12/4/9 Exam 2 Soln

1) $\psi = xy$

- $\xi_1 : \nabla^2\psi = \frac{\partial^2 y}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} = 0 + 0$

A YES

or $u = \frac{\partial \psi}{\partial y} = x$

$v = -\frac{\partial \psi}{\partial x} = -y$

$\xi_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 - 0 \rightarrow$ YES

2) $\rho = x^2 + y^2$

$\nabla\rho)_x = \frac{\partial \rho}{\partial x} = 2x$

@ $x = y = z$: $\nabla\rho)_x = 4$

C

3) $\vec{u} = \nabla \varphi$

$\therefore \vec{\xi} = \nabla \times \vec{u} = 0$

$\therefore$ **b**

4) $U_\infty = 30 \text{ m/s}$

$\rho = 1 \text{ kg/m}^3$

$p + \frac{1}{2}\rho u^2 = \text{const}$

$= p_\infty + \frac{1}{2}\rho U_\infty^2$

$p' = p - p_\infty$

$= \frac{1}{2}\rho(U_\infty^2 - u^2)$

$p'|_{max} = \frac{1}{2}\rho U_\infty^2$

$= \frac{1}{2} \cdot 1 \cdot 30^2$

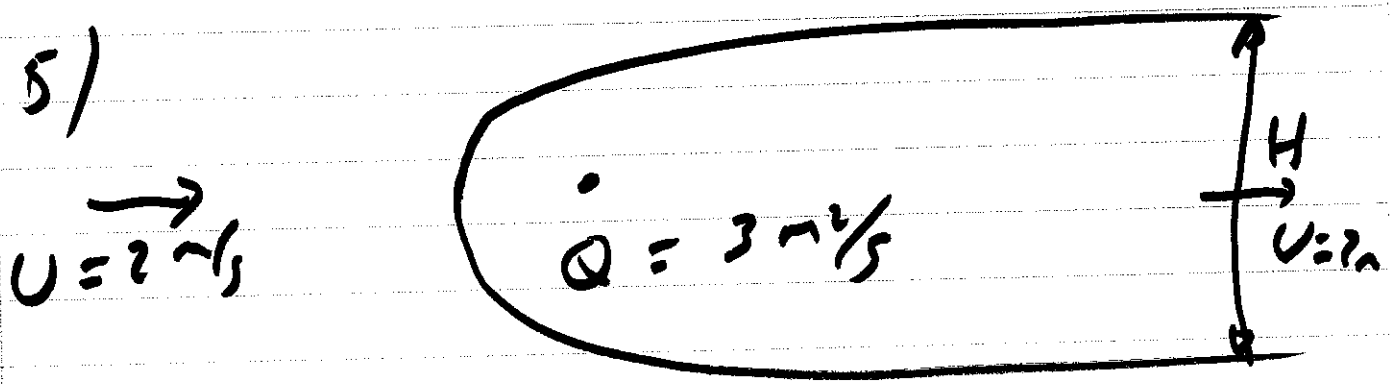
$= 450 \text{ Pa}$

so $\sigma'_{atm} = \frac{45}{10^5}$

$= .00045$

D

5)

