NAME:_____ PUID:_____

**SECTION: Circle one: Alam Lundstrom**

**SOLUTIONS: ECE 305 Exam 4: Spring 2016**
**March 28, 2016**

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This is a closed book exam. You may use a calculator and the formula sheet at the end of this exam. Following the ECE policy, the calculator **must** be a Texas Instruments TI-30X IIS scientific calculator.

There are three equally weighted questions. To receive full credit, you must **show your work and explain your answers.**

The exam is designed to be taken in 60 minutes.

At the top of the page, be sure to fill in your name, Purdue student ID and identify the section you are in.

DO NOT open the exam until told to do so, and stop working immediately when time is called.

The last page is an equation sheet, which you may remove if you want to, after the exam begins.

**75 points possible, 25 per question**

1) 25 points (5 point per part)

2) 25 points (5 points per part)

3) 25 points (5 points per part)

---------- Course policy ----------

I understand that if I am caught cheating in this course, I will earn an F for the course and be reported to the Dean of Students.

Read and understood: _____
signature

Answer the **five multiple choice questions** below by **drawing a circle** around the **one, best answer**.

1a) Which of the following is true of the QFL's, $F_n(x)$ and $F_n(x)$, in a **forward-biased** NP junction?

a) $F_n(x) = F_p(x)$ everywhere.

**b) $F_n(x) \geq F_p(x)$ everywhere.**

c) $F_n(x) \geq F_p(x)$ only on the N-side.

d) $F_n(x) \geq F_p(x)$ only in the transition region.

e) $F_n(x) \geq F_p(x)$ only on the P-side.

1b) Assume that $np = n_i^2 e^{qV_A/k_BT}$ holds at the beginning of the P-region ($x = x_p$) and that low-level injection applies. Which of the following is true?

a) $n = n_0 = n_i^2/N_A$, $p = p_0 \approx N_A$.

**b) $n = \left(n_i^2/N_A\right)e^{qV_A/k_BT}$, $p = p_0 \approx N_A$.**

c) $n = n_0 = n_i^2/N_A$, $p = p_0 e^{qV_A/k_BT} \approx N_A e^{qV_A/k_BT}$.

d) $n = N_A$, $p = p_0 \approx n_i^2/N_A$.

e) $n = N_A$, $p = \left(n_i^2/N_A\right)e^{qV_A/k_BT}$.

1c) Ion implantation is used to:

**a) Dope a semiconductor.**

b) Deposit dielectric thin films on a semiconductor.

c) Deposit metallic thin films on a semiconductor.

d) Grow crystalline layers on a semiconductor.

e) Pattern thin films on a semiconductor.

1d) Consider an NP junction with $N_A >> N_D$. Which one of the following is true?

a) Most of the depletion layer is on the P-side .

b) Most of the forward-biased current is due to electrons.

c) Most of the reverse-biased current is due to electrons.

d) Most of the bandbending is on the P-side.

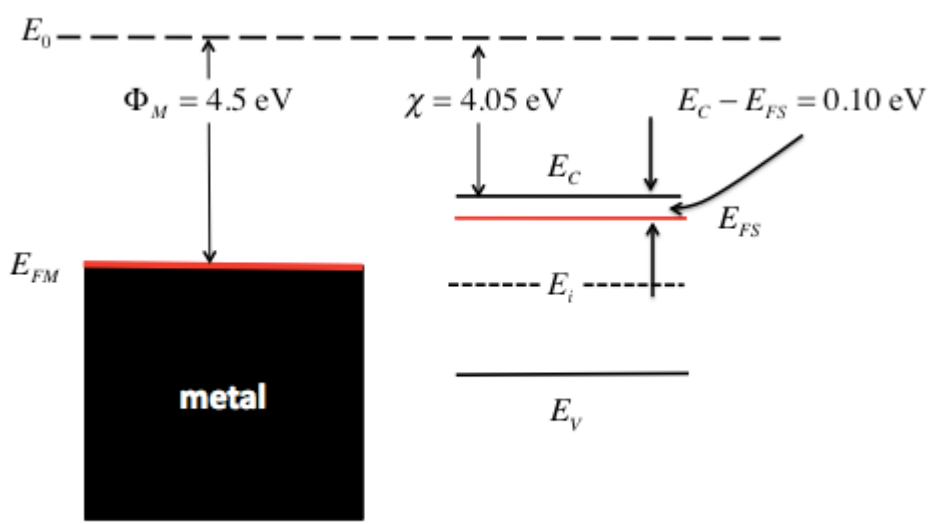
**e) The electric field peaks at the NP junction.**

