Instructions

1. DO NOT START UNTIL TOLD TO DO SO.
2. Write your name, division, professor, and student ID# (PUID) on both your scantron sheet and your work-out sheet.
3. This is a CLOSED BOOKS and CLOSED NOTES exam.
4. This exam has two parts:
   - Part A contains 9 multiple-choice questions, each worth 10 points. There is only one correct answer to each question. Only answers marked on your scantron sheet will be graded. Partial credits will NOT to be given to Part A.
   - Part B contains 1 work-out problem worth 10 points. Only answers written on the work-out sheet will be graded. Clearly show intermediate steps in order to receive partial credits.
5. Calculators are allowed (but not necessary). Please clear any formulas, text, or other information from your calculator memory prior to the exam.
6. If extra scratch paper is needed, use back of test pages. If extra work-out sheet is needed, ask the instructor.
7. Cheating will not be tolerated. Cheating in this exam will result in an F in the course.
8. If you cannot solve a question, be sure to look at the other ones and come back to it if time permits.
9. As described in the course syllabus, we must certify that every student who receives a passing grade in this course has satisfied each of the course outcomes. On this exam, you have the opportunity to satisfy the following outcomes. (See the course syllabus for a complete description of each outcome.) On the chart below, we list the criteria we use for determining whether you have satisfied these course outcomes. You only need to satisfy the outcomes once during the course, so any outcomes that you satisfied previously will remain satisfied, independent of your performance on this exam.

<table>
<thead>
<tr>
<th>Course Outcome</th>
<th>Exam Questions</th>
<th>Minimum correct answers required to satisfy the course outcome</th>
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<tbody>
<tr>
<td>i</td>
<td>3,4,5.</td>
<td>2</td>
</tr>
<tr>
<td>ii</td>
<td>6,7,8,10</td>
<td>2</td>
</tr>
<tr>
<td>iii</td>
<td>1,2,9.</td>
<td>2</td>
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PART A: **9 Multi-Choice Questions**

**IMPORTANT:** Only answers marked on your scantron sheet will be graded!!!

One correct answer for each question.
1. The switch in the RLC circuit below has been at position A for a long time. It turns to position B at $t = 0$ and remains there afterward. The capacitor voltage $V_c(t)$ is known to be zero before $t = 0$. What is the inductor current $i_L(t)$ for $t \geq 0$?

\[ i_L(t) \]

(1) $4 \cos \left( \frac{3}{4} t \right)$  
(2) $4 \cos \left( \frac{1}{4} t \right)$  
(3) $0.8 \cos \left( \frac{3}{4} t \right)$  
(4) $0.8 \sin \left( \frac{3}{4} t \right)$  
(5) $4 \cos \left( \frac{9}{16} t \right)$  
(6) $0.8 \cos \left( \frac{9}{16} t \right)$  
(7) $0.8 \sin \left( \frac{9}{16} t \right)$  
(8) $0.8 \cos \left( \frac{2}{3} t \right)$  
(9) None of the above

\[
\begin{align*}
\text{At } t = 0^- & \\
\text{4A} & \uparrow \quad 5Ω \quad \text{16H} \\
\text{After } t = 0 & \\
\frac{1}{4}Ω & \quad \frac{1}{16}\text{H} \quad \omega = \frac{1}{\sqrt{LC}} = \sqrt{16 \times \frac{1}{9}} = \frac{3}{4} \\
\dot{i}_L(+) & = A \cos \frac{3}{4} t + B \sin \frac{3}{4} t \\
\dot{i}_L(0^-) & = A = 4 \\
\dot{v}_c(0^-) & = \frac{1}{C} v_c(0) = \frac{1}{C} V_c(0) = 0 \\
\frac{d}{dt}v_c(0) & = \frac{3}{4} B \quad \Rightarrow \quad B = 0 \\
\text{Hence, } i_L(+) & = 4 \cos \frac{3}{4} t
\end{align*}
\]
2. In the following RLC circuit, find $i(0^+)$, $i_C(0^+)$, and $i(\infty)$.

