AAE 333 **Fluid Mechanics** **Fall 2004**

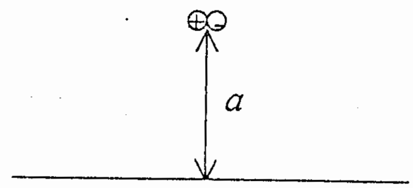
Exam 2 (12/2/04)
2hrs, open book/notes

1. Find the formula for the stream function for a flow with doublet strength k at a distance a from the wall, as shown in the figure.

a. Sketch the streamlines (10 points)

b. Are there any stagnation points? If yes, then find them (10 points)

c. Find the maximum velocity at the wall and its position (10 points)

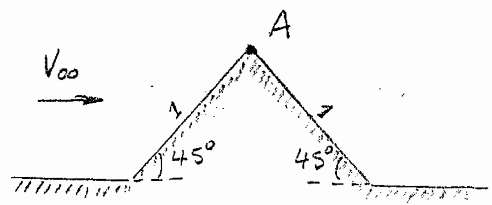


2. We want to estimate the flow around the sketched two-dimensional shape.

a) Set up a crude panel method for that flow. Each panel should be represented a concentrated source at the mid-point, except the target panel. Take advantage of the symmetry and do something for the ground. What is the minimum number of unknowns needed? (15 points)

b) Evaluate the unknown source strengths. (10 points)

c) Evaluate velocity and pressure coefficient at point A. (5 points)



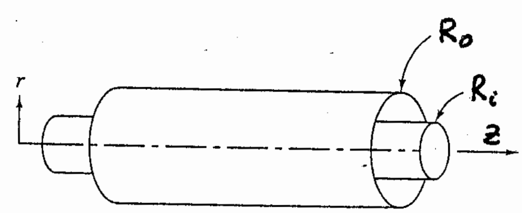
3. Consider the fully developed laminar incompressible flow between 2 concentric stationary cylinders. Assume a constant pressure gradient dp/dz. The outer cylinder has a radius R_o=R and the inner cylinder has a radius R_i=kR (k<1)

a. Find the velocity field u(r) as a function of dp/dz, R, k, and μ (10 points)

b. Obtain an expression for the location of the maximum velocity (5 points)

c. Evaluate the volume flow rate as a function of dp/dz, R, k and μ (7 points)

d. Find the limits for k-> 0 and compare with the circular pipe results (8 points)

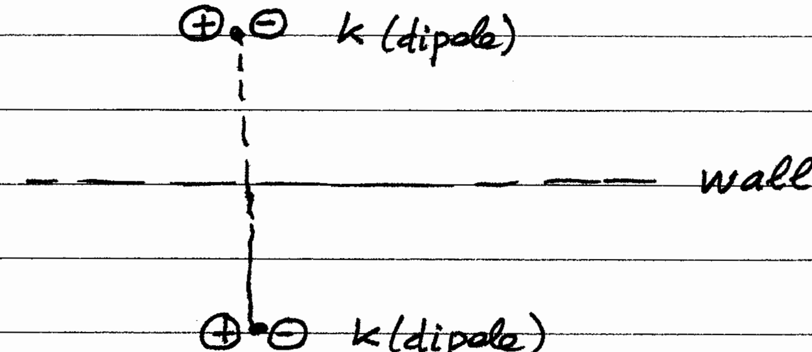


4. Please, indicate whether the following statements are true or false: (2 points each)

a. Panel codes can produce a good estimate for the flow outside the boundary layer.
b. The boundary layer approximation can not be used at low Reynolds numbers.
c. An airfoil has a favorable pressure gradient near the trailing edge, as its thickness decreases.
d. Transition to turbulence increases the skin friction.
e. The displacement boundary layer thickness is larger than the boundary layer thickness based on the 99% of the external velocity.

