08 November 2013 **Name:**__________________

ECE 638 **Exam No. 2** **Fall 2013**

This exam is closed book and closed notes. However, calculators are permitted. You have 50 minutes to work the following three problems. To obtain maximum partial credit, be sure to show the complete derivation of your answers. Also, be sure to budget your time; so that you have time to look at each problem.

1. (40 pts.) A print contains an ideal periodic, clustered-dot halftone pattern $g(x,y)$ with average absorptance 0.25 that consists of square dot-clusters placed with period 100 cycles/inch on a square lattice. Here ideal means that the paper substrate is assumed to have reflectance 1 and the colorant dots are assumed to have reflectance 0. You scan this print at 120 dpi, and display it using a continuous-tone display with resolution 120 dpi that can be modeled as a zero-order hold process, i.e. if $g(mX,nX)$ represents the scanned halftone print, where $X = 1/120$ inch, then the continuous-parameter displayed image is given by

$$g_r(x,y) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} g(mX,nX)\,\mathrm{rect}\left(\frac{x-mX}{X}, \frac{y-nX}{X}\right)$$

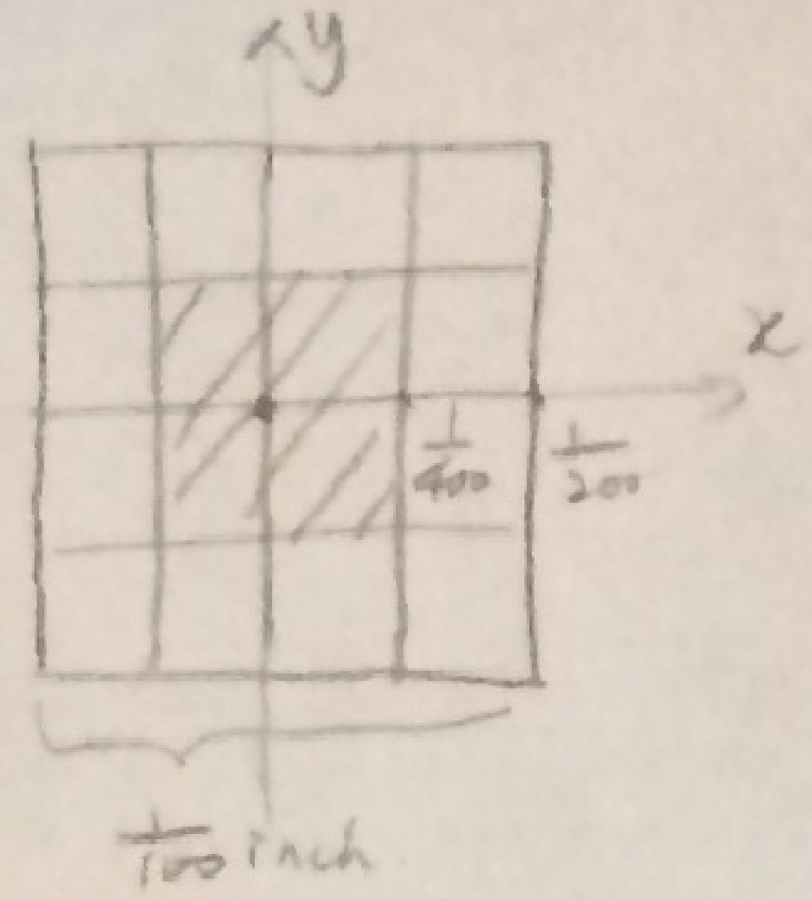
a. (5) Sketch the continuous-parameter halftone pattern $g(x,y)$.

b. (20) Find an expression for the 2-D continuous-space Fourier transform CSFT $G_r(u,v)$ of the continuous-parameter displayed scan $g_r(x,y)$ of the halftone image.

c. (10) Carefully sketch $G_r(u,v)$.

d. (5) Based on your answer to part c), comment on the expected appearance of $g_r(x,y)$.

