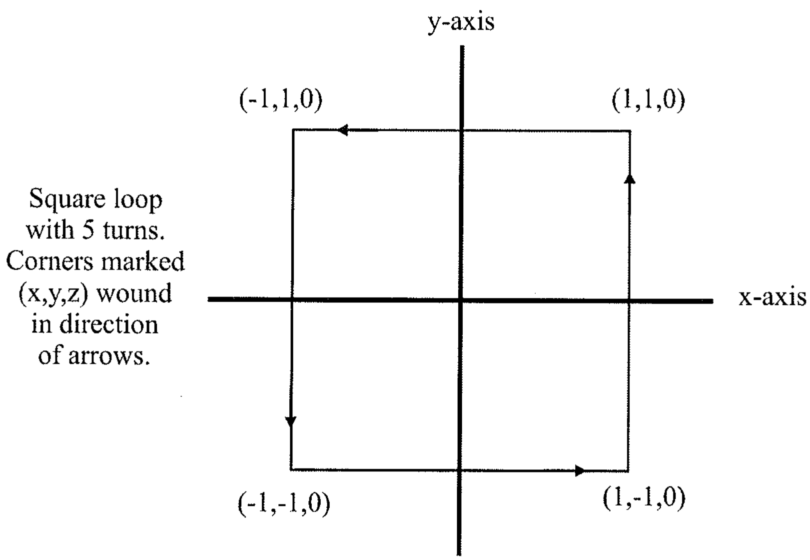
EE321 Exam 1
Spring 2011

Notes: **You must show work for credit on Problems 1-3.**
The last page of exam has a piece of extra paper if needed.

Good luck!

SOLUTIONS

1) 20 pts. Consider a square loop of wire with 5 turns in a Cartesian coordinate system as shown below. The z-component of the flux density is denoted B_z. The open circuit voltage associated with the coil is given by $v = 20000\cos(1000t)$. Given that the flux density is spatially uniform, determine an expression for B_z as a function of time. You may assume B_z has no dc component.



z-axis is out of page

$v = \frac{d\lambda}{dt} \Rightarrow \lambda = \int v\,dt = \int 20000 \cos 1000t \, dt$

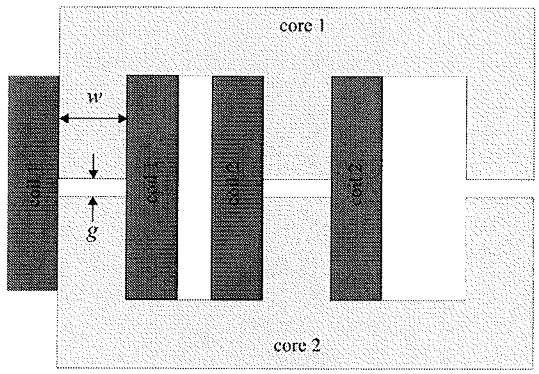
$\lambda = 20 \sin 1000t + c$

No dc component $\Rightarrow c = 0$

$\lambda = N\phi = NBA$

$\Rightarrow B = \frac{\lambda}{NA} = \frac{20 \sin 1000t}{5 \times 4} = 1 \sin 1000t \quad T$

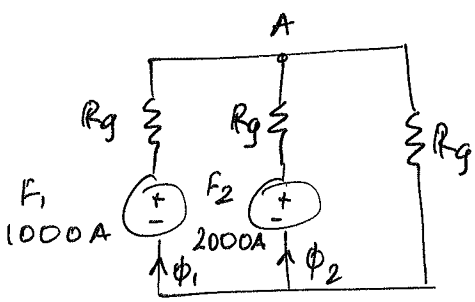
2) 21 pts. Consider the magnetic system below. The permeability of the cores is infinite, and you may neglect fringing and leakage flux. The air gap $g = 1$ mm, $w = 2$ cm, and the depth into the page is 10 cm. Coil 1 has 100 turns (with a direction such that positive current tends to cause flux to go up in the leftmost leg) and a current of 10 A. Coil 2 has 100 turns (with a direction such that positive current tends to cause positive flux go up in the center leg) and a current of 20 A. What is the total flux linkage for winding 2 (i.e. λ_2)?



Side View

$R_{iron} = 0$ $\qquad R_g = \dfrac{10^{-3}}{\mu_0\, 20\times10^{-4}} = 397.89\times10^3$

MEC



Using KCL at node A

$$\frac{F_1 - F_A}{R_g} + \frac{F_2 - F_A}{R_g} - \frac{F_A}{R_g} = 0$$

$$F_A = \frac{F_1/R_g + F_2/R_g}{3/R_g} = \frac{1000+2000}{3} = 1000A$$

$$\Rightarrow \quad \phi_2 = \frac{F_2 - F_A}{R_g} = \frac{1000}{397.89\times10^3} = 2.513\ mWb$$

$$\lambda_2 = N_2 \phi_2 = 0.251\ Vs$$

3.) 21 pts. Consider an electromechanical device with the flux linkage equations given below. Suppose $i_1 = 4$ A, $i_2 = -2$ A, $\theta_{rm} = 1$ (rad), and the units of the coefficients are such that when the currents are in A, and the position is in radians the flux linkages are in Vs, as normal. What is the numerical value for the electromagnetic torque in Nm.