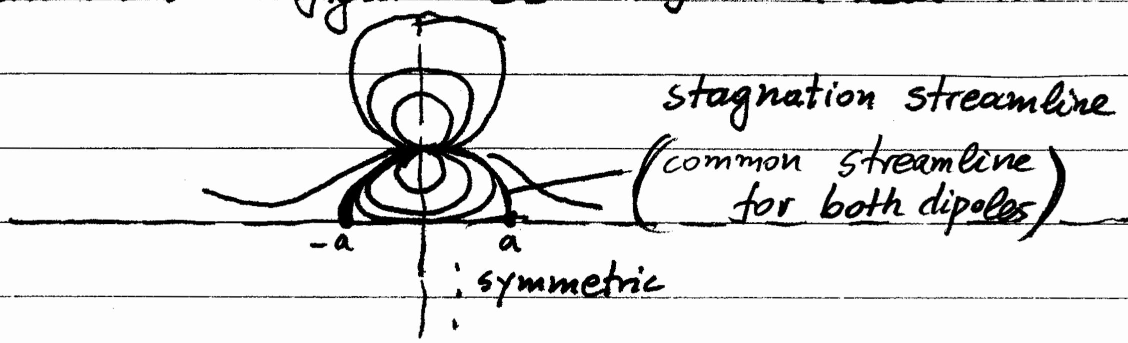
AAE 333 Exam 2 (solutions) Fall 2004

1 a)



use a mirror image

strealines : 2 figure - 8s - flatten near wall



b) $$\psi = -\frac{k}{2\pi}\frac{y+a}{x^2+(y+a)^2} - \frac{k}{2\pi}\frac{(y-a)}{x^2+(y-a)^2}$$

any stagnation point should be at $y=0$ (wall) because of symmetry

$$u_{wall} = \left.\frac{\partial\psi}{\partial y}\right|_{y=0} = \frac{k}{2\pi}\left(-\frac{1}{x^2+a^2} + \frac{2a^2}{(x^2+a^2)^2} - \frac{1}{x^2+a^2} + \frac{2a^2}{(x^2+a^2)^2}\right)$$

$$u_{wall} = 0 \Rightarrow -(x^2+a^2) + 2a^2 = 0 \Rightarrow \underline{x = \pm a}$$

c) for max $\frac{\partial u_{wall}}{\partial x} = 0 \Rightarrow -\frac{-2x}{(x^2+a^2)^2} - \frac{(x^2+a^2)\,2x\cdot 2a^2}{(x^2+a^2)^4} = 0$

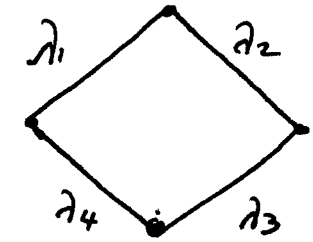
$$\Rightarrow 2x\left(\frac{1}{x^2+a^2} - \frac{1}{(x^2+a^2)^3}\right) = 2x\left(\frac{x^2+a^2-2a^2}{(x^2+a^2)^3}\right) \Rightarrow \boxed{x=0},\ \pm a$$

max min (stag. points)

$$\Rightarrow \underline{u_{wall\ max} = \frac{k}{\pi}\cdot\frac{1}{a^2}}$$

2

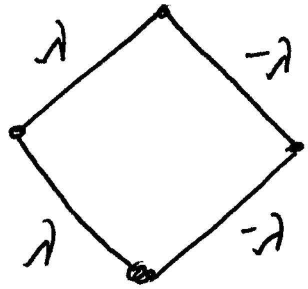
(2)



a) symmetry: $\lambda_1 = \lambda_4$, $\lambda_2 = \lambda_3$ (7 points)

$\Sigma \lambda's = 0 \Rightarrow \lambda_2 = -\lambda_1$, $\lambda_3 = -\lambda_4$

