Solutions to Exam 1

Electromechanical Motion Devices

ECE 321

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1 Problem 1

The H field can be written as

$$\vec{H} = 10\hat{a_x} + 20\hat{a_y} - 5\hat{a_z} \tag{1}$$

The vector connecting the point $[5, 2, 1]$ to $[4, 2, 4]$ is given by

$$\vec{l} = -1\hat{a_x} + 0\hat{a_y} + 3\hat{a_z} \tag{2}$$

The MMF drop is given by

$$F_{a,b} = \int_a^b \vec{H}.\vec{dl} \tag{3}$$

which in this case becomes $\vec{H}.\vec{l}$. Thus the MMF drop is

$$\vec{H}.\vec{l} = 10 \times -1 + 20 \times 0 + -5 \times 3 = -25 \;\; A - t \tag{4}$$

2 Problem 2

An iron core which is a toroid with a rectangular cross section is given. The following parameters are given.

- $g = 1mm = 1 \times 10^{-3}$ m
- $r = 10cm = 1 \times 10^{-1}$ m
- $w = 1cm = 1 \times 10^{-2}$ m
- $d = 2cm = 2 \times 10^{-2}$ m
- $N_1 = 100$ turns and $N_2 = 20$ turns
- $i_1 = 5A$

We are also given μ_B as a function of B.

$$\mu_B(B) = \mu_0 \frac{5000}{1 + |B|} \tag{5}$$

The reluctance $R = \frac{l}{\mu A}$. Then,

$$R_{mag} = \frac{2\pi r}{\mu_B wd} \qquad R_{airgap} = \frac{2g}{\mu_0 wd} \qquad R_{eq} = R_{mag} + R_{airgap} \tag{6}$$

The equations relating flux linkage to current are as follows:

$$\lambda_1 = L_{11}i_1 + L_{12}i_2 \qquad \lambda_2 = L_{12}i_1 + L_{22}i_2 \tag{7}$$

From the way the windings have been wound, we can see that the mutual inductance will be negative.

$$L_{11} = \frac{N_1^2}{R_{eq}} \qquad L_{22} = \frac{N_2^2}{R_{eq}} \qquad L_{12} = -\frac{N_1 N_2}{R_{eq}} \tag{8}$$

We know that $B = \frac{\phi}{wd}$ and $\phi = \frac{\lambda_2}{N_2}$.
So we can calculate B as λ_2 is given from which μ_B can be calculated and using the flux linkage equations, current can be calculated.

We get the answer to be $i_2 = 114.57A$ and $B = 1Wb/m^2$.

3 Problem 3

Given that the relative permeability of the material in the toroid is a constant and is equal to μ_r. Then, we get,

$$R_{mag} = \frac{2\pi r}{\mu_r \mu_0 wd} \qquad R_{airgap} = \frac{2x}{\mu_0 wd} \qquad R_{eq} = R_{mag} + R_{airgap} \tag{9}$$

Now,

$$\mathbf{i} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \qquad \mathbf{L} = \begin{bmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{bmatrix} \tag{10}$$

The co-energy W_c can be written as

$$W_c = i^t L i \tag{11}$$

The force is

$$force = \frac{\partial W_c}{\partial x} \tag{12}$$

$$R_{eq} = \frac{2\pi r}{\mu_r \mu_0 A} + \frac{2x}{\mu_0 A} \tag{13}$$

$$W_c = \frac{1}{2}\left[\frac{N_1^2 i_1^2}{\frac{2\pi r}{\mu_r \mu_0 A} + \frac{2x}{\mu_0 A}} + \frac{N_2^2 i_2^2}{\frac{2\pi r}{\mu_r \mu_0 A} + \frac{2x}{\mu_0 A}} - \frac{2N_1 N_2 i_1 i_2}{\frac{2\pi r}{\mu_r \mu_0 A} + \frac{2x}{\mu_0 A}}\right] \tag{14}$$

$$\frac{\partial W_c}{\partial x} = -\frac{\frac{1}{\mu_0 A}}{\frac{2\pi r}{\mu_r \mu_0 A} + \frac{2x}{\mu_0 A}}\left[\frac{N_1^2 i_1^2}{\frac{2\pi r}{\mu_r \mu_0 A} + \frac{2x}{\mu_0 A}} + \frac{N_2^2 i_2^2}{\frac{2\pi r}{\mu_r \mu_0 A} + \frac{2x}{\mu_0 A}} - \frac{2N_1 N_2 i_1 i_2}{\frac{2\pi r}{\mu_r \mu_0 A} + \frac{2x}{\mu_0 A}}\right] \tag{15}$$

4 Problem 4

Do not forget lower limits!

Step 1: $W_{c1} = \int_0^{i_{1f}} \lambda_1 di_1$ with $i_2 = 0$.

Step 2: $W_{c2} = \int_0^{i_{2f}} \lambda_2 di_2$ with $i_1 = i_{1f}$.

Step 3: $W_c = W_{c1} + W_{c2}$

The answer obtained is as follows.

$$W_c = i_1^2 + \frac{3}{2}i_2^2 + 5sin(4\theta_{rm}) \left[2i_1 + i_2 + e^{-i_2-2i_1} - 1\right] \quad (16)$$

5 Problem 5

From the L equation, we get the number of rotor teeth, RT = 4.
N = 5

$TP = \frac{2\pi}{RT}$ $SL = \frac{TP}{N}$

Thus, $SL = \frac{\pi}{10}$

Holding Torque = 0.032 Nm

6 Problem 6

a) False - Depends on the material.

b) False - Stored in the air gap.

c) False

d) False

e) True

f) True

g) True

h) True

i) False - They need to satisfy the conditions for a conservative field.

j) True