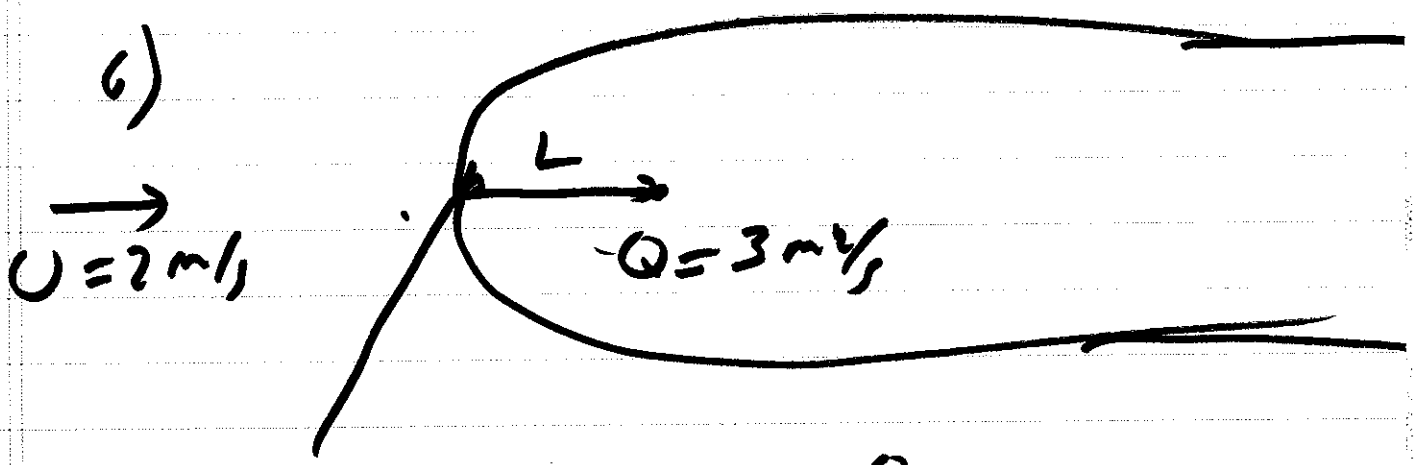


$vH = Q$

$H = Q/v = \frac{3}{2}$

B

6)



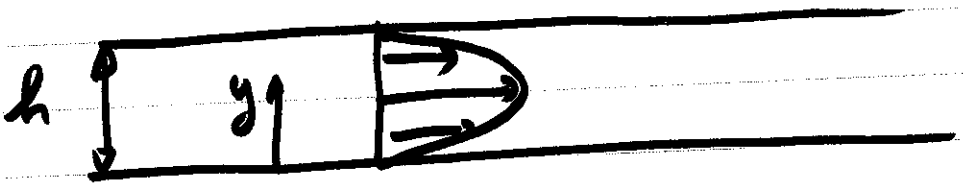
$$u = 0 = U - \frac{Q}{2\pi L}$$

$$\therefore L = \frac{Q}{2\pi U}$$

$$L = \frac{3}{4\pi}$$

D

7)



$h = 1\,cm = .01\ m$

$Q = 2\ m^2/s$

$\mu = 1.2 \cdot 10^{-5}\ \frac{kg}{m \cdot s}$

$u = U_{cl} \cdot 4\bar{y}(1-\bar{y})$, $\bar{y} = y/h$

$$Q = \int_0^h u\,dy = h\,U_{cl} \int_0^1 4\bar{y}(1-\bar{y})\,d\bar{y}$$

$$= h \cdot U_{cl} \cdot \frac{2}{3} \rightarrow U_{cl} = \frac{3}{2}\frac{Q}{h}$$

$$\left.\frac{\partial u}{\partial y}\right|_0 = \left.\frac{U_{cl}}{h} \cdot 4(1-2\bar{y})\right|_{\bar{y}=0}$$

I dropped factor of 4 in class!

$$= 4\,U_{cl}/h$$

$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_0 = 4\mu\frac{U_{cl}}{h}$$

Or $\tau_w = 4\mu \cdot \frac{1}{l} \cdot \frac{3}{2} \frac{Q}{l}$

$= 6\mu \frac{Q}{l^2}$

(or via Eq's 103, 104, 105 & 108 in text)

