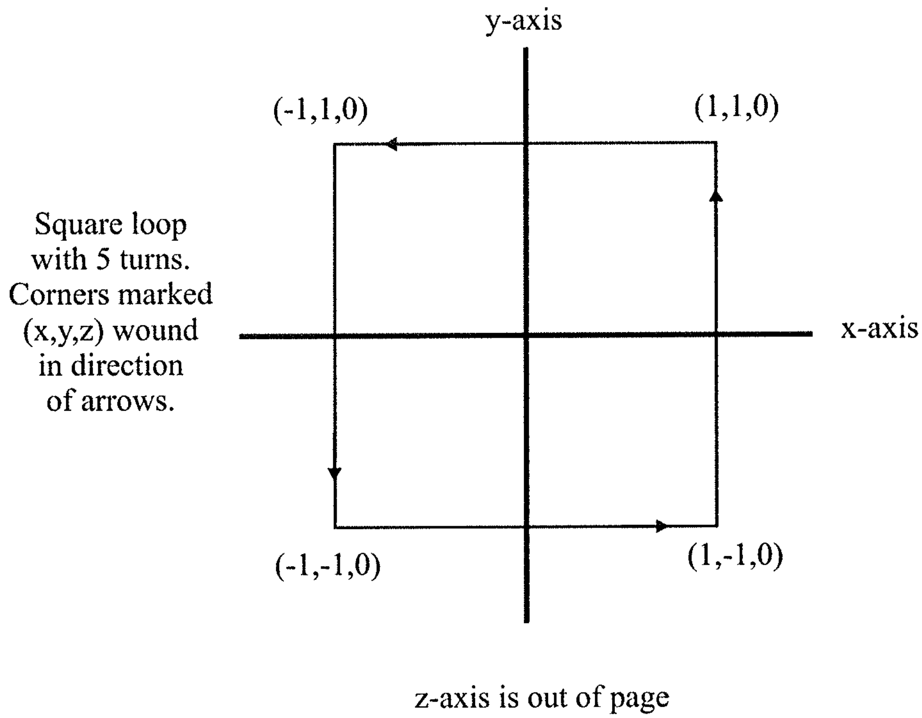
EE321 Exam 1
Spring 2010

Notes: **You must show work for credit.**
The last page of exam has a piece of extra paper if needed.

Good luck !

Consider a square loop of wire with 5 turns in a Cartesian coordinate system as shown below. The x-, y-, and z-components of the flux density are denoted B_x, B_y, and B_z respectively and will be given in units of Tesla.



1a) 10 pts. Suppose the flux density is constant in space and time with $B_x = 0.5$ T, $B_y = -0.5$ T, and $B_z = 1$ T. What is the flux, Φ, through the coil ? What is the flux linkage, λ, of the winding ?

$$\phi = \int B \cdot ds$$

$$= \iint B_z \, dx \, dy$$

$$= B_z A$$

$$= (1T)(4m^2) = 4 \text{ Wb}$$

$$\lambda = N\phi = 20 \text{ Vs}$$

1b) 8 pts. Suppose the flux density is constant in space but varies with time as $B_x = \sin(100t)$ T, $B_y = \sin(200t)$ T, and $B_z = \sin(300t)$ T. What is the voltage across at the terminals of the open-circuited coil at $t = 0.2s$.

$$\phi = B_z A = \sin(300t)\,(4m^2)$$

$$\lambda = N\phi = 20\sin(300t)$$

$$V = \frac{d\lambda}{dt} = \frac{d}{dt}(20\sin(300t))$$

$$= 6000\cos(300t)$$

$$V\big|_{t=0.2s} = 6000\cos(300\times 0.2)$$

$$= -5714\ V$$

1c) 7 pts. Suppose the B-field is constant in time but varies in space as function of the x-coordinate. In particular, $B_x = 0.25x^2$ T, $B_y = 0.5x^2$ T, and $B_z = 0.5x^2$ T. What is the flux linkage associated with the coil (i.e. λ).