(8 points)



b) flow tagency at mid point of panel 1

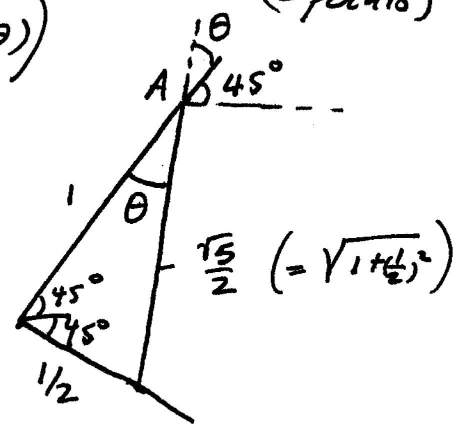
$$-V_\infty \frac{\sqrt{2}}{2} + \underset{①}{\lambda/2} + \underset{②}{\frac{-\lambda}{2\pi \cancel{\frac{\sqrt{2}}{2}}} \cancel{\frac{\sqrt{2}}{2}}} + \underset{③}{\frac{-\lambda}{2\pi 1}} + \underset{④}{\frac{\lambda}{2\pi \cancel{\frac{\sqrt{2}}{2}}} \cancel{\frac{\sqrt{2}}{2}}} = 0$$

(8 points)

$$\Rightarrow V_\infty \frac{\sqrt{2}}{2} = \lambda \left(\frac{1}{2} - \frac{1}{2\pi}\right) \Rightarrow \lambda = 2.07457\, V_\infty$$

(2 points)

c) $V_A = V_\infty + \overset{\text{symmetry}}{2}\lambda \left(\underset{①+②}{\frac{\lambda}{2\pi \frac{1}{2}} \frac{\sqrt{2}}{2}} + \underset{③+④}{\frac{\lambda}{2\pi \frac{\sqrt{5}}{2}} \cos(45° + \theta)} \right)$



$$\Rightarrow V_A = V_\infty + 2\lambda \left(\frac{\sqrt{2}}{2\pi} + \frac{1}{\sqrt{5}} \cos(45° + \theta) \right)$$

$$\theta = \sin^{-1}\left(\frac{1/\cancel{2}}{\sqrt{5}/\cancel{2}}\right) = 26.565°$$

$$\Rightarrow V_A = 2.521\, V_\infty$$

$$C_p = 1 - \left(\frac{V_A}{V_\infty}\right)^2 = -5.35$$

(4)

a. True (the flow is ≈ inviscid/potential outside the b.l.)
b. True (only works for <u>high</u> Reynolds numbers)
c. False (it is adverse)
d. True (τ_w is increased)
e. False ($\delta_1 < \delta$)

Problem 3: a) For Poiseuille flow from the notes:

$$V_z(r) = \frac{dp}{dz}\frac{r^2}{4\mu} + \frac{a}{\mu}\ln r + b \quad (1)$$

the boundary conditions are: $V_z = 0$ at $r = R$

$V_z = 0$ at $r = kR$

substitute: $$0 = \frac{R^2}{4\mu}\frac{dp}{dz} + \frac{a}{\mu}\ln R + b \quad (2)$$

$$0 = \frac{k^2R^2}{4\mu}\frac{dp}{dz} + \frac{a}{\mu}\ln kR + b \quad (3)$$

subtracting: $$0 = \frac{R^2}{4\mu}\frac{dp}{dz}(1-k^2) + \frac{a}{\mu}(\ln R - \ln kR)$$

$$\Rightarrow a = -\frac{R^2}{4}\frac{dp}{dz}\frac{(1-k^2)}{\ln(1/k)}$$

put in (2) $\Rightarrow$ $$b = -\frac{R^2}{4\mu}\frac{dp}{dz} + \frac{R^2}{4\mu}\frac{dp}{dx}\frac{(1-k^2)}{\ln(1/k)}\ln R$$

substitute a, b in (1):

$$V_z = \frac{r^2}{4\mu}\frac{dp}{dz} - \frac{R^2}{4\mu}\frac{dp}{dz}\frac{(1-k^2)}{\ln(1/k)}\ln r - \frac{R^2}{4\mu}\frac{dp}{dz} + \frac{R^2}{4\mu}\frac{dp}{dz}\frac{(1-k^2)}{\ln(1/k)}\ln R$$

$$\Rightarrow V_z = \frac{1}{4\mu}\frac{dp}{dz}\left[r^2 - R^2 - \frac{R^2(1-k^2)}{\ln(1/k)}(\ln r - \ln R)\right.$$