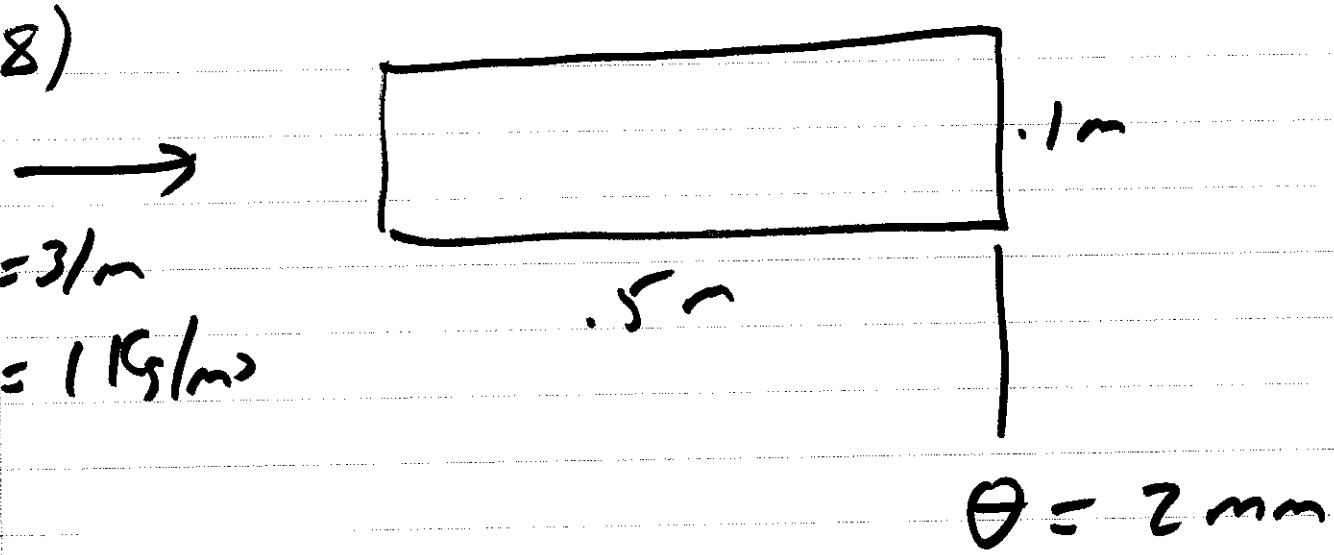
so $\tau_w = 6 \cdot 1.2 \cdot 10^{-5} \cdot \frac{2}{(.01)^2}$

$= 1.44 \cdot$ N/m²

Ans is **D**

(Note: in class I missed the factor $\frac{1}{4}$ in $\frac{\partial u}{\partial y}\Big|_0$)

8)



