Solution to Problem 1.

Review. The free body diagram, shear force diagram, bending moment diagram, and torque diagram of the shaft are shown in Figures 1, 2, 3, and 4, respectively.

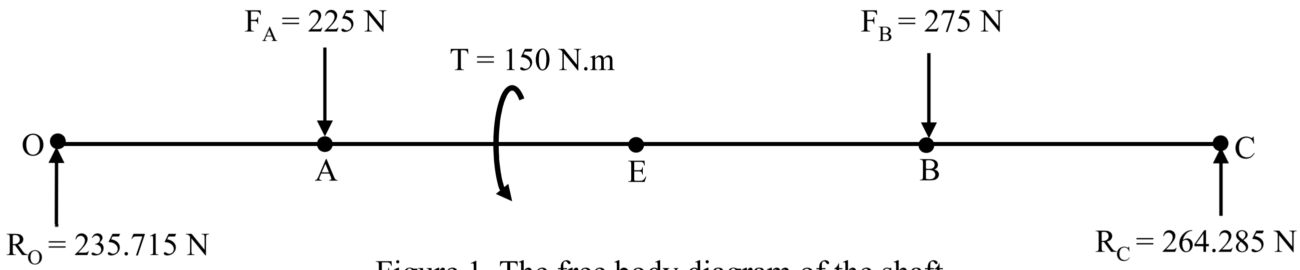

Figure 1. The free body diagram of the shaft.

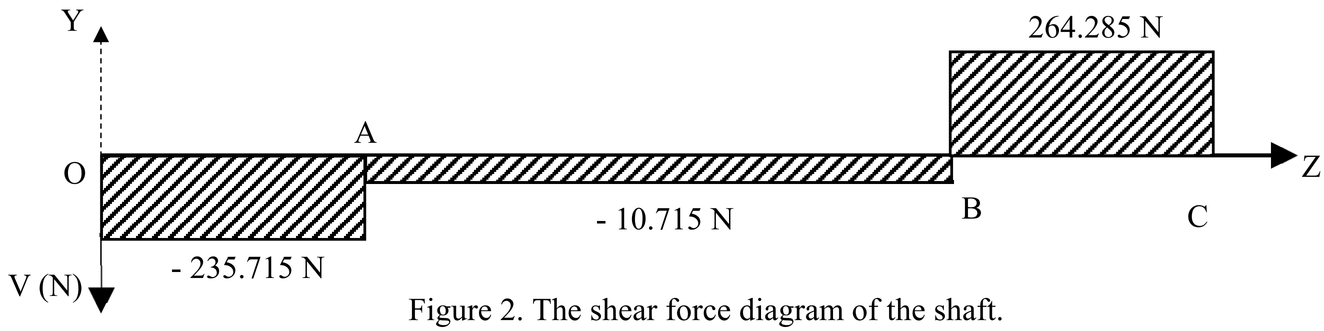

Figure 2. The shear force diagram of the shaft.

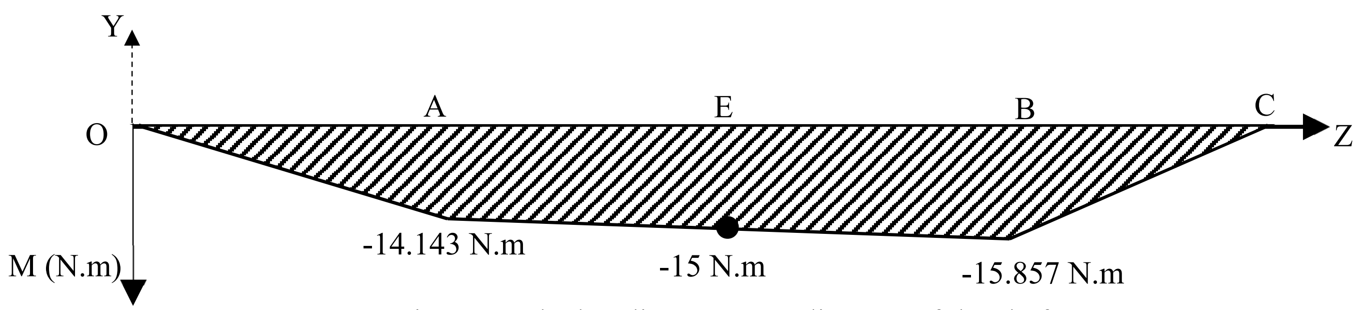

Figure 3. The bending moment diagram of the shaft.