a) $g(x,y)$



(b) $g(x,y) = \mathrm{rep}_{\frac{1}{100},\frac{1}{100}}\left[\mathrm{rect}\left(\frac{x}{\frac{1}{200}}, \frac{y}{\frac{1}{200}}\right)\right]$

$$G(u,v) = 100^2\, \mathrm{comb}_{100,100}\left[\frac{1}{200^2}\,\mathrm{sinc}\left(\frac{1}{200}u, \frac{1}{200}v\right)\right].$$

$$= \frac{1}{4}\,\mathrm{comb}_{100,100}\left[\mathrm{sinc}\left(\frac{1}{200}u, \frac{1}{200}v\right)\right]$$

$$= \frac{1}{4}\sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty} \mathrm{sinc}\left(\frac{1}{200}k\cdot 100, \frac{1}{200}l\cdot 100\right)\delta(u-100k, v-100l)$$

$$= \frac{1}{4}\sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty} \mathrm{sinc}\left(\frac{1}{2}k, \frac{1}{2}l\right)\delta(u-100k, v-100l)$$

$$g_r(x,y) = \mathrm{comb}_{X,X}[g(x,y)] ** \mathrm{rect}\left(\frac{x}{X}, \frac{y}{X}\right)$$

$$G_r(u,v) = \frac{1}{X^2}\mathrm{rep}_{\frac{1}{X},\frac{1}{X}}[G(u,v)]\cdot X^2\cdot \mathrm{sinc}(Xu, Xv)$$

$$= \mathrm{rep}_{\frac{1}{X},\frac{1}{X}}[G(u,v)]\cdot \mathrm{sinc}(Xu, Xv).$$

$$= \sum_m\sum_n G\left(u-\frac{m}{X}, v-\frac{n}{X}\right)\mathrm{sinc}(Xu, Xv)$$

$$= \sum_m\sum_n\left(\frac{1}{4}\sum_k\sum_l \mathrm{sinc}\left(\frac{1}{2}k, \frac{1}{2}l\right)\delta\left(u-\frac{m}{X}-100k, v-\frac{n}{X}-100l\right)\right)$$

$$\mathrm{sinc}(Xu, Xv)$$

$$= \frac{1}{4}\mathrm{sinc}(Xu, Xv)\sum_m\sum_n\sum_k\sum_l \mathrm{sinc}\left(\frac{1}{2}k, \frac{1}{2}l\right)\delta\left(u-\frac{m}{X}-100k, v-\frac{n}{X}-100l\right)$$

$$= \frac{1}{4}\mathrm{sinc}\left(\frac{u}{120}, \frac{v}{120}\right)\sum_m\sum_n\sum_k\sum_l \mathrm{sinc}\left(\frac{1}{2}k, \frac{1}{2}l\right)\delta(u-120m-100k, v-120n-100l)$$

$$= \frac{1}{4} \operatorname{sinc}\left(\frac{u}{120}, \frac{v}{120}\right) \sum_m \sum_n \sum_k \sum_l \operatorname{sinc}\left(\tfrac{1}{2}k, \tfrac{1}{2}l\right) \delta(u-120m-100k, v-120n-100l)$$

c) $m, n, k, l \in \mathbb{Z}$

$\operatorname{sinc}(p, q) = \operatorname{sinc}(p)\operatorname{sinc}(q)$ and $\operatorname{sinc}(p) = \frac{\sin(\pi p)}{\pi p}$ for $p \in \mathbb{Z}$ $\operatorname{sinc}(p) = 0$