$$\Rightarrow V_z = -\frac{R^2}{4\mu}\frac{dp}{dz}\left[1 - \left(\frac{r}{R}\right)^2 + \frac{(1-k^2)}{\ln(1/k)}\ln\frac{r}{R}\right]$$ (10 points)

b) for the max $$\frac{dV_z}{dr} = 0 \Rightarrow -\frac{R^2}{4\mu}\frac{dp}{dz}\left[-\frac{2r}{R^2} + \frac{1-k^2}{\ln(1/k)}\frac{1}{r}\right] = 0$$

$$\Rightarrow \frac{2r}{R^2} = \frac{1-k^2}{\ln(1/k)}\frac{1}{r^2} \Rightarrow r^2 = \frac{(1-k^2)R^2}{2\ln(1/k)} \Rightarrow r = \left(\frac{1-k^2}{2\ln(1/k)}\right)^{1/2} R$$ (5 points)

c. $$Q = \int_A V_z \, dA = \int_{kR}^{R} V_z \, 2\pi r \, dr =$$

$$= 2\pi \left(-\frac{R^2}{4\mu} \frac{dp}{dz} \right) \int_{kR}^{R} \left[r - \frac{r^3}{R^2} + \frac{(1-k^2)}{\ln(1/k)} r \ln\frac{r}{R} \right] dr$$

$$= \left(-\frac{\pi R^4}{2\mu} \frac{dp}{dz} \right) \int_k^1 \left[\frac{r}{R} - \left(\frac{r}{R}\right)^3 + \frac{1-k^2}{\ln(1/k)} \frac{r}{R} \ln\frac{r}{R} \right] d\left(\frac{r}{R}\right)$$

$$= \left(-\frac{\pi R^4}{2\mu} \frac{dp}{dz} \right) \left[\frac{1}{2}\left(\frac{r}{R}\right)^2 - \frac{1}{4}\left(\frac{r}{R}\right)^4 + \frac{1-k^2}{\ln(1/k)} \left\{ \left(\frac{r}{R}\right)^2 \left[\frac{1}{2} \ln\left(\frac{r}{R}\right) - \frac{1}{4} \right] \right\} \right]_k^1$$

$$= \left(-\frac{\pi R^4}{2\mu} \frac{dp}{dz} \right) \left[\frac{1}{2} - \frac{k^2}{2} - \frac{1}{4} + \frac{k^4}{4} + \frac{1-k^2}{\ln(1/k)} \left\{ -\frac{1}{4} - k^2 \left[\frac{1}{2} \ln k - \frac{1}{4} \right] \right\} \right]$$

$$= \left(-\frac{\pi R^4}{2\mu} \frac{dp}{dz} \right) \left[\frac{1-2k^2+k^4}{4} + \frac{1-k^2}{\ln(1/k)} \frac{k^2-1}{4} - \frac{1-k^2}{\ln(1/k)} k^2 \frac{1}{2} \ln k \right]$$

$$= \left(-\frac{\pi R^4}{2\mu} \frac{dp}{dz} \right) \left[\frac{1 - \cancel{2k^2} + k^4 + \cancel{2k^2} - 2k^4}{4} - \frac{(1-k^2)^2}{4\ln(1/k)} \right]$$

$$\Rightarrow \quad Q = -\frac{\pi R^4}{8\mu} \frac{dp}{dx} \left[(1-k^4) - \frac{(1-k^2)^2}{\ln(1/k)} \right] \quad \text{(7 points)}$$

d) for $k \to 0$ (circular) pipe

$$V_z = -\frac{R^2}{4\mu} \frac{dp}{dz} \left(1 - \frac{r^2}{R^2} \right)$$

max V_z at $r = 0$ (centerline)

$$Q = -\frac{\pi R^4}{8\mu} \frac{dp}{dz} \quad \left(= \pi R^2 \bar{V}, \ \bar{V} = -\frac{R^2}{8\mu} \frac{dp}{dz} \right)$$