$U = 3 \text{ m/s}$

$\rho = 1 \text{ kg/m}^3$

VK Mom Int: $\frac{\tau_w}{\rho} = U_\infty^2 \cdot \frac{d\theta}{dx}$

for flat plate

$$\Rightarrow \int_0^L \tau_w \, dx = \rho U_\infty^2 \, \theta_{TE}$$

$$= 1 \cdot 3^2 \cdot 2 \cdot 10^{-3}$$

$$= 18 \cdot 10^{-3}$$

$$= .018$$

$$F = .018 \cdot (.1) = .0018$$

$$= \underline{1.8 \cdot 10^{-3}}$$ D

9) $T = x^2 + y^2$

$$\frac{DT}{Dt} = u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}$$

$$= u \cdot 2x + v \cdot 2y$$

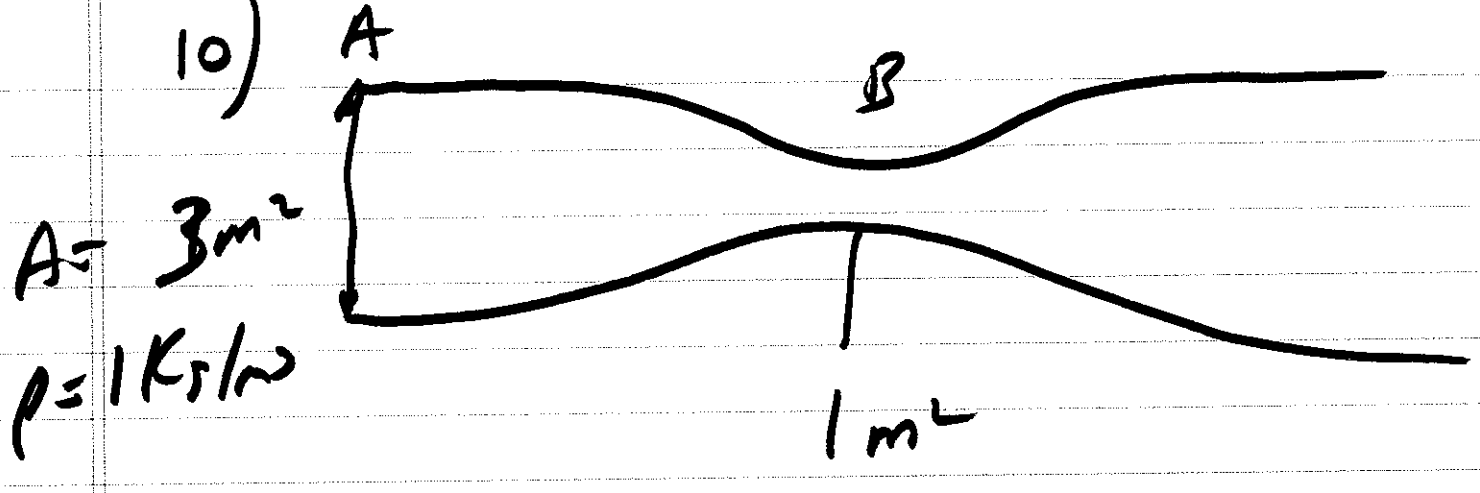
$$= 2y \cdot 2x + 3x \cdot 2y$$

$$= 10xy$$

@ $x = y = 1$: $\underline{10}$

B

10)



$$P_A - P_B = \Delta P$$

$$= \frac{1}{2}\rho\left(v_B^2 - v_A^2\right)$$

$$v_B A_B = v_A \cdot A_A$$

$$v_B = v_A \cdot \frac{3}{1} = 3 v_A$$

$$P_A - P_B = 4 \rho v_A^2$$

$$\dot{m} = \rho v_A \cdot A_A$$

$$P_A - P_0 = 4 \cdot 1 \, v_A^2 = 9$$

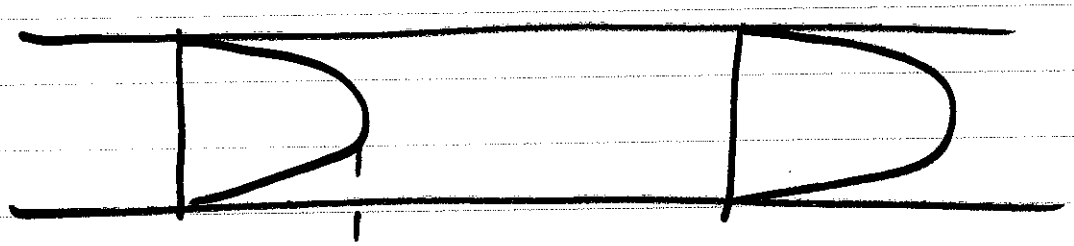
$$v_A = \sqrt{9/4} = 3/2 = 1.5$$

$$\dot{m} = 1 \cdot 1.5 \cdot 3$$

$$\boxed{\dot{m} = 4.5 \ kg/s}$$

(D)

11)



U_{cc} const

θ falls

$$P_{stag} = \theta + \tfrac{1}{2} \rho U_{cc}^2$$

(θ: falls; $\tfrac{1}{2}\rho U_{cc}^2$: const)

$\therefore$ P_{stag} falls

B

12)

