Solve the following problems. The number of points for each problem is shown in the table below. The outcome corresponding to each problem is also shown in the table.

Use only the space provided to solve each problem, and copy the answers to the space marked "Answer..." Do not forget to specify units.

Show all the steps of your solution. Final answers alone will not be considered.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>2</td>
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<tr>
<td>4</td>
<td>15</td>
<td>2</td>
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<tr>
<td>5</td>
<td>20</td>
<td>3</td>
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<tr>
<td>6</td>
<td>15</td>
<td>4</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td></td>
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</tbody>
</table>
Reminder:

The energy absorbed by an inductor in the time interval \([t_0,t_1]\) is equal to 
\[w_L(t_0,t_1) = \frac{1}{2}L[i_L^2(t_1) - i_L^2(t_0)].\]

The energy absorbed by a capacitor in the time interval \([t_0,t_1]\) is equal to 
\[w_C(t_0,t_1) = \frac{1}{2}Cv_C^2(t_1) - \frac{1}{2}Cv_C^2(t_0).\]

The solution to \[\frac{dx(t)}{dt} + ax(t) = A\] is \[x(t) = \frac{A}{a} + Ke^{-at}.\]

For a source-free RL circuit, \(i_L(t) = i_L(t_0)e^{-\frac{R}{L}(t-t_0)}\)

For a source-free RC circuit, \(v_C(t) = v_C(t_0)e^{-\frac{1}{RC}(t-t_0)}\)

For a series RL circuit with an independent voltage source \(V_s\), \(i_L(t) = \frac{V_s}{R} + \frac{V_s}{R} \left[ i_L(t_0) - \frac{R}{L}e^{-\frac{R}{L}(t-t_0)} \right]\)

For a series RC circuit with an independent voltage source \(V_s\), \(v_C(t) = V_s + \left[ v_C(t_0) - V_s \right] e^{-\frac{1}{RC}(t-t_0)}\)

For a voltage or current in an RC or RL first-order linear circuit with constant input, 
\[x(t) = X_e + \left[ x(t_0^-) - X_e \right] e^{-\frac{t-t_0}{\tau}}\]
Problem 1

In the following circuit, $R_L$ represents the load, $R_1 = 3\Omega$, $V_1 = 3V$, $R_2 = 6\Omega$, and $V_2 = 6V$. Find the value of $R_L$ that will lead to maximum power transfer to the load, and find the maximum power $P_{L,\text{max}}$ delivered to the load.

\[ i_X = \frac{\omega x - 3}{3} + \frac{\omega x - 6}{6} \]

\[ 6i_X = 2(\omega X - 6) + \omega x - 6 = 3\omega x - 12 \]

\[ 3\omega x = 6i_X + 12 \]

\[ \omega x = 2i_X + \frac{6}{3} \]

\[ u_{oc} \]

\[ P_{L,\text{max}} = \frac{u_{oc}^2}{4R_{th}} = \frac{16}{4 \cdot 2} = 2\omega \]

Answer:

$R_L = R_{th} = 2\Omega$

$P_{L,\text{max}} = 2\omega$
Problem 2

In the following circuit, $R$, $C$ and $v_C(0)$ are known. Answer the following questions with respect to this circuit:

(a) What is the energy stored in the capacitor at $t = 0$?

Answer: \( \frac{1}{2} C v_C^2(0) \)

(b) What is the energy stored in the capacitor at $t = \infty$?

Answer: 0

(c) What happened to the difference in energy you found in (a) and (b)? Keep your answer to five or less words.

Answer: dissipated by the resistor

(d) Write an expression for $i_R(t)$, where $t \geq 0$, in terms of $R$, $C$, and $v_C(0)$.

The expression should be as simple as possible.

\[
\begin{align*}
    v_C(t) &= v_C(0) e^{-\frac{t}{RC}} \\
    i_R(t) &= -i_C(t) = -\frac{1}{C} \frac{dv_C}{dt} = -C \frac{dv_C}{dt} = -C v_C(0) \left( -\frac{1}{RC} \right) e^{-\frac{t}{RC}}
\end{align*}
\]

Answer: $i_R(t) = \frac{v_C(0)}{R} e^{-\frac{t}{RC}}$
Problem 3

In the following circuit, $C = 1F$, $R = 1\Omega$, $g_m = 2S$ and $v_{c}(0) = 1V.$

(a) Write an expression for $v_x(t)$ as a function of time only ($R$, $g_m$ and $C$ should be replaced by their values and not appear in the expression).