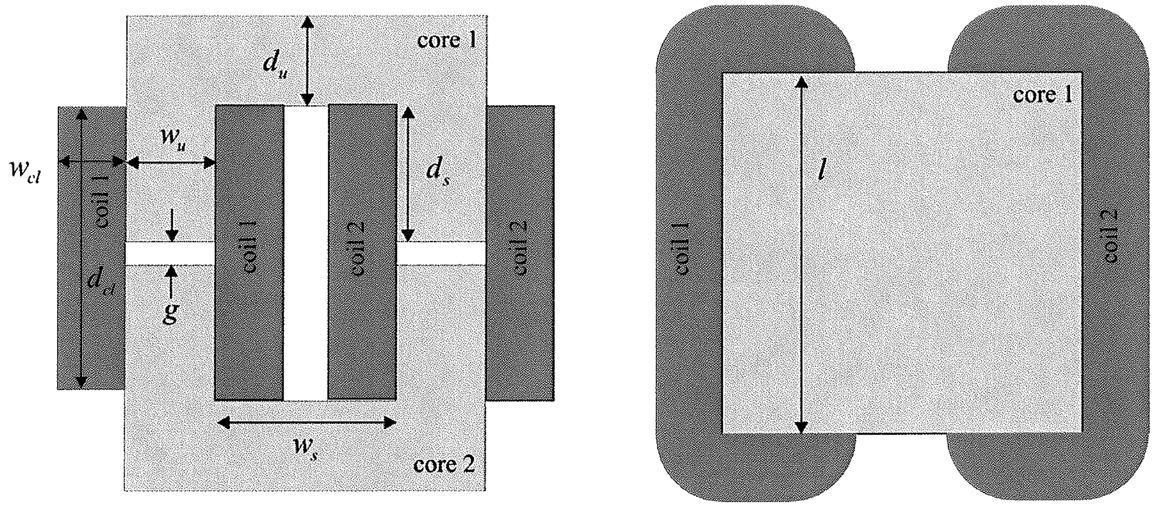
$$\phi = \int_{-1}^{1}\int_{-1}^{1} B_z\,dx\,dy$$

$$= \int_{-1}^{1} B_z\left(y\Big|_{-1}^{1}\right)dx$$

$$= \int_{-1}^{1} 2B_z\,dx = \int_{-1}^{1} x^2\,dx = \frac{x^3}{3}\Big|_{-1}^{1} = \frac{2}{3}\ Wb$$

$$\lambda = N\phi = \frac{10}{3}\ Vs$$

Consider the UU core inductor below. This inductor has two coils. Each coil is wound in a direction such that positive current will cause flux to flow in a clockwise direction. Each coil has N turns and has a packing factor of p_f. The permeability of air is denoted μ_0, the permeability of the magnetic material is $\mu_r \mu_0$, where μ_r is the relative permeability of the material. The conductivity of the conductor in the coil is σ.

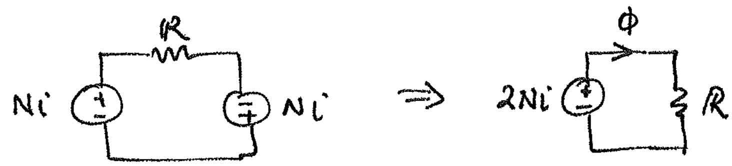


Side View   Top View

2a) 15 pts. Using MEC techniques, derive an expression for the inductance of the winding circuit consisting of the two coils connected in series in terms of N, μ_0, μ_r and the dimensions in the figure.

Current through coil 1&2 = i

MEC



$$R = \underbrace{\frac{2g}{\mu_0 w_u l}}_{\text{airgap reluctance}} + \underbrace{2\left\{ \underbrace{\frac{2d_s + d_u}{\mu_0 \mu_r w_u l}}_{\text{side leg}} + \underbrace{\frac{w_s + w_u}{\mu_0 \mu_r d_u l}}_{\text{horizontal part of U-core}} \right\}}_{\text{U-core reluctance}}$$

$L = \frac{\lambda}{i} = \frac{N\phi}{i}$ Here No of turns (total) - $2N$

$\phi = \frac{2Ni}{R}$

$\Rightarrow L = \frac{4N^2}{R}$

$$R = \frac{2}{\mu_0 \mu_r W_u d_u l}\left(\left(2d_s + d_u + \frac{\mu_r g}{2}\right) d_u + (W_s + W_u) W_u\right)$$

$$\Rightarrow \quad L = \frac{2 \mu_0 \mu_r W_u d_u l N^2}{\left(2d_s + d_u + \frac{\mu_r g}{2}\right) d_u + (W_s + W_u) W_u}$$

2b) 10 pts. Derive an expression for the resistance of the series connections of the two coils in terms of p_f, σ, N and the dimensions in the figure.