1e) The lithography process typically consists of several steps - first resist application, then prebake, then exposure, and then what?

a) Etching.
b) Hardbake.
c) Resist stripping.
d) Epitaxy.
**e) Develop**

2) Answer the following questions about Metal-Silicon (MS) junctions.

2a) If we form an MS diode from the metal and semiconductor shown below, what is the magnitude of the built in voltage in volts? (A numerical answer is required.)

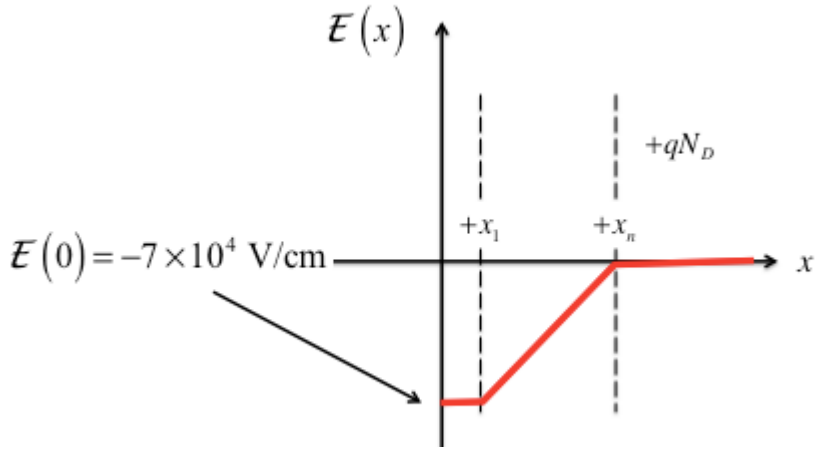



**Solution:**

$$qV_{bi} = \Phi_M - \Phi_S = 4.5 - (4.05 + 0.10) = 0.35\,\text{eV}$$

$$\boxed{V_{bi} = 0.35\ \text{V}}$$

Consider a metal-semiconductor diode. The equilibrium electric field vs. position in the semiconductor, $\mathcal{E}(x)$, is sketched below where $x_1 = 0.05\times10^{-4}$ cm and $x_n = 0.20\times10^{-4}$ cm.




2b) Sketch the space charge density in the semiconductor, $\rho(x)$, on the axes below. **Explain your sketch.**

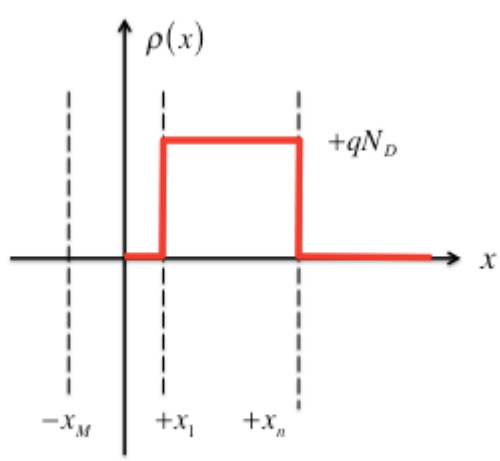
**Solution:**
Begin with the Poisson equation:

$$\frac{d\mathcal{E}}{dx} = \frac{\rho}{K_S\varepsilon_0}$$

$0 < x < x_1, \mathcal{E}(x)$ constant $\Rightarrow \rho(x) = 0$

$x_1 < x < x_n, \mathcal{E}(x)$ constant, positive slope $\Rightarrow \rho(x) > 0$ and constant

$x > x_n, \mathcal{E}(x)$ constant $\Rightarrow \rho(x) = 0$



2c) What is the numerical value of the doping density for $0 < x < x_1$?

