R_{Th} \cdot C = 2 \times 10^3 \times 0.1 \times 10^{-3} = 0.2$ sec

Step 5: $V_c(0) = V_c(\infty) + [V_c(0^+) - V_c(\infty)]e^{-\frac{t-0}{0.2}}$

$= 0 + [7 - 0]e^{-5t}$

$= 7e^{-5t}$
Question 10

In the circuit below, the switch moves from the left position to the right position at \( t = 0 \) sec. Find \( v_c(t) \) for \( t \geq 0 \) (in V):

(1) \( 40 - e^{2t} \)
(2) \( 20 + 20e^{-0.5t} \)
(3) \( 30 + 20e^{-0.5t} \)
(4) \( 40 + 10e^{-2t} \)
(5) \( 30 + 20e^{-2t} \)
(6) \( 30 + 5e^t \)
(7) \( 20 + 10e^{-t} \)
(8) None of the above

\[
\text{Step 1: Replace C w/ open circuit}
\]
\[
\text{Step 2: at } t = \infty, \text{ replace C w/ open circuit}
\]

Notice that the voltage across the 30\( \Omega \) resistor is zero because no current flows through it:

\[
v_c(\infty) = 40 \cdot \frac{20}{20+20} = 20 \text{ V}
\]

Step 3: Disable the 40 V source to find \( R_{th} \)

\[
R_{th} = 30 + \frac{(10/10)}{10} = 40 \text{ \Omega}
\]

Step 4:

\[
v_c(t) = v_c(\infty) + \left[ v_c(0^+) - v_c(\infty) \right] e^{-\frac{t-0}{\tau}} = 20 + \left[ \frac{30-20}{10} \right] e^{-\frac{t}{10}} = 20 + 10e^{-\frac{t}{10}}
\]
Potentially Useful Formulas

\[ x(t) = x(\infty) + \left[ x(t'_0) - x(\infty) \right] e^{-(t-t'_0)/\tau}, \text{ where } \tau = \frac{R M C}{R M} \]

\[ \nu_L(t) = L \frac{d i_L(t)}{dt} \]

\[ i_L(t) = i_L(t_0) + \frac{1}{L} \int_{t_0}^{t} v_L(t')dt' \]

\[ W_L(t_0, t_1) = \frac{L}{2} \left[ (i_L(t_1))^2 - (i_L(t_0))^2 \right] \]

\[ \nu_C(t) = C \frac{d v_C(t)}{dt} \]

\[ v_C(t) = v_C(t_0) + \frac{1}{C} \int_{t_0}^{t} i_C(t')dt' \]

\[ W_C(t_0, t_1) = \frac{C}{2} \left[ (v_C(t_1))^2 - (v_C(t_0))^2 \right] \]