Resistance of one coil $R = \frac{l}{a_w \sigma} = \frac{V}{a_w^2 \sigma}$

$$V = (2 W_{cl} d_{cl} l + 2 W_{cl} d_{cl} W_u + \pi W_{cl}^2 d_{cl}) pf$$

$$= W_{cl} d_{cl} (2l + 2W_u + \pi W_{cl}) pf$$

$$a_w = \frac{W_{cl} d_{cl} pf}{N}$$

Total resistance $R_t = 2R = \frac{2 W_{cl} d_{cl} (2l + 2W_u + \pi W_{cl}) pf N^2}{W_{cl}^2 d_{cl}^2 pf^2 \sigma}$

$$R_t = \frac{2 (2l + 2W_u + \pi W_{cl}) N^2}{W_{cl} d_{cl} pf \sigma}$$

3.) 25 pts. Find an expression for force in terms of x and λ_1 and λ_2 for the system with the following current equations.

$$i_1 = 5\lambda_1 + \frac{10}{1+x}(\lambda_1 + \lambda_2)^3$$

$$i_2 = 8\lambda_2 + \frac{10}{1+x}(\lambda_1 + \lambda_2)^3$$

$\lambda_1 = 0 \rightarrow \lambda_1 \quad , \lambda_2 = 0$

$$W_{f1} = \int i_1 \, d\lambda_1 + \int i_2 \, \cancel{d\lambda_2}^{\,0}$$

$$= \int_0^{\lambda_1} i_1 \, d\xi = \int_0^{\lambda_1} 5\xi + \frac{10}{1+x}\xi^3 \, d\xi$$

ξ - dummy variable

$$= \frac{5}{2}\xi^2 + \frac{10}{4}\frac{1}{1+x}\xi^4 \Big|_0^{\lambda_1} = \frac{5}{2}\lambda_1^2 + \frac{10\lambda_1^4}{4(1+x)}$$

λ_1 - constant, $\lambda_2 = 0 \rightarrow \lambda_2$

$$W_{f2} = \int i_1 \, \cancel{d\lambda_1}^{\,0} + \int i_2 \, d\lambda_2$$

$$= \int_0^{\lambda_2} 8\xi + \frac{10}{1+x}(\xi + \lambda_1)^3 \, d\xi$$

$$= 4\xi^2 + \frac{10}{4(1+x)}(\xi + \lambda_1)^4 \Big|_0^{\lambda_2}$$

$$= \cancel{4\xi} \; 4\lambda^2 + \frac{10}{4(1+x)}\left\{(\lambda_1 + \lambda_2)^4 - \lambda_1^4\right\}$$

$$W_f = W_{f1} + W_{f2} = \frac{5}{2}\lambda_1^2 + 4\lambda_2^2 + \frac{10}{4(1+x)}(\lambda_1 + \lambda_2)^4$$

$$f_e = -\frac{\partial W_f}{\partial x} = \frac{10}{4(1+x)^2}(\lambda_1 + \lambda_2)^4$$

4.) 25 pts. The flux linkage equations for a certain 2-phase induction machine are shown below. At a given instant of time $\theta_{rm} = \pi/2$, $i_{as} = 1$, $i_{bs} = 0$, $i_{ar} = 2$, and $i_{br} = -2$. What is the electromagnetic torque at that instant of time.

$$\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{ar} \\ \lambda_{br} \end{bmatrix} = \begin{bmatrix} 4 & 0 & 3\cos\theta_{rm} & 3\sin\theta_{rm} \\ 0 & 4 & -3\sin\theta_{rm} & 3\cos\theta_{rm} \\ 3\cos\theta_{rm} & -3\sin\theta_{rm} & 5 & 0 \\ 3\sin\theta_{rm} & 3\cos\theta_{rm} & 0 & 5 \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{ar} \\ i_{br} \end{bmatrix}$$

For linear magnetic systems $\lambda = \bar{\bar{L}} i$

$W_c = \frac{1}{2} i^T \bar{\bar{L}} i$

$\bar{\bar{L}}$ - inductance Matrix

$$T_e = \frac{\partial W_c}{\partial \theta_{rm}} = \frac{1}{2} i^T \frac{\partial \bar{\bar{L}}}{\partial \theta_{rm}} i$$

$$= \frac{1}{2} \begin{bmatrix} 1 & 0 & 2 & -2 \end{bmatrix} \begin{bmatrix} 0 & 0 & -3\sin\theta_{rm} & 3\cos\theta_{rm} \\ 0 & 0 & -3\cos\theta_{rm} & -3\sin\theta_{rm} \\ -3\sin\theta_{rm} & -3\cos\theta_{rm} & 0 & 0 \\ 3\cos\theta_{rm} & -3\sin\theta_{rm} & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 2 \\ -2 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 1 & 0 & 2 & -2 \end{bmatrix} \begin{bmatrix} 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & -3 \\ -3 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 2 \\ -2 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 1 & 0 & 2 & -2 \end{bmatrix} \begin{bmatrix} -6 \\ 6 \\ -3 \\ 0 \end{bmatrix}$$

$$= \frac{1}{2} [-6-6] = -6 \text{ Nm}$$