ECE321/ECE595 Exam 1
Spring 2012

Notes: **You must show work for credit.**
This exam has 5 problems and 12 pages.

Note that problems 1, 3, and 5 have different specifications depending on if you are in ECE321 or ECE595

Good luck!

1) 15 pts. Consider a H-field given by either

$$\mathbf{H} = 10\mathbf{a}_x + 5\mathbf{a}_y \quad \text{[ECE321 Students Use This]}$$

$$\mathbf{H} = 10x\mathbf{a}_x + 5y^2\mathbf{a}_y \quad \text{[ECE595 Students Use This]}$$

where $\mathbf{a}_x$ and $\mathbf{a}_y$ are unit vectors in the x- and y- directions, respectively. What is the MMF drop between the points (x=1,y=2) and (x=4,y=3).

$$F = \int H \cdot dl$$

$$= \int_1^4 H|_{y=2}\,dx + \int_2^3 H|_{x=4}\,dy$$

$$= \int_1^4 10\,dx + \int_2^3 5\,dy$$

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$$= 30 + 5 = 35$$

$$= \int_1^4 10x\,dx + \int_2^3 5y^2\,dy$$

$$= 5x^2\Big|_1^4 + \frac{5}{3}y^3\Big|_2^3$$

$$= 5(16-1) + \frac{5}{3}(27-8)$$

$$= \frac{320}{3}$$

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2) 20 pts. Consider a toroid with a rectangular cross section of 1 cm by 1 cm. The mean radius is 8 cm. The toroid has 100 turns of wire. **[If you are in ECE595, there is also a 0.5 mm gap in the core. If you are in ECE321, there is no gap]**. The magnetic material has a permeability of

$$\mu_B(B) = \mu_0 \frac{1000}{B+1}$$

where $\mu_0 = 4\pi 10^{-7}$ H/m. Ignoring fringing and leakage, if the flux linkage λ is 3 mVs, what is the current ?

$$\lambda = 0.003$$

$$\Phi = \frac{\lambda}{N} = 3 \times 10^{-5}$$

$$B = \frac{\Phi}{A} = \frac{\Phi}{(10^{-2})(10^{-2})} = 0.3$$

$$\mu = 769.2\,\mu_0 = 9.666 \times 10^{-4}$$

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$$R = \frac{2\pi r}{A\mu} = 5.2 \times 10^{6}\ H^{-1}$$

$$F = R\Phi = 156$$

$$i = \frac{F}{N} = 1.56\,A$$

595

$$R = \underbrace{\frac{2\pi r - 0.5\times 10^{-3}}{A\,\mu}}_{5.19\times 10^{6}} + \underbrace{\frac{0.5\times 10^{-3}}{A\,\mu_0}}_{3.98\times 10^{6}} = 9.17\times 10^{6}$$

$$F = R\Phi = 275\,A$$

$$i = \frac{F}{N} = 2.75\,A$$

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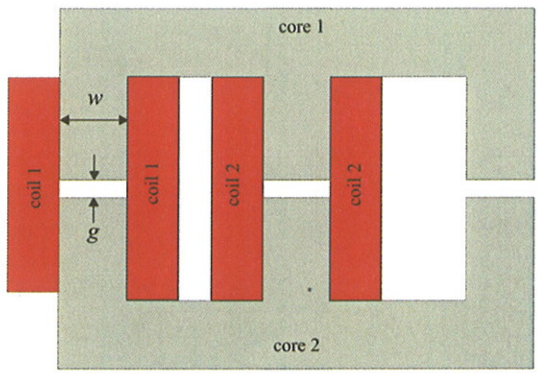
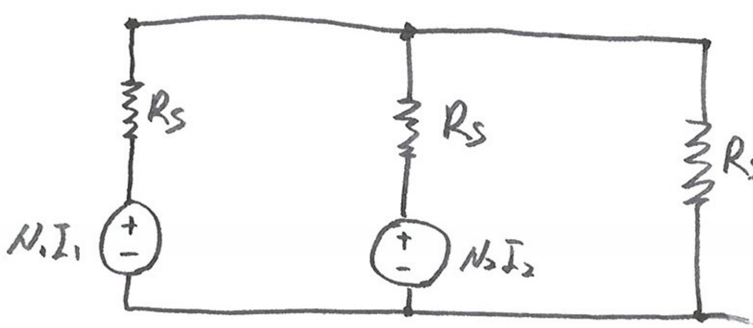
3) 20 pts. Consider the magnetic system below. The permeability of the cores is infinite, and you may neglect fringing and leakage flux. The air gap $g = 1$ mm, $w = 2$ cm, and the depth into the page is 5 cm. Winding 1 has 50 turns, wound so as to cause flux to travel upward in the diagram; winding 2 has 25 turns, also wound so as to cause flux to travel upwards. What is the self-inductance of winding 1 ? What is the mutual inductance between winding 1 and winding 2 ? Note this problem is magnetically linear so there is not distinction between incremental and absolute inductance.