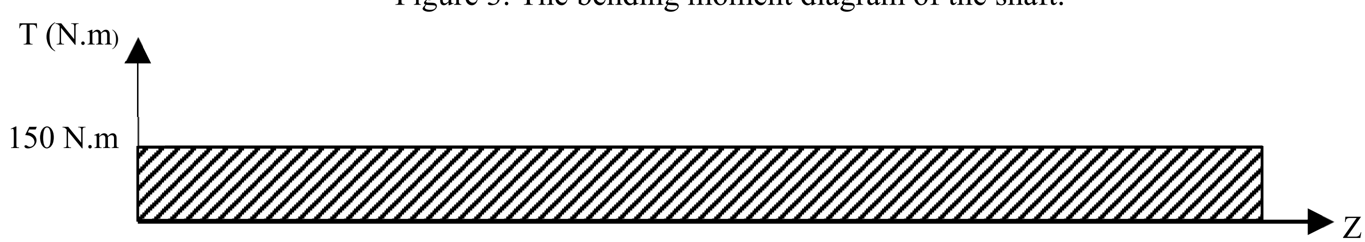

Figure 4. The torque diagram of the shaft.

From the sum of the moments about the bearing O and the sum of the forces in the Y-direction, the reaction forces at bearings C and O are

$$R_C = \frac{225 \text{ x } 60 + 275 \text{ x } 220}{280} = 264.285 \text{ N} \qquad \text{and} \qquad R_O = 500 - 264.285 = 235.715 \text{ N} \qquad (1)$$

(i) The shaft diameter at the groove can be written from the Goodman criterion, see Eq. (7.8), page 382, as

$$d = \left(\frac{16\ n_f}{\pi} \left(\frac{A}{S_e} + \frac{B}{S_{ut}} \right) \right)^{1/3} \tag{2}$$

The coefficients in Eq. (2) are given by Eq. (7.6), see page 381, namely

$$A = \sqrt{4\left(K_f M_a\right)^2 + 3\left(K_{fs} T_a\right)^2} \tag{3a}$$

and

$$B = \sqrt{4\left(K_f M_m\right)^2 + 3\left(K_{fs} T_m\right)^2} \tag{3b}$$

The rotating shaft is subjected to fully reversed bending, therefore, the mean component of the bending moment at the mid-point of the shaft, point E, is

$$M_m = 0 \tag{4}$$

The alternating component of the bending moment acting at the mid-point of the shaft, point E, can be written as

$$M_a = -\text{OE x } R_O + \text{AE x } F_A \tag{5a}$$

Substituting Eq. (1) into Eq. (5a), the alternating component of the bending moment is

$$M_a = \left| -140 \text{ x } 235.715 + 80 \text{ x } 225 \right| = \left| -15000 \text{ N.mm} \right| = 15 \text{ N.m} \tag{5b}$$

The mean and alternating components of the constant torque, see Figure 4, are

$$T_m = 150 \text{ N.m} \qquad \text{and} \qquad T_a = 0 \tag{6}$$

The fatigue stress concentration factors for the normal stress and the shear stress are

$$K_f = 2.25 \qquad \text{and} \qquad K_{fs} = 1.95 \tag{7}$$

Substituting Eqs. (4), (5), (6), and (7) into Eqs. (3), the coefficients are

$$A = \sqrt{4(2.25 \text{ x } 15)^2 + 3(1.95 \text{ x } 0)^2} = 67.5 \text{ N.m} \tag{8a}$$

and

$$B = \sqrt{4(2.25 \text{ x } 0)^2 + 3(1.95 \text{ x } 150)^2} = 506.625 \text{ N.m} \tag{8b}$$

Substituting $n_f = 3$, $S_e = 125$ MPa, $S_{ut} = 335$ MPa, and Eqs. (8) into Eq. (2), the diameter of the shaft at the groove can be written as

$$d = \left(\frac{16 \text{ x } 3}{\pi} \left(\frac{67.5}{125 \text{ x } 10^6} + \frac{506.625}{335 \text{ x } 10^6} \right) \right)^{1/3} \text{ m} \tag{9a}$$

that is, the diameter of the shaft at the groove can be written as

$$d = \left(\frac{16\,\mathrm{x}\,3}{\pi\,\mathrm{x}\,10^6}(0.54+1.512) \right)^{1/3} = \left(\frac{16\,\mathrm{x}\,3\,\mathrm{x}\,2.052}{\pi\,\mathrm{x}\,10^6} \right)^{1/3} \text{ m} \tag{9b}$$

Therefore, the diameter of the shaft at the groove, see Figure 1, is

$$d = 31.53 \text{ mm} \tag{10}$$

Check: The diameter of the shaft at the groove can also be written from Eq. (2) as

$$d = \left(\frac{16 n_f}{\pi} \left\{ \frac{1}{S_e} \left[4\left(K_f M_a\right)^2 + 3\left(K_{fs} T_a\right)^2 \right]^{1/2} + \frac{1}{S_{ut}} \left[4\left(K_f M_m\right)^2 + 3\left(K_{fs} T_m\right)^2 \right]^{1/2} \right\} \right)^{1/3} \tag{11}$$