**Solution:**

No charge – implies no doping and negligible carrier density (i.e. depletion approx.).

$$\frac{d\mathcal{E}}{dx} = 0 = \frac{\rho}{K_S \varepsilon_0} \Rightarrow \rho = 0 \Rightarrow N_D = 0$$

$$\boxed{N_D = 0}$$

2d) What is the numerical value of the doping density for $x_1 < x < x_n$? (Hint. The linearity of the electric field allows you to calculate this quantity easily).

**Solution:**
Positive charge – implies N-type doping and negligible carrier density (i.e. depletion approximation).

$$\frac{d\mathcal{E}}{dx} = \frac{\rho}{K_S \varepsilon_0} = \frac{qN_D}{K_S \varepsilon_0}$$

$$N_D = \frac{K_S \varepsilon_0}{q} \frac{d\mathcal{E}}{dx} = \frac{11.8 \times 8.854 \times 10^{-14}}{1.6 \times 10^{-19}} \times \frac{\left(0 - \left(-7 \times 10^4\right)\right)}{0.15 \times 10^{-4}} = 3.05 \times 10^{16}$$

$$\boxed{N_D = 3.05 \times 10^{16} \text{ cm}^{-3}}$$

2e) What is the numerical value of the built-in potential, $V_{bi}$? Note that you cannot use the p-n junction formula, $V_{bi} = \left(k_B T / q\right) \ln\left(N_A N_D / n_i^2\right)$ for MS diodes.

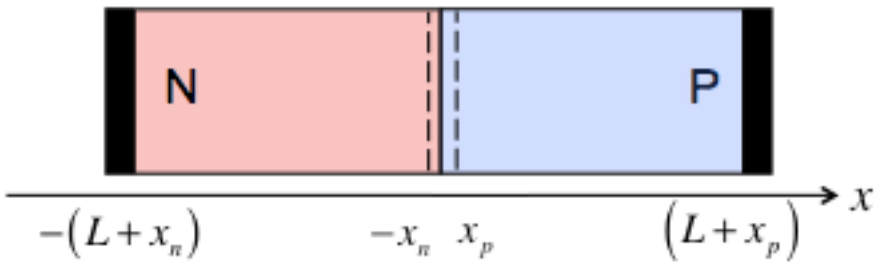
**Solution:**

$$V_{bi} = \left|\mathcal{E}(0)\right| x_1 + \frac{\left|\mathcal{E}(0)\right|}{2}\left(x_n - x_1\right)$$

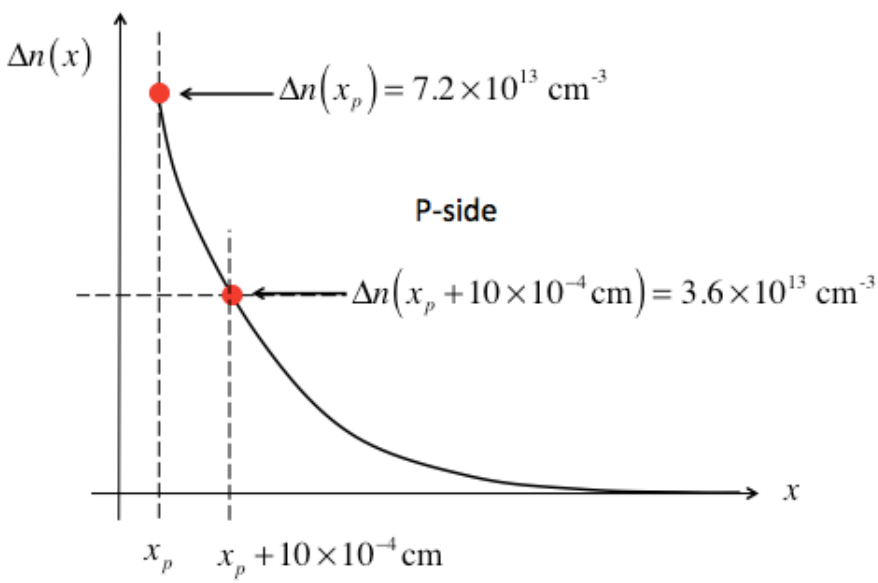
$$V_{bi} = \frac{\left|\mathcal{E}(0)\right|}{2}\left(x_1 + x_n\right) = \frac{7 \times 10^4}{2}\left(0.05 + 0.20\right) \times 10^{-4} = 0.875$$