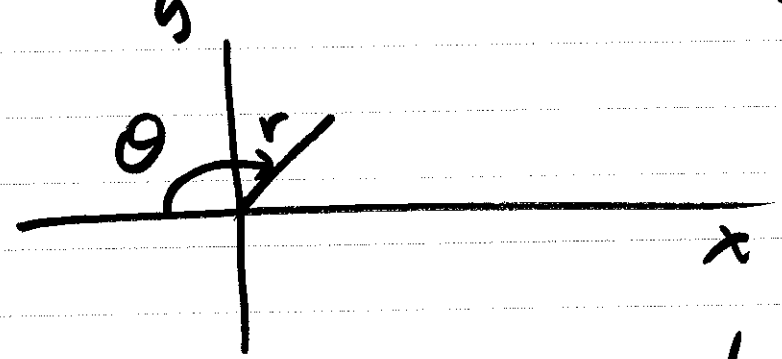
C $\quad x_{max\,camb} = .4$

13) $\frac{\text{thickness}}{\text{chord}} = \frac{.4}{2} = .2$

$\frac{U_{max}}{V_\infty} = 1.2$

B

14) $\varphi = 3x + 17\ln r + \frac{.6}{2\pi}\theta$



uniform stream + point source + vortex

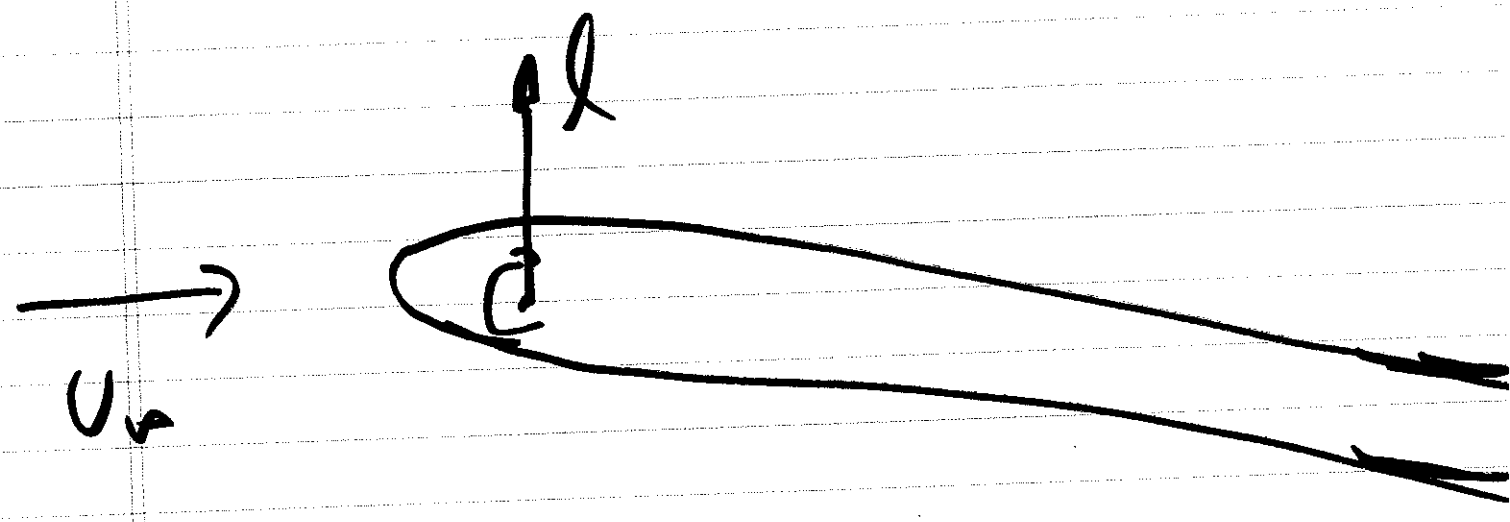
$U_\infty = 3$

$\Gamma = .6$

$l = \rho \Gamma U_\infty$

$= 1.2 \cdot .6 \cdot 3$

$\boxed{l = 2.16}$ (C)



15) $\tau_w \sim \frac{1}{\sqrt{\kappa}}$

$\rightarrow \omega$ at $\kappa \rightarrow 0$

(B)

16) c) vortices