Substituting the infinite life fatigue factor of safety, the endurance limit, the ultimate tensile strength, and Eqs. (4), (5), and (6) into Eq. (11) the diameter of the shaft at the groove is

$$d = 31.53 \text{ mm} \tag{12}$$

Note that this answer is in complete agreement with Eq. (10).
(ii) The von Mises mean stress for critical element at E can be written from Eq. (7.4) see page 381, as

$$\sigma'_{\mathrm{m}} = \left[\sigma_m^{\ 2} + 3\tau_m^{\ 2} \right]^{1/2} \tag{13a}$$

and the von Mises alternating stress can be written from Eq. (7.5) see page 381, as

$$\sigma'_a = \left[\sigma_a^{\ 2} + 3\tau_a^{\ 2} \right]^{1/2} \tag{13b}$$

The mean component of the normal stress and the alternating component of the shear stress, respectively, are

$$\sigma_m = 0 \qquad \text{and} \qquad \tau_a = 0 \tag{14a}$$

The alternating component of the normal stress is

$$\sigma_a = \frac{32\ \mathrm{K_f} M_a}{\pi d^3} = \frac{32(2.25)(15000)}{\pi(31.53)^3} = 10.967 \text{ MPa} \tag{14b}$$

The mean component of the shear stress is

$$\tau_m = \frac{16\ K_{\mathrm{fs}} T_m}{\pi d^3} = \frac{16(1.95)(150000)}{\pi(31.53)^3} = 47.525 \text{ MPa} \tag{14c}$$

Substituting Eqs. (14a) and (14c) into Eq. (13a), the von Mises mean stress for the critical element at E is

$$\sigma'_{\mathrm{m}} = \left[0 + 3\,\mathrm{x}\,47.525^2 \right]^{1/2} = 82.316 \text{ MPa} \tag{15a}$$

Substituting Eqs. (14a) and (14b) into Eq. (13b), the von Mises alternating stress at E is

$$\sigma'_a = \left[10.967^2 + 0\right]^{1/2} = 10.967 \text{ MPa} \tag{15b}$$

The maximum von Mises stress at E can be written from Eq. (7.15), see page 382, as

$$\sigma'_{\max} = \left[\sigma^2_{\max} + 3\,\tau^2_{\max}\right]^{1/2} = \left[(\sigma_m + \sigma_a)^2 + 3(\tau_m + \tau_a)^2\right]^{1/2} \tag{16a}$$

or as

$$\sigma'_{\max} = \left[\left(\frac{32K_f(M_m + M_a)}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs}(T_m + T_a)}{\pi d^3}\right)^2\right]^{1/2} \tag{16b}$$

Substituting Eqs. (4) and (5) into Eq. (16b), the maximum von Mises stress at E is

$$\sigma'_{\max} = \left[10.967^2 + 3(47.525)^2\right]^{1/2} = 83.043 \text{ MPa} \tag{17}$$

(iii) The factor of safety guarding against first-cycle yielding for the critical element at E using the Langer line, see Eq. (7.16), page 382, can be written as

$$n_y = \frac{S_{yt}}{\sigma'_{\max}} \qquad \text{or as} \qquad n_y = \frac{S_{yc}}{\sigma'_{\max}} \tag{18}$$

Since the tensile yield strength is less than the compressive yield strength then the static factor of safety for the critical element guarding against yielding is given by the first equation of Eq. (18).

Substituting the tensile yield strength $S_{yt} = 190$ MPa and Eq. (17) into Eq. (18), the factor of safety guarding against first-cycle yielding for the critical element at E is

$$n_y = \frac{190}{83.043} = 2.28 \tag{19}$$

Using the conservative approach, see page 383. The factor of safety guarding against first-cycle yielding for the critical element at E can be written as

$$n_y = \frac{S_{yt}}{\sigma'_m + \sigma'_a} \tag{20}$$

Substituting the tensile yield strength and Eqs. (15) into Eq. (20), the factor of safety guarding against first-cycle yielding for the critical element at E is

$$n_y = \frac{190}{82.316 + 10.967} = \frac{190}{93.283} = 2.03 \tag{21}$$

Note that this answer given is, indeed, more conservative than the answer given by Eq. (19).