$$\boxed{V_{bi} = 0.875 \text{ V}}$$

3) A silicon diode is asymmetrically doped at $N_D = 10^{19}$ cm$^{-3}$ and $N_A = 10^{17}$ cm$^{-3}$. Answer the following questions assuming room temperature ($k_B T/q = 0.026$ V). Assume that the lengths of the neutral N and P regions are $L = 1000\ \mu$m. Two ideal metal contacts on the ends of the N- and P-regions maintain equilibrium carrier densities under all conditions.




The minority electron concentration on the P-side is sketched below for a certain forward bias.




3a) Is the P-side in low-level injection? **Explain why or why not.**

**Solution:**

**YES**

$\Delta n(x > x_p) \le 7.2\times10^{13} << p_0 = N_A = 10^{17}$

Excess minority electron concentration is much less than the majority concentration.

3b) Determine the forward bias in volts using the law of the junction.

**Solution:**

$$\Delta n\left(x_p\right)=\frac{n_i^2}{N_A}\left(e^{qV_A/k_BT}-1\right)\approx\frac{n_i^2}{N_A}e^{qV_A/k_BT}=7.2\times10^{13}$$

$$e^{qV_A/k_BT}=\frac{7.2\times10^{13}}{n_i^2/N_A}=\frac{7.2\times10^{13}}{10^3}=7.2\times10^{10}$$

$$V_A=0.026\ln\left(7.2\times10^{10}\right)=0.65$$

$$\boxed{V_A=0.65\text{ V}}$$

3c) Determine the minority carrier diffusion length (in cm) based on the pair of carrier concentrations at $x_p$ and $x_p+10\times10^{-4}$ cm as indicated above.

**Solution:**

$$\Delta n\left(x\right)=\Delta n\left(x_p\right)e^{-\left(x-x_p\right)/L_n}$$

$$\Delta n\left(x_p+10\times10^{-4}\text{cm}\right)=\frac{\Delta n\left(x_p\right)}{2}=\Delta n\left(x_p\right)e^{-\left(10\times10^{-4}\right)/L_n}$$

$$L_n=\frac{10\times10^{-4}}{\ln 2}=14.4\times10^{-4}$$

$$\boxed{L_n=14.4\times10^{-4}\text{ cm }=14.4\ \mu\text{m}}$$

3d) Determine a numerical value for the hole quasi-Fermi level, $E_i\left(x\right)-F_p\left(x\right)$, at $x=x_p+10\times10^{-4}\text{cm}$. (Hint. Find the formula that relates carrier concentration to the intrinsic Fermi level of the semiconductor).

**Solution:**

$$p\left(x\right)\approx p_0=n_ie^{\left(E_i-F_p\left(x\right)\right)/k_BT}\quad\text{(low level injection)}$$

$$E_i-F_p\left(x\right)=0.026\ln\left(10^{17}/10^{10}\right)=0.419\text{ eV}$$

$$\boxed{E_i-F_p\left(x_p+10\times10^{-4}\text{cm}\right)=0.419\text{ eV}}$$

3e) Determine a numerical value for the electron quasi-Fermi level, $F_n(x) - E_i(x)$, at $x = x_p + 10 \times 10^{-4}\,\text{cm}$.

**Solution:**

$n(x) \approx \Delta n(x) = n_i e^{\left(F_n(x) - E_i\right)/k_B T}$ (low level injection)

$F_n(x) - E_i = 0.026 \ln\left(3.6 \times 10^{13}/10^{10}\right) = 0.213\,\text{eV}$

$$\boxed{F_n\left(x_p + 10 \times 10^{-4}\,\text{cm}\right) - E_i = 0.213\,\text{eV}}$$

Note also that $F_n\left(x_p + 10 \times 10^{-4}\,\text{cm}\right) - F_p\left(x_p + 10 \times 10^{-4}\,\text{cm}\right) = 0.632\,\text{eV}$ is less than the applied bias, so the QFL's are split by less than they are at $x_p$ .