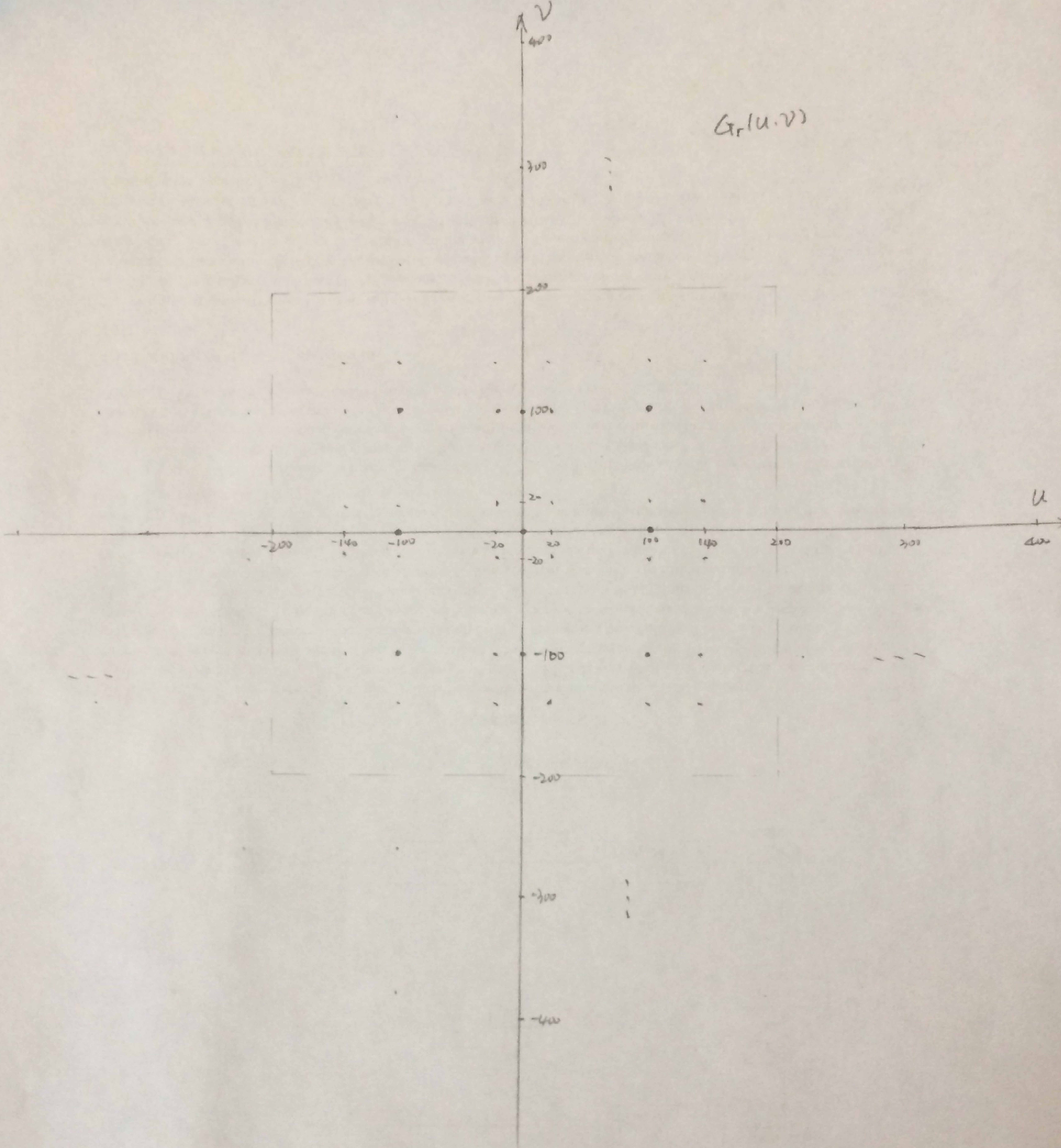
If $p \in \mathbb{Z}$ or $q \in \mathbb{Z}$, $\operatorname{sinc}(p, q) = 0$

1° if k $\neq 0$ is even or l $\neq 0$ is even, then $\operatorname{sinc}(\tfrac{1}{2}k, \tfrac{1}{2}l) = 0$

2° if $\frac{u}{120} \in \mathbb{Z}$ or $\frac{v}{120} \in \mathbb{Z}$, then $\operatorname{sinc}\left(\frac{u}{120}, \frac{v}{120}\right) = 0$