Solution to Problem 2.
(i) For the given position, the cam and the follower can be modeled as two cylinders. Therefore, the half-width of the contact patch can be written from Eq. (3-73), see page 148, as

$$b = \sqrt{\left(\frac{2F}{\pi l}\right)\left(\frac{\frac{(1-\nu_1^2)}{E_1} + \frac{(1-\nu_2^2)}{E_2}}{\frac{1}{d_1} + \frac{1}{d_2}}\right)} \tag{1}$$

The subscript 1 is used here to denote the cam and the subscript 2 is used to denote the follower. Since the follower is flat-faced then the diameter of the follower is infinite, that is

$$d_2 = \infty \tag{2}$$

The elastic material properties of the carbon steel cam, see Table A-5, page 1023, are

$$E_1 = 207 \text{ GPa} \qquad \text{and} \qquad \nu_1 = 0.292 \tag{3a}$$

The elastic material properties of the titanium alloy follower, see Table A-5, page 1023, are

$$E_2 = 114 \text{ GPa} \qquad \text{and} \qquad \nu_2 = 0.340 \tag{3b}$$

Substituting and the given geometry, the force F = 8000 N, and Eqs. (2) and (3) into Eq. (1), the half-width of the contact patch can be written as

$$b = \left(\left(\frac{2\times 8000}{\pi\times 40}\right)\left(\frac{(1-0.292^2)/(207\times 10^3)+(1-0.340^2)/(114\times 10^3)}{1/100+1/\infty}\right)\right)^{1/2} \text{ mm} \tag{4a}$$

Therefore, the half-width of the contact patch is

$$b = 0.394 \text{ mm} \tag{4b}$$

(ii) The maximum pressure on the contact patch can be written from Eq. (3-74), see page 148, as

$$p_{\max} = \left(\frac{2}{\pi}\right)\left(\frac{F}{bl}\right) \tag{5a}$$

Substituting the given geometry, the force $F = 8000$ N, and Eq. (4b) into Eq. (5a), the maximum pressure acting on the contact patch is

$$p_{\max} = \frac{2\times 8000}{\pi\times 0.394\times 40} = 323.157 \text{ MPa} \tag{5b}$$

(iii) The x, y, and z components of the normal stress acting on the element of the contact patch on the Z-axis can be written from Eqs. (3-75), (3-76), and (3-77), see page 148, as

$$\sigma_x = -2\nu p_{\max}\left\{\sqrt{1+\left(\frac{z}{b}\right)^2} - \left|\frac{z}{b}\right|\right\} \tag{6a}$$

$$\sigma_y = -p_{\max}\left\{\left[\frac{1+2\left(\frac{z}{b}\right)^2}{\sqrt{1+\left(\frac{z}{b}\right)^2}}\right] - 2\left|\frac{z}{b}\right|\right\} \tag{6b}$$

and

$$\sigma_Z = -\frac{p_{\max}}{\sqrt{1+\left(\frac{z}{b}\right)^2}} \tag{6c}$$

(a) Substituting $Z = 0$, $b = 0.394$ mm, Poisson's ratio for the cam, that is, and $\nu = \nu_1 = 0.292$, and Eq. (5b), into Eqs. (6a), the x component of the normal stress on element O of the cam on is

$$\sigma_x = -2\times0.292\times323.157(1-0) = -188.724\,\text{MPa} \tag{7}$$

Substituting $Z = 0$, $b = 0.394$ mm, and Eq. (5b), into Eqs. (6b) and (6c), the y and z components of the normal stress acting on element O of the cam are

$$\sigma_y = -323.157\left\{\left[\frac{1+2(0)^2}{\sqrt{1+(0)^2}}\right] - 2(0)\right\} = -323.157\text{ MPa} \tag{8a}$$

and

$$\sigma_z = -\frac{323.157}{\sqrt{1+(0)^2}} = -323.157\,\text{MPa} \tag{8b}$$

The principal normal stresses acting on element O of the cam, written in ordered form, are

$$\sigma_1 \geq \sigma_2 \geq \sigma_3 \tag{9}$$

Therefore, from Eqs. (7) and (9), the maximum principal normal stress acting on element O of the cam is

$$\sigma_1 = \sigma_x = -188.724\,\text{MPa} \tag{10a}$$

and the minimum principal normal stresses acting on element O of the cam is

$$\sigma_2 = \sigma_3 = \sigma_y = \sigma_z = -323.157 \text{ MPa} \tag{10b}$$

Check. From Eqs. (5b), (10a), and (10b), the ratios of the principal normal stresses and the maximum pressure are

$$\frac{\sigma_1}{p_{\max}} = -0.584 \qquad \text{and} \qquad \frac{\sigma_2}{p_{\max}} = \frac{\sigma_3}{p_{\max}} = -1 \tag{11}$$

The maximum shear stress acting on the element O of the cam on the Z-axis, see Eq. (3.72), page 147, can be written as