Side View



$$R_S = \frac{g}{w d \mu_0} = 7.96 \times 10^5$$

$$\lambda_1 = N_1 \left[\frac{N_1 I_1}{\frac{3}{2} R_S} - \frac{N_2 I_2}{\frac{3}{2} R_S} \frac{1}{2} \right]$$

$$= \underbrace{\frac{2}{3} \frac{N_1^2}{R_S}}_{L_{11}} I_1 - \underbrace{\frac{N_1 N_2}{3 R_S}}_{L_{12}} I_2$$

$L_{11} = 2.09$ mH $\qquad L_{12} = -0.524$ mH

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4) 25 pts. Consider an electromechanical device with the flux linkage equations given below. Compute an expression for force f_e in terms of i_1, i_2, and x. The currents are restricted to positive values.

$$\lambda_1 = 4i_1 + \left(\frac{1}{1+x^2}\right)\left(1-e^{-(i_1+2i_2)}\right) \qquad i_1 + 2i_2 \geq 0$$

$$\lambda_2 = 10i_2 + \left(\frac{2}{1+x^2}\right)\left(1-e^{-(i_1+2i_2)}\right) \qquad i_1 + 2i_2 \geq 0$$

$$W_{C1} = \int_0^{i_1} \lambda_1 \big|_{i_2=0} \, di_1$$

$$= \int_0^{i_1} 4i_1 + \left(\frac{1}{1+x^2}\right)\left(1-e^{-i_1}\right) di_1$$

$$= 2i_1^2 + \left(\frac{1}{1+x^2}\right)\left(i_1 + e^{-i_1}\right)\Big|_0^{i_1}$$

$$= 2i_1^2 + \left(\frac{1}{1+x^2}\right)\left(i_1 + e^{-i} - 1\right)$$

$$W_{C2} = \int_0^{i_2} \lambda_2 \big|_{i_1 \text{ fixed}} \, di_2$$

$$= 5i_2^2 + \left(\frac{2}{1+x^2}\right)\left(i_2 + \frac{1}{2}e^{-(i_1+2i_2)}\right)\Big|_0^{i_2}$$

$$= 5i_2^2 + \left(\frac{2}{1+x^2}\right)\left(i_2 + \frac{1}{2}e^{-(i_1+2i_2)} - \frac{1}{2}e^{-i_1}\right)$$

$$\therefore W_C = 2i_1^2 + 5i_2^2 + \left(\frac{1}{1+x^2}\right)\left(i_1 + 2i_2 + e^{-(i_1+2i_2)} - 1\right)$$

$$f_e = \frac{\partial W_C}{\partial x} = -\frac{2x}{(1+x^2)^2}\left(i_1 + 2i_2 + e^{-(i_1+2i_2)} - 1\right)$$

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5) 20 pts. Consider a 3-phase variable reluctance stepper motor. The a-phase flux linkage equation may be expressed

$$\lambda_{as} = \frac{5 + 4\cos(4\theta_{rm})}{1000} i_{as}$$

Following a step, the a-phase current is given by

$$i_{as} = 2\left(1 - e^{-1000t}\right)$$

and the rotor position

$$\theta_{rm} = -0.1e^{-1000t}$$

where t is time. The winding resistance is 1 Ohm.

(a) How many rotor teeth does the machine have ?
(b) What is L_B ?
(c) What is the step length in radians ?
(d) At the instant t=1 ms, what is the electromagnetic torque?
(e) At the instant t=1 ms, what is a-phase voltage ? [ECE595 Students Only].

a) $RT = 4$

b) $L_B = 4\,mH$

c) $SL = \frac{2\pi}{RT \cdot N} = \frac{\pi}{6}$

d) $T_e = -\frac{1}{2} RT \cdot L_B \cdot \sin(4\theta_{rm})\, i_{as}^2$

$t = 1e{-3}$ s

$I_{as} = 1.26$ A

$\theta_{rm} = -0.0368$ (rad)

$\therefore T_e = 1.87 \times 10^{-3}$ Nm

e) $V_{as} = r_s i_{as} + \frac{d\lambda_{as}}{dt}$

$$\frac{d\lambda_{as}}{dt} = L_{as}\frac{di_{as}}{dt} + i_{as}\frac{\partial L}{\partial \theta_m}\underbrace{\omega_{rm}}_{\frac{\partial \theta_m}{\partial t}}.$$

$L_{as}|_t = 8.96\ mH$

$p\,i_{as}|_t = 735.8\ A/s$

$$\frac{\partial L}{\partial \theta_m}\Big|_t = \frac{-16 \sin(4\theta_m)}{1000} = -2.346\ \frac{mH}{rad}$$

$\omega_{rm} = 36.78$

$$\frac{d\lambda_{as}}{dt} = 6.69\ V$$

$$V_{as} = r\,i_{as} + \frac{d\lambda_{as}}{dt} = 7.96\ V$$

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