ECE20100 Linear Circuit Analysis I  
Fall 2019, Section 015  
Exam 2, October 31, 2019  
THIS EXAM SHOULD BE TAKEN ONLY BY  
STUDENTS REGISTERED FOR ECE20100 SECTION 015

Last Name: ___________________ First Name: ____________________

As a boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together - we are Purdue.

I certify that I have neither given nor received unauthorized aid on this exam.

Signed: ___________________

This exam corresponds to learning outcome 2.

Solve the following problems. The number of points for each problem is shown in the table below.

Use only the space provided to solve each problem, and copy the answers to the space marked "Answer:..." Do not forget to specify units.

Show all the steps of your solution. Final answers alone will not be considered.

Non-integer answers can be written as $\frac{a}{b}$ or as $c.d$ where $d$ can be rounded to 2-3 digits.

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Reminder:

The energy absorbed by an inductor in the time interval \([t_0, t_1]\) is equal to
\[
w_L(t_0, t_1) = \frac{1}{2} L [i_L^2(t_1) - i_L^2(t_0)].
\]

The energy absorbed by a capacitor in the time interval \([t_0, t_1]\) is equal to
\[
w_C(t_0, t_1) = \frac{1}{2} C v_C^2(t_1) - \frac{1}{2} C v_C^2(t_0).
\]

The solution to \(\frac{dx(t)}{dt} + ax(t) = A\) is \(x(t) = \frac{A}{a} + K_2 e^{-at}\).

For a source-free RL circuit, \(i_L(t) = i_L(t_0) e^{-\frac{R}{L}(t-t_0)}\).

For a source-free RC circuit, \(v_C(t) = v_C(t_0) e^{-\frac{1}{RC}(t-t_0)}\).

For a series RL circuit with an independent voltage source \(V_S\),
\[
i_L(t) = \frac{V_S}{R} + [i_L(t_0) - \frac{V_S}{R}] e^{-\frac{R}{L}(t-t_0)}.
\]

For a series RC circuit with an independent voltage source \(V_S\),
\[
v_C(t) = V_S + [v_C(t_0) - V_S] e^{-\frac{1}{RC}(t-t_0)}.
\]

For a voltage or current in an RC or RL first-order linear circuit with constant input,
\[
x(t) = X_e + [x(t_0) - X_e] e^{-\frac{t-t_0}{\tau}}
\]
Problem 1

Answer the questions below with respect to the following circuit.

(a) Without solving differential equations, find $v_C(\infty)$.

$$v_C(\infty) = 1 \cdot \frac{2}{1+2} = \frac{2}{3} \text{V}$$

(10 points) Answer: $v_C(\infty) = \frac{2}{3} \text{V}$

(b) Using nodal or loop analysis as learned in class, write a first-order differential equation for $v_C(t)$. Express the equation in the standard form we have seen in class.

(10 points) Nodal or loop equations:

$$\frac{v_C - 1}{\frac{1}{3}} + 3 \frac{dv_C}{dt} + \frac{v_C}{2} = 0$$

$$3 \frac{dv_C}{dt} + \frac{3}{2} v_C = 1$$

(10 points) Answer (differential equation):

$$\frac{dv_C}{dt} + \frac{1}{2} v_C = \frac{1}{3}$$

(c) Write an expression for $v_C(t)$, where $t \geq 0$, assuming that $v_C(0) = 0$.

$$v_C(t) = \frac{1}{3} \cdot \frac{2}{1} + K_2 e^{-\frac{1}{2}t}$$

$$0 = \frac{2}{3} + K_2, K_2 = -\frac{2}{3}$$

(10 points) Answer: $v_C(t) = \frac{2}{3} - \frac{2}{3} 2e^{-\frac{1}{2}t}$
Problem 2

Using nodal or loop analysis as learned in class, write a first-order differential equation for $v_c(t)$ in the following circuit. Express the equation in the standard form we have seen in class.

(10 points) Nodal or loop equations:

\[-5000i_c + 101v_d - v_d = 0\]
\[-40i_C - v_c + v_d - 1 = 0\]
\[i_C = 0.1 \frac{dv_C}{dt} \]

\[100v_d = 5000i_c\]
\[v_d = 50i_c = 50 \cdot 0.1 \frac{dv_C}{dt} = 5 \frac{dv_C}{dt}\]
\[-40 \cdot 0.1 \frac{dv_C}{dt} - v_C + 5 \frac{dv_C}{dt} - 1 = 0\]

(30 points) Answer (differential equation for $v_c(t)$):

\[\frac{dv_C}{dt} - v_c = 1\]
Problem 3

In the following circuit, \( v_S(t) = 10u(t) \text{V}, \) \( i_S(t) = 2u(t) \text{A}, \) \( i_L(0) = 5 \text{A}, \) \( R_1 = 10 \Omega, \) \( R_2 = 20/3 \Omega \) and \( L = 2 \text{H}. \) Analysis of the circuit yielded the equation
\[
i_L(t) = 3 + 2e^{-2t} \text{A} \quad \text{for} \quad t \geq 0.
\]

Find \( v_L(t), v_{R1}(t), i_{R1}(t) \) and \( i_{R2}(t) \) for \( t > 0. \)

\[
v_L(t) = 2 \frac{di_L}{dt} = 2(-2)2e^{-2t} = -8e^{-2t} \text{V}
\]
\[
v_{R1}(t) = 10 - v_L = 10 + 8e^{-2t} \text{V}
\]
\[
i_{R1} = \frac{v_{R1}}{R_1} = \frac{10 + 8e^{-2t}}{10} \text{A}
\]
\[
i_{R2} = \frac{v_L}{R_2} = -\frac{8e^{-2t}}{20/3} \text{A}
\]

Answer:

(5 points) \( v_L(t) = -8e^{-2t} \text{V} \)

(5 points) \( v_{R1}(t) = 10 + 8e^{-2t} \text{V} \)

(5 points) \( i_{R1}(t) = 1 + 0.8e^{-2t} \text{A} \)

(5 points) \( i_{R2}(t) = -1.2e^{-2t} \text{A} \)