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} \tag{12a}$$

Substituting Eqs. (10a) and (10b) into Eq. (12a), the maximum shear stress acting on element O of the cam on the Z-axis is

$$\tau_{max} = \frac{-188.724 - (-323.157)}{2} = 67.217 \text{ MPa} \tag{12b}$$

(b) Substituting $Z = 0$, $b = 0.394$ mm, $\nu = \nu_2 = 0.340$, and Eq. (5b) into Eq. (6a), the x component of the normal stress acting on element O of the follower is

$$\sigma_x = -2 \times 0.340 \times 323.157\,(1-0) = -219.747 \text{ MPa} \tag{13a}$$

Substituting $Z = 0$, $b = 0.394$ mm, and Eq. (5b) into Eqs. (6b) and (6c), the y and z components of the normal stress acting on element O of the follower are the same as Eqs. (8a) and (8b), that is

$$\sigma_y = -323.157 \text{ MPa} \qquad \text{and} \qquad \sigma_z = -323.157 \text{ MPa} \tag{13b}$$

Therefore, the maximum principal normal stresses acting on element O of the follower is

$$\sigma_1 = \sigma_x = -219.747 \text{ MPa} \tag{14a}$$

and the minimum principal normal stresses acting on element O of the follower on the Z-axis are

$$\sigma_2 = \sigma_3 = \sigma_y = \sigma_z = -323.157 \text{ MPa} \tag{14b}$$

Check: Substituting Eqs. (14a) and (14b) into Eq. (12a), the maximum shear stress acting on element O of the follower is

$$\tau_{max} = \frac{-219.747 - (-323.157)}{2} = 51.705 \text{ MPa} \tag{15}$$

Equations (12b) and (15) show that the maximum shear stress at the surface of the cam (at point O) is greater than the maximum shear stress at the surface of the follower (at point O). The reason is that Poisson's ratio for the cam material is less than Poisson's ratio for the follower material.

(iv) Since the depth $\mathrm{OB} = 0.25\,\mathrm{mm}$ does not correspond to the location of the maximum shear stress, that is, $Z = 0.786\,\mathrm{b} = 0.310\,\mathrm{mm}$, see page 148, then Figure 3-40, see page 149, cannot be used in the solution to this part of the problem.

Substituting $Z = \mathrm{OB} = 0.25\,\mathrm{mm}$, $b = 0.394$ mm, and $\nu = \nu_1 = 0.292$ into Eq. (7a), the X-component of the normal stress on element B is

$$\sigma_x = -2\times 0.292\times 323.157\left\{\sqrt{1+\left(\frac{0.25}{0.394}\right)^2} - \left|\frac{0.25}{0.394}\right|\right\} = -103.761\,\mathrm{MPa} \qquad (16a)$$

Substituting $Z = 0.25\,\mathrm{mm}$ into Eq. (7b), the Y-component of the normal stress on element B is

$$\sigma_y = -\,323.157\left\{\left[\frac{1+2\left(\frac{0.25}{0.394}\right)^2}{\sqrt{1+\left(\frac{0.25}{0.394}\right)^2}}\right] - 2\left|\frac{0.25}{0.394}\right|\right\} = -\,82.482\ \mathrm{MPa} \qquad (16b)$$

Substituting $Z = 0.25\,\mathrm{mm}$ into Eq. (7c), the Z-component of the normal stress on element B is

$$\sigma_z = -\frac{323.157}{\sqrt{1+\left(\frac{0.25}{0.394}\right)^2}} = -272.863\,\mathrm{MPa} \qquad (16c)$$

Again, the principal normal stresses on element B are written in ordered form, see Eq. (9), that is, $\sigma_1 \geq\ \sigma_2 \geq\ \sigma_3$. Therefore, the minimum principal normal stress on element B is

$$\sigma_3 = \sigma_z = -272.863\,\mathrm{MPa} \qquad (17a)$$

the maximum principal normal stress on element B is

$$\sigma_1 = \sigma_y = -\,82.482\,\mathrm{MPa} \qquad (17b)$$

and the third principal normal stress acting on element B on the Z-axis is

$$\sigma_2 = \sigma_x = -103.760\,\mathrm{MPa} \qquad (17c)$$

Substituting Eqs. (17a) and (17b) into Eq. (12a), the maximum shear stress acting on element B is

$$\tau_{max} = \frac{-\,82.482-(-272.863)}{2} = 95.191\,\mathrm{MPa} \qquad (18)$$

Comparing Eq. (18) with Eq. (12b), the observation is that the maximum shear stress acting on element B of the cam is greater than the maximum shear stress acting on element O of the cam. This agrees with the Hertzian theory of contact stress.

Solution to Problem 3.