The expression should be as simple as possible.

\[
i_x = \frac{v_x}{R} - g_m v_x = v_x - 2v_x = -v_x.
\]

\[R_{th} = -1 \Omega.
\]

\[v_x(t) = v_{c}(0) e^{-\frac{t}{R_{th}C}} = 1 \cdot e^{-\frac{t}{(\cdot)}1}.
\]

Answer: $v_x(t) = e^t \checkmark$

(b) What happens to the circuit as $t \rightarrow \infty$? Keep your answer to five or less words.

Answer: saturates
Problem 4

In the following circuit, \( v_s(t) = 1\text{V} \) and \( v(0) = 1\text{V} \). Find \( v(t) \) for \( t \geq 0 \).

\[
\frac{\omega}{1} + 1 \cdot \frac{d\omega}{dt} + \frac{\omega - 1}{1} + 1 \cdot \frac{d}{dt} (\omega - 1) = 0
\]

\[
2\omega + 2 \frac{d\omega}{dt} = 1
\]

\[
\frac{d\omega}{dt} + \omega = 0.5
\]

\[
\omega(t) = 0.5 + k_2 e^{-t}
\]

\[
v(t) = 0.5 + 0.5 e^{-t}
\]

Answer: \( v(t) = 0.5 + 0.5 e^{-t} \) ✔
Problem 5

In the following circuit, $R_1 = 10\Omega$, $R_2 = 20\Omega$ and $L = 2\text{H}$.
The following information is known about this circuit:  
With $i_L(0) = 0$, $v_s(t) = 10\text{V}$ and $i_s(t) = 0$, one obtains that $i_L(t) = 1-e^{-2t} \text{ A}$ for $t \geq 0$. 
With $i_L(0) = 0$, $v_s(t) = 0$ and $i_s(t) = 2\text{A}$, one obtains that $i_L(t) = 2-2e^{-2t} \text{ A}$ for $t \geq 0$. 
Find $i_L(t)$, where $t \geq 0$, for the following case: $i_L(0) = 2\text{A}$, $v_s(t) = 20\text{V}$, and $i_s(t) = 3\text{A}$.

\[
\begin{aligned}
V_s = 10\text{V}: & \quad i_L(t) = 1-e^{-2t} \\
V_s = 20\text{V}: & \quad i_L(t) = 2-2e^{-2t} \\
I_s = 2\text{A}: & \quad i_L(t) = 2-2e^{-2t} \\
I_s = 3\text{A}: & \quad i_L(t) = 2-3e^{-2t} \\
i_L(0) = 2\text{A}: & \quad R_{eq} = R_{L1}||R_2 = \frac{1}{\frac{1}{10} + \frac{2}{20}} = \frac{20}{3} = 4\Omega \\
 & \quad i_L(t) = i_L(0) e^{-\frac{R_{eq}t}{L}} = 2 e^{-2t} \\
 & \quad \frac{5 - 3e^{-2t}}{}
\end{aligned}
\]

Answer: $i_L(t) = \frac{5 - 3e^{-2t}}{} \text{ A}$
Problem 6

In the following circuit, \( R_1 = 1\Omega, \ R_2 = 1\Omega, \ C = 1F, \ v_{in}(t) = -2V \) for \( t < 0 \) and \( v_{in}(t) = 2V \) for \( t \geq 0 \).

(a) Find \( i_{R1}(0-) \).

\[
\frac{-2}{1+1} = -1A
\]

Answer: \( i_{R1}(0-) = -1A \)

(b) Find \( i_{R1}(0+) \).

\[
v_C(0-) = -1V = v_C(0+);
\]

\[
i_{R1}(0+) = \frac{2-(-1)}{1} = 3A
\]

Answer: \( i_{R1}(0+) = 3A \)

(c) Find \( i_{R1}(\infty) \).

\[
\frac{2}{1+1} = 1A
\]

Answer: \( i_{R1}(\infty) = 1A \)