$u = 120m + 100k$ and $v = 120n + 100l$.

if $\underbrace{k = 0 \text{ and } m \neq 0}_{\text{duplicates of points on } u \text{ coordinate}}$, or $\underbrace{l = 0 \text{ and } n \neq 0}_{\text{duplicates of points on } v \text{ coordinate}}$, $\Rightarrow \operatorname{sinc}\left(\frac{u}{120}, \frac{v}{120}\right) = 0$



v
400
300
200
100
20
-20
-100
-200
-300
-400
u
-200
-140
-100
-20
20
100
140
200
300
400
$G_r(u,v)$

2. (continued - 1)

a. From the given figure we have

$a_C = \frac{3}{16}$, $a_Y = \frac{3}{16}$, $a_M = \frac{2}{16}$, $a_{CM} = \frac{1}{16}$, $a_{YM} = \frac{1}{16}$, $a_W = \frac{6}{16}$

$$\therefore R(\lambda) = a_C R_C(\lambda) + a_M R_M(\lambda) + a_Y R_Y(\lambda) + a_{CM} R_C(\lambda) R_M(\lambda) + a_{YM} R_Y(\lambda) R_M(\lambda) + a_W R_W(\lambda)$$
$$= \frac{3}{16} R_C(\lambda) + \frac{2}{16} R_M(\lambda) + \frac{3}{16} R_Y(\lambda) + \frac{1}{16} R_C(\lambda) R_M(\lambda) + \frac{1}{16} R_Y(\lambda) R_M(\lambda) + \frac{6}{16} R_W(\lambda)$$

where $R_W(\lambda) = 1$, $400 \le \lambda < 700$, since the paper substrate has unity spectral reflectance across the band of visible wavelengths.

1°. $400 \le \lambda < 500$

$$R(\lambda) = \frac{3}{16} \times 1 + \frac{2}{16} \times 1 + \frac{3}{16} \times 0 + \frac{1}{16} \times 1 \times 1 + \frac{1}{16} \times 0 \times 1 + \frac{6}{16} \times 1 = \frac{3+2+0+1+0+6}{16} = \frac{12}{16} = \frac{3}{4}$$

2°. $500 \le \lambda < 600$

$$R(\lambda) = \frac{3}{16} \times 1 + \frac{2}{16} \times 0 + \frac{3}{16} \times 1 + \frac{1}{16} \times 1 \times 0 + \frac{1}{16} \times 1 \times 0 + \frac{6}{16} \times 1 = \frac{3+0+3+0+0+6}{16} = \frac{12}{16} = \frac{3}{4}$$

3°. $600 \le \lambda < 700$

$$R(\lambda) = \frac{3}{16} \times 0 + \frac{2}{16} \times 1 + \frac{3}{16} \times 1 + \frac{1}{16} \times 0 \times 1 + \frac{1}{16} \times 1 \times 1 + \frac{6}{16} \times 1 = \frac{0+2+3+0+1+6}{16} = \frac{12}{16} = \frac{3}{4}$$

$$\therefore R(\lambda) = \begin{cases} \frac{3}{4}, & 400 \le \lambda < 500 \\ \frac{3}{4}, & 500 \le \lambda < 600 \\ \frac{3}{4}, & 600 \le \lambda < 700 \end{cases}$$

Note: It looks like neutral grey.

b.

C	M
	Y

It's possible that the three colors have no overlap at all. The above solution is only one possibility.