<table>
<thead>
<tr>
<th></th>
<th>$i(0^+)$</th>
<th>$i_C(0^+)$</th>
<th>$i(\infty)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>1 A</td>
<td>2 A</td>
<td>1 A</td>
</tr>
<tr>
<td>(2)</td>
<td>1 A</td>
<td>1 A</td>
<td>1.4 A</td>
</tr>
<tr>
<td>(3)</td>
<td>2.5 A</td>
<td>0 A</td>
<td>2 A</td>
</tr>
<tr>
<td>(4)</td>
<td>1 A</td>
<td>0 A</td>
<td>2 A</td>
</tr>
<tr>
<td>(5)</td>
<td>1 A</td>
<td>0 A</td>
<td>1.4 A</td>
</tr>
<tr>
<td>(6)</td>
<td>−1 A</td>
<td>−1 A</td>
<td>1 A</td>
</tr>
<tr>
<td>(7)</td>
<td>2.5 A</td>
<td>0 A</td>
<td>2 A</td>
</tr>
<tr>
<td>(8)</td>
<td>−1 A</td>
<td>−1 A</td>
<td>1 A</td>
</tr>
</tbody>
</table>

\[ i(0^+) = \begin{cases} 
1 A & \text{(1)} \\
1 A & \text{(2)} \\
2.5 A & \text{(3)} \\
1 A & \text{(4)} \\
1 A & \text{(5)} \\
−1 A & \text{(6)} \\
2.5 A & \text{(7)} \\
−1 A & \text{(8)} 
\end{cases} \]

\[ i_C(0^+) = \begin{cases} 
2 A & \text{(1)} \\
1 A & \text{(2)} \\
0 A & \text{(3)} \\
2 A & \text{(4)} \\
0 A & \text{(5)} \\
−1 A & \text{(6)} \\
0 A & \text{(7)} \\
1 A & \text{(8)} 
\end{cases} \]

\[ i(\infty) = \begin{cases} 
1 A & \text{(1)} \\
1 A & \text{(2)} \\
1.4 A & \text{(3)} \\
2 A & \text{(4)} \\
1.4 A & \text{(5)} \\
1 A & \text{(6)} \\
2 A & \text{(7)} \\
1 A & \text{(8)} 
\end{cases} \]
3. Assume ideal op-amp. For \( v_{in} = 0.5 \) V, find the output voltage \( v_{out} \)

\[
\begin{align*}
V_{+} &= 0, \quad V_{-} = 0 \\
I_{1} &= \frac{V_{in}}{1k} = \frac{0.5}{1k} \\
I_{2} &= I_{1} \\
V_{out} &= V_{-} - I_{2} \cdot 5k \\
&= 0 - \frac{0.5}{1k} \cdot 5k = -2.5 V
\end{align*}
\]

(1) 5 V
(2) –5 V
(3) –2.5 V
(4) 2.5 V
(5) 3 V
(6) –3 V
(7) –3.5 V
(8) None of the above
4. Assume ideal op-amp. To obtain $v_{out} = -v_{in1} + v_{in2}$, the ratio $R/R_g$ must be:

\begin{align*}
(1) & \quad 1 \\
(2) & \quad 0.5 \\
(3) & \quad 2 \\
(4) & \quad 3/2 \\
(5) & \quad 2/3 \\
(6) & \quad 0 \\
(7) & \quad \text{None of the above}
\end{align*}

**Solution:**

Using the KCL at $V_-$:

\[ \frac{N_- - V_{in1}}{1k} + \frac{V_- - V_{out}}{1k} = 0 \]

\[ \frac{N_- - V_{in1}}{1k} + \frac{V_-}{R} + \frac{V_+ - V_{out}}{R_g} = 0 \]

\[ N_- = V_+ = V_{in2} \frac{R_g}{R + R_g} \]

\[ V_{out} = 2V_- - V_{in1} = \frac{2R_g}{R + R_g} \left( V_{in2} - V_{in1} \right) \]

\[ \Rightarrow \quad \frac{2R_g}{R + R_g} = 1 \quad \Rightarrow \quad R = R_g \]
5. Assume ideal op-amp. For \( v_{in} = 5 \text{ V} \), the Thevenin equivalent circuit looking into terminals a & b is:

(1) A 10 V source in series with a 2/3 kΩ resistor
(2) A 15 V source in series with a 1 kΩ resistor
(3) A -10 V source in series with a 2/3 kΩ resistor.
(4) A -15 V source in series with a 1 kΩ resistor
(5) A -5 V source in parallel with a 1 kΩ resistor
(6) A 15V source
(7) A 5 V source
(8) A 1 kΩ resistor

\[ V_+ = 5 \text{ V} \]
\[ V_- = 5 \text{ V} \]
\[ \frac{V_-}{3k} + \frac{V_- - V_a}{6k} = 0 \]
\[ V_a = 3V_- = 15 \text{ V} \]
\[ \Rightarrow V_{ab} = 15 \text{ V}, \text{ independently of the load.} \]
6. In the circuit below, find the phasor for the current $I_R$ through the $2\Omega$ resistor. ($\omega=10$ rad/s)

$$V_S(t) = 4 \cos(\omega t + \frac{2\pi}{3}) \ V$$

$$\vec{V}_S = 4 \angle \frac{2\pi}{3}$$

$$\frac{\vec{I}_R}{V_S} = \frac{\frac{4 \angle \frac{2\pi}{3}}{2}}{4} = \frac{2 \angle \frac{2\pi}{3}}{4}$$

$$= 2 \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)$$

$$= -1 + j\sqrt{3}$$

(1) $-1 + j\sqrt{3}$

(2) $-1 - j\sqrt{3}$

(3) $1 + j\sqrt{3}$

(4) $1 - j\sqrt{3}$

(5) $-\sqrt{3} + j$

(6) $-\sqrt{3} - j$

(7) $\sqrt{3} + j$

(8) $\sqrt{3} - j$
7. The following circuit operates at sinusoidal steady state with $\omega = 10 \text{ rad/sec}$. The values next to the independent sources are the phasor values. Find the phasor $V_0$ across the independent current source.

\[ V_0 = 6 + j6 \]

\[ V_0 = 6 - j6 \]

\[ V_0 = -2 + j6 \]

\[ V_0 = -2 - j6 \]

\[ V_0 = -1 + j7 \]

\[ V_0 = -1 - j7 \]

\[ V_0 = 5 + j \]

\[ V_0 = 5 - j \]

\[ 0.1 \text{H} \Rightarrow \quad j\omega L = j \times 10 \times 0.1 = j \Omega \]

\[ \mathcal{Z}_L = (3 + j4) \cdot (j) = -4 + 3j \]

By KVL

\[ 2 = j3 + V_0 + V_L \]

\[ V_0 = 2 - j3 - V_L \]

\[ = 2 - j3 - (-4 + 3j) \]

\[ = 6 - j6 \]
8. Find the phasor for the voltage $V_C$ (in Volts) across the 0.1F capacitor, assuming sinusoidal steady state.

\[ V_C = 50 \cos(t) \text{ V} \]

\[ 10\Omega \]

\[ 0.1F \] \((-j_{10}\pi)\]

\[ V_C = \frac{j}{j\omega C} = \frac{j}{j \times 1 \times 0.1} = -j_{10}\pi \]

\[ V_C = 50 \cdot \frac{-j_{10}}{10 + (-j_{10})} \]

\[ = \frac{-j_{50}}{1 - j} \cdot \frac{1 + j}{1 + j} \]

\[ = j_{25}(1 + j) \]

\[ = 2.5 - j_{25} \]

\[(1) \ 25+j25 \]

\[(2) \ 25-j25 \]

\[(3) \ -25+j25 \]

\[(4) \ -25-j25 \]

\[(5) \ 50+j50 \]

\[(6) \ 50-j50 \]

\[(7) \ -50+j50 \]

\[(8) \ -50-j50 \]
9. In the following RLC circuit, the switch, which was closed for a long time prior to \( t = 0 \), opens at \( t = 0 \). The capacitor voltage \( v_C(t) \) for \( t > 0 \) has been measured and found to be that shown in the figure below.

The capacitance of the capacitor is known to be \( C = 1 \text{ mF} \), and the inductance \( L \) is \( 1 \text{ mH} \). Find the best estimation of \( R \).