(i) The Sommerfeld number from Eq. (12-10), see page 632, can be written as

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} \tag{1}$$

The absolute viscosity of the SAE 50 lubricant at an operating temperature of 100°F, see Figure 12-2, page 628, is

$$\mu = 40\ \mu\text{reyn} \tag{2}$$

The nominal bearing pressure (that is, the load acting on the projected area of the bearing), see Eq. (12.7), page 630, can be written as

$$P = \frac{W}{2\,r\,l} \tag{3a}$$

Substituting the load $W = 400\,\text{lbs}$, the radius of the journal $r = 2\,\text{in}$, and the length of the bearing $l = 5\,\text{in}$ into Eq. (3a), the nominal bearing pressure is

$$P = \frac{400\ \text{lb}}{2 \times 2\ \text{in} \times 5\ \text{in}} = 20\,\text{psi} \tag{3b}$$

The speed of the journal specified in the problem statement is 1200 rpm, that is

$$N = \frac{1200}{60} = 20\ \text{rev}/\text{s} \tag{4}$$

Substituting $r = 2\,\text{in}$, the radial clearance $c = 0.02$ in, and Eqs. (2), (3), and (4), into Eq. (1), the Sommerfeld number can be written as

$$S = \left[\frac{2\,\text{in}}{0.02\ \text{in}}\right]^2 \left[\frac{(40\times 10^{-6}\ \text{reyn}) \times (20\,\text{rev/s})}{20\,\text{psi}}\right] \tag{5a}$$

Therefore, the Sommerfeld number is

$$S = 0.400 \tag{5b}$$

(ii) The friction torque acting on the journal can be written from Eq. (12.6), see page 629, as

$$T = \frac{4\pi^2\, r^3\, l\, \mu\, N}{c} \tag{6a}$$

Substituting the given geometry and Eqs. (2) and (4) into Eq. (6a), the friction torque acting on the journal is

$$T = \frac{4\pi^2 \text{ x } 2^3 \text{x } 5 \text{ x } 40 \text{ x} 10^{-6} \text{ x } 20}{0.02} = 63.165\ \text{lbs.in} \tag{6b}$$

Check: The friction torque acting on the journal can be written from Eq. (12.8), see page 630, as

$$T = 2\,r^2 f\,l\,P = f\,W\,r \tag{7}$$

The coefficient of friction variable specified in the problem statement (based on Figure 12.17, see page 642) is

$$f\,\frac{r}{c} = 7.7 \tag{8a}$$

Substituting $r = 2\,\text{in}$ and $c = 0.02$ in into Eq. (8a), and rearranging, the coefficient of friction is

$$f = 7.7 \times \frac{0.02}{2} = 0.077 \tag{8b}$$

Check: Substituting $r = 2\,\text{in}$, $W = 400\,\text{lbs}$, and Eq. (8b) into Eq. (7), the friction torque acting on the journal is

$$T = 0.077 \times 400 \times 2 = 61.60 \text{ lbs.ins} \tag{9a}$$

This answer is in good agreement with Eq. (6b). Also, the coefficient of friction can be written from Eq. (12.9), see page 631, as

$$f = 2\,\pi^2 (\frac{\mu\,N}{P})(\frac{r}{c}) \tag{9b}$$

Substituting $r = 2\,\text{in}$, $c = 0.02$ in, and Eqs. (2), (3b), and (4) into Eq. (9b), the coefficient of friction is

$$f = 2\,\pi^2 (\frac{40 \times 10^{-6} \times 20}{20})(\frac{2}{0.02}) = 0.079 \tag{9c}$$

Note this answer is in good agreement with Eq. (8b). However, this answer is based on Petroff's equation and does not agree with the given problem statement.

(iii) The thermal energy loss at steady state (that is, the heat rate) can be written from page 648 as

$$H_{\text{Loss}} = \frac{2\,\pi\,N\,T}{J} \tag{10a}$$

where for common petroleum lubricants, the Joulean heat equivalent, see page 648, is

$$J = 778 \text{ ft.lb / Btu} = 9336 \text{ in.lb / Btu} \tag{10b}$$

Substituting Eqs. (4), (6b), and (10b) into Eq. (10a), the thermal energy loss at steady state is

$$H_{\text{Loss}} = \frac{2\,\pi \times 20 \times 63.165}{9336} = 0.850 \text{ Btu} \tag{11a}$$

Substituting Eqs. (4), (9a), and (10b) into Eq. (10a), the thermal energy loss at steady state is

$$H_{\text{Loss}} = \frac{2\,\pi \times 20 \times 61.60}{9336} = 0.829 \text{ Btu} \tag{11b}$$

Check: The thermal energy loss at steady state can also be written from Eq. (b) on page 648 as

$$H_{\text{Loss}} = \left(\frac{4\pi P r l N c}{J}\right)\left(\frac{f r}{c}\right) \tag{12a}$$

Substituting Eqs. (3b), (4), (9c), and (10b) into Eq. (12a), the thermal energy loss at steady state is

$$H_{\text{Loss}} = \frac{4\pi\times 20\times 2\times 5\times 20\times 0.02}{9336}(\frac{0.0079\times 2}{0.02}) = 0.851 \text{ Btu} \tag{12b}$$

Substituting Eqs. (3b), (4), and (10b) and the flow variable specified in the problem statement into Eq. (12a), the thermal energy loss at steady state is

$$H_{\text{Loss}} = \frac{4\pi\times 20\times 2\times 5\times 20\times 0.02}{9336}(7.7) = 0.8295 \text{ Btu} \tag{12c}$$

Note that Eq. (12b) is in good agreement with Eq. (11a) and Eq. (12c) is in good agreement with Eq. (11b).