2. (continued - 2)

c. There is no overlap among the three colors. We have

$a_W = \frac{1}{4}$, $a_C = \frac{1}{4}$, $a_M = \frac{1}{4}$, $a_Y = \frac{1}{4}$

The new spatially averaged spectral reflectance function is

$$R(\lambda) = \frac{1}{4}R_W(\lambda) + \frac{1}{4}R_C(\lambda) + \frac{1}{4}R_M(\lambda) + \frac{1}{4}R_Y(\lambda)$$

$$= \begin{cases} \frac{1}{4}+\frac{1}{4}+\frac{1}{4}+0 \\ \frac{1}{4}+\frac{1}{4}+0+\frac{1}{4} \\ \frac{1}{4}+0+\frac{1}{4}+\frac{1}{4} \end{cases} = \begin{cases} \frac{3}{4}, & 400 \leq \lambda < 500 \\ \frac{3}{4}, & 500 \leq \lambda < 600 \\ \frac{3}{4}, & 600 \leq \lambda < 700 \end{cases}$$

$$\therefore \quad R(\lambda) = \begin{cases} \frac{3}{4}, & 400 \leq \lambda < 500 \\ \frac{3}{4}, & 500 \leq \lambda < 600 \\ \frac{3}{4}, & 600 \leq \lambda < 700 \end{cases}$$

Note: We have the same amount of each color and white, which is expected to look like neutral grey.

Problem 3

a. Use the approximation from the lecture,

$$\varepsilon = \frac{\Delta^2}{12}$$

Since there are three output levels $-1, 0, 1$,

$$\Delta = 1.$$

Hence:

$$\varepsilon = 1/12.$$

b. Optimum quantizer:

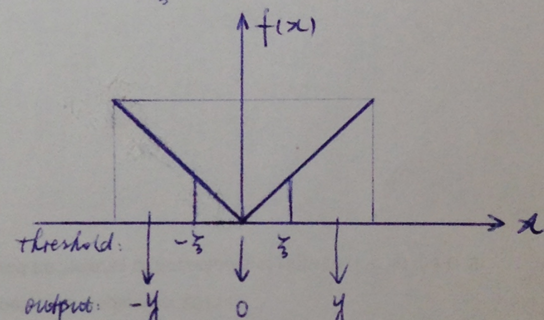
Considering the symmetry of the density function, the three output levels will be: $-y$ 0 y,

and the threshold: $-\xi, \xi$

For the pos. half:

$$y = \frac{\int_\xi^1 x f_x(x)\,dx}{\int_\xi^1 f_x(x)\,dx} = \frac{\int_\xi^1 x^2 dx}{\int_\xi^1 x\,dx} = \frac{\frac{1}{3}x^3|_\xi^1}{\frac{1}{2}x^2|_\xi^1} = \frac{2(1-\xi^3)}{3(1-\xi^2)}$$

$$= \frac{2(1+\xi+\xi^2)}{3(1+\xi)} \quad ①$$



The threshold condition:

$$\xi = \frac{0+y}{2} \quad ②$$

$$①+② \Rightarrow 2\xi = \frac{2(1+\xi+\xi^2)}{3(1+\xi)}$$

$$\Rightarrow 2\xi^2 + 2\xi - 1 = 0$$

$$\Rightarrow \xi = \frac{-2 \pm \sqrt{4+8}}{4} = \frac{-1 \pm \sqrt{3}}{2}$$

$\because \xi > 0$ for the pos. half.

$$\therefore \xi = \frac{\sqrt{3}-1}{2} \approx 0.366$$

$$y = \frac{2}{\sqrt{3}+1} \approx 0.732$$

By symmetry, output levels: 0.732, 0, -0.732

threshold: -0.366, 0.366.

c.

$$\varepsilon = E\{|y-x|^2\}$$

$$= 2\left[\int_0^\xi (x-0)^2 x\,dx + \int_\xi^1 (y-x)^2 f(x)\,dx\right]$$

$$= 2\left[\frac{1}{4}\xi^4 + \int_\xi^1 (x^3 - 2x^2 y + y^2 x)\,dx\right]$$

$$= 2\left[\frac{1}{4}\xi^4 - \frac{1}{4}\xi^4 + \frac{1}{4} + \frac{1}{2}y^2x^2|_\xi^1 - \frac{2}{3}yx^3|_\xi^1\right]$$

$$= 2[0.25 + 0.232 - 0.464] = 0.036 < \frac{1}{12}$$

The optimal quantizer has a smaller MSE compared to the approximation.