(1) \( R = 0.01 \Omega \)
(2) \( R = 0.2 \Omega \)
(3) \( R = 0.7 \Omega \)
(4) \( R = 2.0 \Omega \)
(5) \( R = 2.5 \Omega \)
(6) \( R = 5.0 \Omega \)
(7) \( R = 10 \Omega \)

After \( t > 0 \), this is a second-order RLC system.

\[
S^2 + \frac{R}{L}S + \frac{1}{LC} = 0
\]

\[
S = -\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} - \frac{4}{LC}}
\]

\[
S = -\frac{R}{2LC} \pm j\omega_L
\]

The response indicates an under-damped system.

Note that the envelope goes down as \( R > \frac{2L}{C} \).

\[
E = \frac{R}{2L} \times 6 \times 10^{-3} = \frac{3}{6}
\]

\[
\frac{R}{2L} \times 6 \times 10^{-3} = \frac{1}{L} \times 0.69 \approx 0.69
\]

\[
R = \frac{2L \times 0.69}{6 \times 10^{-3}} = \frac{2 \times 10^{-3} \times 0.69}{6 \times 10^{-3}} = 0.28 \Omega
\]
2) We may also try to match with \( \omega_d \).

From the figure, the period of the (decayed) sinusoid

\[ T = 6 \text{ ms} \]

\[ \omega_d = 2\pi \frac{T}{T} = \frac{2\pi}{1} = 1.04 \times 10^3 \text{ rad/s} \]

From the earlier formula

\[ \omega_d = \sqrt{\frac{1}{LC} - \frac{R^2}{4LC}} \]

If \( R = 0.01 \Omega \) ⇒

\[ \omega_d = \sqrt{\frac{1}{10^{-3} \times 10^{-3}} - \frac{0.01^2}{(4 \times 10^{-3})^2}} \approx 1 \times 10^3 \text{ rad/s} \]

If \( R = 0.2 \Omega \) ⇒

\[ \omega_d = \sqrt{\frac{1}{10^{-3} \times 10^{-3}} - \frac{0.2^2}{(4 \times 10^{-3})^2}} \approx 1 \times 10^3 \text{ rad/s} \]

If \( R = 0.7 \Omega \) ⇒

\[ \omega_d = \sqrt{\frac{1}{10^{-3} \times 10^{-3}} - \frac{(0.7)^2}{(4 \times 10^{-3})^2}} \approx 0.94 \times 10^3 \text{ rad/s} \]

All of the three are very close (cannot distinguish)

If \( R = 2 \Omega \) ⇒

\[ \omega_d = 0 \] (critically damped)

If \( R > 2 \Omega \) ⇒

over-damped.

In summary, cannot distinguish \( R \) accurately from \( \omega_d \).
PART B: 1 Work-Out Question

IMPORTANT: Only answers written on your workout sheet will be graded!!!

Clearly show intermediate steps in order to receive partial credits.

Make sure you write your name, division, professor, PUID on the work-out sheet.
1. Assume ideal op-amp. If $v_{in}(t) = \cos(\pi t) \text{ V}$, find the value of $v_{out}(t)$ at $t = 1/2 \text{ sec}$. (*Hint:* You don't have to use SSS analysis to solve this problem.)

**Alternate solution:** Assume SSS (when $v_{in}(t)$ is turned off).

\[
\frac{1}{0.001 \text{ F}} = \frac{1}{2 \times 0.001} = -\frac{j \omega_0}{C} \\
V_{in} = \cos(\pi t) \Rightarrow \text{phasor } V_{in} = 1
\]
\[ \vec{V}_{+} = 0 \Rightarrow \vec{V}_{-} = 0 \]

\[ \vec{I}_{C} = \frac{\vec{V}_{in} - \vec{0}}{-j\frac{1}{1000} \vec{2}} = \frac{\vec{2}}{1000} \]

\[ \vec{I}_{R} = \frac{\vec{V}_{in} - \vec{0}}{1500} = \frac{1}{1500} \]

\[ \vec{I}_{Z} = \vec{I}_{C} + \vec{I}_{R} = \frac{1}{1000} + j\frac{2}{1500} \]

\[ V_{out} = V_{-} - j\vec{I}_{R} \cdot 1k = -1 - j2 \]

\[ N_{out}(t) = -e^{-2t} + 2e^{-2t} \]

\[ N_{out}\left(\frac{1}{2}\right) = 2. \]