Solution

This exam should be taken only by students registered for section 2

November 28, 2012

Name ____________________________

ECE201 Linear Circuit Analysis I
Fall 2012, Section 2, MWF 7:30-8:20am
Exam 3

This exam corresponds to learning objective 3.

Solve the following problems. The number of points for each problem is shown in the table below.

Use only the space provided to solve each problem, and copy the answers to the space marked "Answer:..." Do not forget to specify units.

Show all the steps of your solution. Final answers alone will not be considered.

Non-integer answers can be written as $a/b$ or as $c.d$ where $d$ is rounded to 2-3 digits.

Complex numbers can be written as $a+jb$ or as $Ae^{j\theta}$.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
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<tr>
<td>1</td>
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<td>4</td>
<td>/ 25</td>
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<td>Total</td>
<td>/ 100</td>
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Reminder:

For a source-free LC circuit, \( \frac{d^2v_C(t)}{dt^2} + \frac{1}{LC}v_C(t) = 0 \). \( v_C(t) = K\cos(\omega t + \theta) \) with \( \omega = \frac{1}{\sqrt{LC}} \).

For a source-free series RLC circuit, \( \frac{d^2v_C(t)}{dt^2} + \frac{R}{L} \frac{dv_C(t)}{dt} + \frac{1}{LC}v_C(t) = 0 \).

\[ \frac{d^2x(t)}{dt^2} + b \frac{dx(t)}{dt} + cx(t) = 0. \]

\( s^2 + bs + c = 0 \) yields \( s_1 = \frac{-b + \sqrt{b^2 - 4c}}{2} \), \( s_2 = \frac{-b - \sqrt{b^2 - 4c}}{2} \).

Case 1: \( b^2 - 4c > 0 \), \( x(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t} \).

Case 2: \( b^2 - 4c < 0 \), \( s_{1,2} = -\sigma \pm j \omega \), \( x(t) = e^{-\sigma t} [A \cos(\omega t) + B \sin(\omega t)] \), or \( x(t) = Ke^{-\sigma t} \cos(\omega t + \theta) \).

Case 3: \( b^2 - 4c = 0 \), \( x(t) = (K_1 + K_2 t) e^{s_1 t} \).

For an RLC circuit with a constant independent source, \( \frac{d^2x(t)}{dt^2} + b \frac{dx(t)}{dt} + cx(t) = F \), \( x(t) = x_0(t) + \frac{F}{c} \).

\( e^{j\phi} = \cos(\phi) + j \sin(\phi) \)

\( j = e^{j90^\circ} \)
\( -j = e^{-j90^\circ} \)
\( 1+j = 1.414e^{j45^\circ} \)
\( 1-j = 1.414e^{-j45^\circ} \)
\( -1+j = 1.414e^{j135^\circ} \)
\( -1-j = 1.414e^{-j135^\circ} \)
Problem 1

Using nodal or loop analysis as learned in class, write a second-order differential equation for $v_c(t)$ in the following circuit. Express the equation in the standard form we have seen in class.

\[ 3i_1 + 2 \frac{di_1}{dt} - v_c + 2(i_1 - 2) = 0 \]
\[ 2 \frac{di_1}{dt} - v_c + 5i_1 = 0 \]
\[ 2 \left( -3 \frac{d^2 v_c}{dt^2} \right) - v_c + 5 \left( 2 - 3 \frac{dv_c}{dt} \right) = 0 \]
\[ -6 \frac{d^2 v_c}{dt^2} - v_c + 10 - 15 \frac{dv_c}{dt} = 0 \]
\[ 6 \frac{d^2 v_c}{dt^2} + 15 \frac{dv_c}{dt} + v_c = 0 \]

(30 points) Answer (differential equation):

\[ \frac{d^2 v_c}{dt^2} + \frac{5}{2} \frac{dv_c}{dt} + \frac{1}{6} v_c = 0 \]
Problem 2

In the following circuit, $v_{C1}(0) = 2 \text{V}$ and $v_{C2}(0) = 4 \text{V}$. Using one of the methods studied in class, find $\frac{dv_{C1}(0^+)}{dt}$ and $\frac{dv_{C2}(0^+)}{dt}$.

\[
\frac{d v_{C1}(0^+)}{d t} = \frac{i_{C1}(0^+)}{2} = \frac{1}{2} (1 - 1) = 0
\]

\[
\frac{d v_{C2}(0^+)}{d t} = \frac{i_{C2}(0^+)}{2} = \frac{1}{2} (-1 - 1) = -\frac{\sqrt{2}}{2}
\]

Answer:

(10 points) $\frac{dv_{C1}(0^+)}{dt} = 0$

(10 points) $\frac{dv_{C2}(0^+)}{dt} = -\frac{\sqrt{2}}{2}$
Problem 3

The following circuit operates in sinusoidal steady state. For this circuit, \( v_s(t) = \cos(t) \) V, \( R = 1 \Omega, L = 1 \text{H}, C = 1 \text{F} \) and \( a = 1 \). Using phasor analysis as studied in class, find the phasor \( I_x \) corresponding to \( i_x(t) \). Then find \( i_x(t) \), and the complex power delivered by the independent voltage source.

\[
I_x = \frac{1 - I_x}{j} + \frac{1}{j + 1} = \frac{1 - I_x}{j} + \frac{1}{1 - j}
\]

\[
I_x \left(1 + \frac{1}{j}\right) = \frac{1}{j} + \frac{1}{1 - j}
\]

\[
I_x \left(1 - j\right) = \frac{1 - j + j}{j(1 - j)} = \frac{1}{1 + j}
\]

\[
I_x = \frac{1}{(1 + j)(1 - j)} = \frac{1}{2} \quad \text{A} \quad , \quad i_x(t) = \frac{1}{2} \cos(t) \quad \text{A}
\]

\[
\vec{S} = V_s e^{\text{eff}} \cdot J_x e^{\text{eff}} = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} - \frac{1}{2} = \frac{1}{4} \quad \text{VA}
\]

Answer:

(15 points) \( I_x = \frac{1}{2} \quad \text{A} \)

(5 points) \( i_x(t) = \frac{1}{2} \cos(t) \quad \text{A} \)

(5 points) Complex Power = \( \frac{1}{4} \quad \text{VA} \)
Problem 4

The following circuit operates in sinusoidal steady-state. For this circuit, \( C_1 = C_2 = 1 \text{F} \) and \( R_1 = R_2 = R_3 = R_4 = 1 \Omega \). Find the frequency response of the circuit. You can express it in the form \( a + jb \) or in the form \( \frac{1}{a + jb} \).

\[
\begin{align*}
\frac{V_{in}}{R_1} + V_{in} j\omega C_1 + \frac{V_1}{R_2} &= 0 \\
\frac{V_1}{R_3} + V_{out} j\omega C_2 + \frac{V_{out}}{R_4} &= 0 \\
V_{in} + j\omega V_1 + V_1 &= 0 \quad \rightarrow \quad V_1 = -\frac{V_{in}}{1+j\omega} \\
V_1 + j\omega V_{out} + V_{out} &= 0 \quad \rightarrow \quad V_{out} = -\frac{V_1}{1+j\omega} \\
V_{out} &= \frac{V_{in}}{(1+j\omega)^2} \\
\frac{V_{out}}{V_{in}} &= \frac{1}{1+j\omega} = \frac{1}{1-\omega^2 + j2\omega}
\end{align*}
\]

(25 points) Answer (frequency response):

\[
H(j\omega) = \frac{1}{1-\omega^2 + j2\omega}
\]