(iv) The thermal energy loss at steady state can also be written in terms of the temperature rise and the flow parameters from Eq. (a), see page 647, as

$$H_{loss} = \rho C_p \, \text{Q} \Delta T\left(1 - 0.5\frac{Q_s}{Q}\right) \tag{13a}$$

Rearranging Eq. (13a), the flow ratio can be written as

$$\frac{Q_s}{Q} = 2\left(1 - \frac{H_{loss}}{\rho C_p \, \text{Q} \, \Delta T}\right) \tag{13b}$$

where the density and the specific heat capacity of common petroleum lubricants are specified on page 648, are

$$\rho = 0.0311 \text{ lbm / in}^3 \qquad \text{and} \qquad C_p = 0.42 \text{ Btu / lbm }^\circ\text{F} \tag{14}$$

The flow variable, see Figure 12.18, page 643, is specified in the problem statement as

$$\frac{Q}{r c N l} = 3.6 \tag{15a}$$

Substituting $r = 2\,\text{in}$, $c = 0.02$ in, $l = 5$ in, and Eq. (4) into Eq. (15a), and rearranging, the flow rate (that is, the volumetric oil flow) is

$$Q = 3.6\,(2\times 0.02\times 20\times 5) = 14.4\,\text{in}^3/\text{s} \tag{15b}$$

Substituting the specified temperature rise $\Delta T = 5°\text{F}$ and Eqs. (11), (12b), and (15b) into Eq. (13b), the flow ratio can be written as

$$\frac{Q_s}{Q} = 2\left(1 - \frac{0.829\,\text{Btu}}{\left(0.0311\,\text{lbm/in}^3\right)\left(0.42 \text{ Btu / lbm}\,°\text{F}\right)\left(14.4 \text{ in}^3\text{ / s}\right)\left(5\,°\text{F}\right)}\right) \tag{16a}$$

Therefore, the flow ratio is

$$\frac{Q_s}{Q} = 0.24 \tag{16b}$$

Check: The flow ratio can also be obtained from Eq. (12-19), see page 648, that is

$$\frac{9.70\,\Delta T_F}{P} = \frac{f\,(r/c)}{\left(1-0.5Q_s/Q\right)\left(Q/r\,c\,N\,l\right)} \tag{17a}$$

Rearranging this equation, the flow ratio can be written as

$$\frac{Q_s}{Q} = 2\left(1 - \frac{f\,(r/c)\,P}{9.70\;\Delta T_F\left(Q/r\,c\,N\,l\right)}\right) \tag{17b}$$

Substituting the temperature rise $\Delta T = 5°\mathrm{F}$, and Eqs. (1b), (7a), and (14), into Eq. (17b), the flow ratio can be written as

$$\frac{Q_s}{Q} = 2\left(1 - \frac{(7.7)(20)}{9.70(5)(3.6)}\right) \tag{18a}$$

Therefore, the flow ratio is

$$\frac{Q_s}{Q} = 0.24 \tag{18b}$$

Note that Eq. (18b) is in complete agreement with Eq. (16b).

Solution to Problem 4.

(i) The transmitted load can be written from Eq. (13-35), see page 712, as

$$W_t = 33000\,\frac{H}{V_t} \tag{1}$$

The pitch line velocity can be written from Eq. (13-34), see page 711, as

$$V_t = \frac{\pi\, d_G\, n_G}{12}\ \text{ft/min} \tag{2a}$$

Substituting the pitch circle diameter of the gear $d_G = 5$ in and the speed of the gear $n_G = 500$ rpm into Eq. (2a), the pitch line velocity is

$$V_t = \frac{\pi\times 5\times 500}{12} = 654.5\ \text{ft/ min} \tag{2b}$$

Substituting the horsepower $H = 5\,\text{hp}$ and Eq. (2b) into Eq. (1), the transmitted load is

$$W_t = \frac{33000\times 5}{654.5} = 252.1\,\text{lb} \tag{3}$$

The radial component of the load can be written from Figure 13-30, see page 713, as

