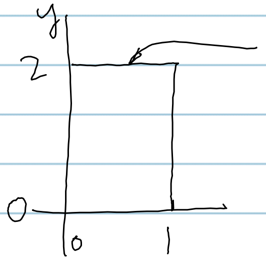
Exam1 Solution

1)

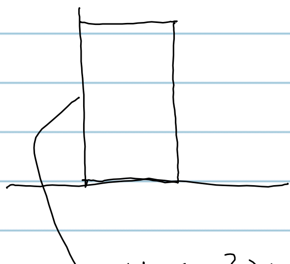


$u = 3x$

$v = -3y = -6$

$Q = v \cdot \Delta x = \boxed{-6}$

2)



$u = 3x = 0 \Rightarrow \boxed{Q = 0}$

3) oil floats on water

density of water ~ 1000 kg/m³

$\Rightarrow \boxed{1000}$

4) $u = 3y + 6x$

$v = -3x - 6y$

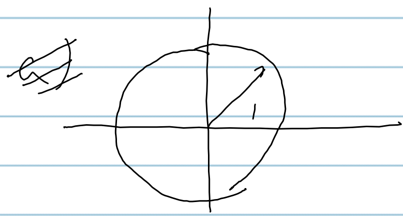
$$\xi_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = -3 - 3 = \boxed{-6}$$

(5) $\frac{\partial u}{\partial y} \sim \frac{\text{vel.}}{\text{length}} = \frac{\text{length}}{\text{length} \cdot \text{time}} = \frac{1}{\text{time}}$

$\Rightarrow \boxed{\frac{1}{s}}$

(6) $u = 3y + 6x$

$v = -3x - 6y$



$\nabla \cdot \vec{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$

$= 6 - 6$

$= 0$

$\Rightarrow \boxed{Q = 0}$

(7) $T = 300 + 4(x^2 + y^2)$

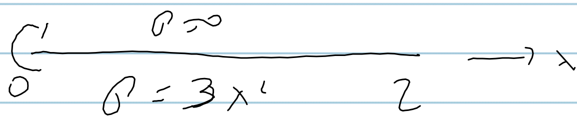
$u = 3y$

$v = -3x$

$$\frac{DT}{Dt} = \underbrace{\frac{\partial T}{\partial t}}_{\to 0} + u \underbrace{\frac{\partial T}{\partial x}}_{8x} + v \underbrace{\frac{\partial T}{\partial y}}_{8y} = 24xy - 24xy$$

$= \boxed{0}$

(8)



$$M_0 = \int_0^2 x \cdot \Delta p \, dx$$

$$\Delta p = p_{top} - p_{bottom}$$

$$= -3x^2$$

$$M_0 = -3 \int_0^2 x^3 \, dx = -\frac{3}{4} \cdot 2^4$$

$$= \boxed{-12}$$

(9)

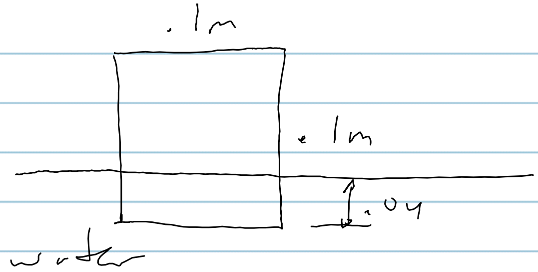
$$\vec{F} = -\int_0^2 \Delta p \, dx \; \hat{e}_y$$

$$= \int_0^2 3x^2 \, dx \; \hat{e}_y$$

$$= \frac{3}{3} x^3 \Big|_0^2 \; \hat{e}_y$$

$$= \boxed{8 \, \hat{e}_y}$$

(10)



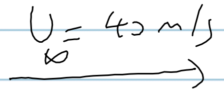
Mass cube = Mass of displaced water

$= \rho_{water} \cdot (.1)^2 (.04)$

$= 1000 \cdot .01 \cdot (.04)$

$= \boxed{.4 \text{ Kg}}$

(11)



$p - p_\infty$ = gage pressure

$= \frac{1}{2}\rho U_\infty^2 - \frac{1}{2}\rho |\vec{u}|^2$

max $\frac{1}{2}\rho U_\infty^2$

$= \frac{1}{2} \cdot 1 \cdot 40^2 = \boxed{800 \text{ Pa}}$

12) $D = 4\ cm = .04\ m$

$Q = 500\ L/m$

$= 500 \cdot 1000\ \frac{cc}{m}$

$A = \pi D^2/4 = 4\pi\ cm^2$

$$\bar{u} = \frac{Q}{A} = \frac{.5 \cdot 10^6}{4\pi} \underbrace{\frac{cm^3}{min} \cdot \frac{1}{cm^2}}_{\frac{cm}{min}}$$

$$= \frac{.5 \cdot 10^6}{4\pi \cdot 60}$$

~ 1000