$$\lambda_1 = 4i_1 + (2 + \sin 4\theta_{rm})(i_1 + i_2)^{\frac{1}{3}}$$

$$\lambda_2 = 4i_2 + (2 + \sin 4\theta_{rm})(i_1 + i_2)^{\frac{1}{3}}$$

$$W_{c1} = \int_0^{i_{1f}} \lambda_1 \, di_1 + \int_0^{0} \lambda_2 \, di_2$$

$$= \frac{4 i_{1f}^2}{2} + \frac{3}{4}(2 + \sin 4\theta_{rm}) \, i_1^{4/3} \Big|_0^{i_{1f}}$$

$$W_{c1} = 2 i_1^2 + \frac{3}{4}(2 + \sin 4\theta_{rm}) \, i_1^{4/3}$$ replace i_{1f} by i_1

$$W_{c2} = \int_{i_{1f}}^{i_{1f}} \lambda_1 \, di_1 + \int_0^{i_{2f}} \lambda_2 \, di_2 = \frac{4 i_{2f}^2}{2} + \frac{3}{4}(2 + \sin 4\theta_{rm})(i_{1f} + i_2) \Big|_0^{i_{2f}}$$

$$= 2 i_2^2 + \frac{3}{4}(2 + \sin 4\theta_{rm}) \left((i_1 + i_2)^{4/3} - i_1^{4/3} \right)$$

$$W_c = W_{c1} + W_{c2} = 2(i_1^2 + i_2^2) + \frac{3}{4}(2 + \sin 4\theta_{rm})(i_1 + i_2)^{4/3}$$

$$\frac{\partial W_c}{\partial \theta_{rm}} = 3(i_1 + i_2)^{4/3} \cos 4\theta_{rm} \quad \text{Nm}$$

$$T_e \Big|_{\substack{i_1 = 4A \\ i_2 = -2A \\ \theta_{rm} = 1 rad}} = 3(4 - 2)^{4/3} \cos 4 = -4.941 \text{ Nm}$$

4.) 4 pts. Which of the following systems is magnetically linear ? (You may circle more than one)

Linear

System 1: $\lambda_1 = i_1 + i_2$, $\lambda_2 = i_2 + i_1$

System 2: $\lambda_1 = \dfrac{5i_1 + 3i_2}{10+x}$, $\lambda_2 = \dfrac{3i_1 + 7i_2}{10+x}$

System 3: $\lambda_1 = \dfrac{5i_1^2 + 3i_2}{10+x}$, $\lambda_2 = \dfrac{3i_1 + 7i_2^2}{10+x}$

System 4: $\lambda_1 = i_1 + i_2 + 2\sin\theta_{rm}$, $\lambda_2 = i_2 + i_1$

5.) 5 pts. In electromagnetic or electromechanical device design, what is the relationship between mass and loss?

$$\text{Mass} \propto \frac{1}{\text{Loss}}$$

6.) 5 pts. The a-phase of a VR stepper motor has a flux linkage equation given by

$$\lambda_{as} = (5 + 3\cos(4\theta_{rm}))i_{as}$$

The b-phase has a maximum inductance at $\theta_{rm} = \pi/8$. Write the b-phase flux linkage equation.

$$\lambda_{bs} = \left(5 + 3\cos 4(\theta_{rm} - \pi/8)\right) i_{bs} \quad \text{Vs}$$

7.) 5 pts. Consider a VR stepper motor with 5 phases and 8 rotor teeth. What is the step length.

$$SL = \frac{2\pi}{5\times 8} = \frac{\pi}{20} \text{ rad} \quad \text{or} \quad \frac{180}{20} = 9^\circ$$

8.) 5 pts. Under what condition is the following statement not true: $J\dfrac{d\omega_{rm}}{dt} = T_e - T_l$?

if J is ~~time~~ varying

9.) 4 pts. What is an advantage of ferromagnetic materials over ferrimagnetic materials. What is an advantage of ferrimagnetic materials over ferromagnetic materials. If you could be a ferromagnetic material or a ferrimagnetic material which would you be and why (you can skip this last part if desired).

Ferromagnetic mtrls have higher saturation flux densities than ferri

Ferrimagnetic mtrls are non conductive relative to ferro

10.) 5 pts. Which of the following hold in a material with hysteresis ? (circle all that apply) Gauss's Law; Ampere's Law; Faraday's Law; Ohm's Property for Magnetic Systems; Lossless field

11.) 5 pts. Why do magnetic materials saturate?

After all domains are aligned with applied field, the material cannot respond to further increase in fields.