$$W_r = W_t\ \tan\phi \tag{4a}$$

Substituting Eq. (3) and pressure angle $\phi = 20^0$ into Eq. (4a), the radial component of the load is

$$W_r = 252.1\times\tan 20^0 = 91.76\ \text{lb} \tag{4b}$$

(ii) The AGMA normal stress due to bending of the pinion teeth can be written in US customary units from Eq. (14-15), see page 751, as

$$\sigma = \frac{W_t\,P}{F\,J}(K_o\,K_v\,K_s\,K_m\,K_B) \tag{5}$$

To determine the bending strength geometry factor J for the pinion. The diametral pitch can be written from Eq. (13-1), see page 684, as

$$P = \frac{N}{d} \tag{6}$$

Substituting the diametral pitch $P = 8\,\text{inches}^{-1}$ and the pitch circle diameter of the gear $d_G = 5$ in into Eq. (6), and rearranging, the number of teeth on the gear is

$$N_G = P\,d_G = 8\times 5 = 40 \tag{7a}$$

Therefore, the number of teeth on the pinion is

$$N_p = P\,d_p = 8 \times 3 = 24 \tag{7b}$$

Since $N_p = 24$ and the loads are applied at the tip of the teeth then the bending strength geometry factor for the pinion teeth from Figure 14-6, see page 759, is

$$J = 0.25 \tag{8}$$

To determine the overload factor. For light shock input and moderate shock output the overload factor from the table on page 772, see Figure 14-17, is

$$K_o = 1.5 \tag{9a}$$

The dynamic factor for the pinion is given in the problem statement as

$$K_v = 1.25 \tag{9b}$$

The size factor for the pinion, see Eq. (a), page 765, is given in the problem statement as

$$K_s = 1.0 \tag{9c}$$

The load distribution factor for the pinion, see Eq. (14-30), page 765, is given in the problem statement as

$$K_m = 1.0 \tag{9d}$$

Since the pinion is solid then the rim thickness factor for the pinion, see Section 14-16, page 770, is

$$K_B = 1.0 \tag{9e}$$

Substituting the face width $F = 1.25\,\text{inches}$, the diametral pitch $P = 8\,\text{inches}^{-1}$, and Eqs. (3), (8), and (9) into Eq. (5), the AGMA normal stress due to bending of the pinion teeth can be written as

$$\sigma = \frac{252.1 \times 8}{1.25 \times 0.25} \times (1.5 \times 1.25 \times 1.0 \times 1.0 \times 1.0)\ \text{kpsi} \tag{10a}$$

Therefore, the AGMA normal stress due to bending of the pinion teeth is

$$\sigma = 12.1\ \text{kpsi} \tag{10b}$$

(iii) The AGMA bending factor of safety for the pinion can be written from Eq. (14-41), see page 771, as

$$S_F = \frac{S_{fb}}{\sigma} \tag{11}$$

where the fully corrected bending fatigue strength can be written as

$$S_{fb} = \left(\frac{Y_N}{K_T\, K_R} \right) \mathrm{S_t} \tag{12}$$

For commercial quality carburized and hardened Grade 1 steel gears, the repeatedly applied bending strength stress-cycle factor, for number of load cycles greater than 10^7, from Figure 14-14, see page 769, can be written as

$$Y_N = 1.3558 N^{-0.0178} \tag{13a}$$

Substituting the number of load cycles $N = 10^9$ cycles into Eq. (13a), the repeatedly applied bending strength stress-cycle factor is

$$Y_N = 1.3558(10^9)^{-0.0178} = 0.937 \tag{13b}$$

Note that the larger the value of Y_N then the larger the bending factor of safety for the pinion. This is consistent with using commercial quality gears. If precision gears are used then a lower factor of safety would be acceptable.

The gearset is operating at room temperature, therefore, the temperature factor from page 770, is

$$K_T = 1 \tag{14}$$

The reliability factor, for $R = 0.995$, can be written from the least-squares regression fit, see Eq. (14-38), page 769, as

$$K_R = 0.50 - 0.109 \ln(1 - R) \tag{15a}$$

Substituting the specified reliability $R = 0.995$ into Eq. (15a), the reliability factor is

$$K_R = 0.50 - 0.109 \ln(1 - 0.995) = 1.077 \tag{15b}$$

Substituting the repeatedly applied bending strength $S_t = 55$ kpsi and Eqs. (13b), (14), and (15b) into Eq. (12), the fully corrected bending strength is

$$S_{fb} = \left(\frac{0.937}{1.0 \times 1.077}\right) \text{x}\, 55 = 47.85 \text{ kpsi} \tag{16}$$

Substituting Eqs. (10b) and (16) into Eq. (12), the AGMA bending factor of safety for the pinion is

$$S_F = \frac{47.85}{12.1} = 3.95 \tag{17}$$