Solution

ECE321/ECE595 Exam 1
Spring 2014

Notes: You must show work for credit.

This exam has 5 problems and 12 pages.

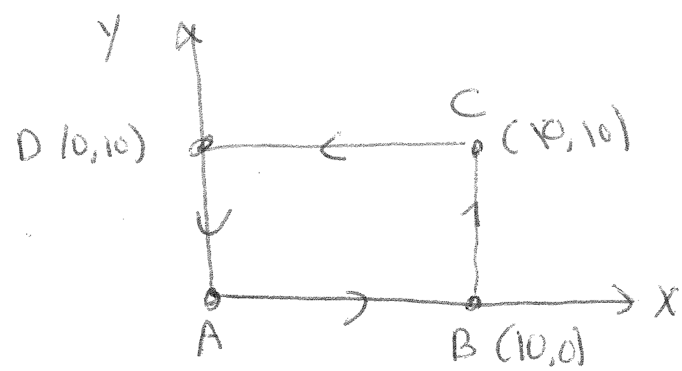
Note that problems 2 and 4 have different specifications depending on if you are in ECE321 or ECE595

Good luck!

1) 22 pts. Consider an H-field given by

$$\mathbf{H} = 10x\mathbf{a}_x + 10xy\mathbf{a}_y$$

where $\mathbf{a}_x$ and $\mathbf{a}_y$ are unit vectors in the x and y directions, respectively. The x-axis is directed towards the right of the page, the y-axis is directed towards the top of this page, and the z-axis is point out of the page towards you. Consider the path from point A (x=0,y=0) to point B (x=10,y=0) to point C (x=10,y=10) to point D (x=0, y=10) and back to point A. How much current is enclosed by the path, and what is its direction.



$$\oint H \cdot dl = I_{enc}$$

$$\oint H \cdot dl = \int_A^B H \cdot dl + \int_B^C H \cdot dl + \int_C^D H \cdot dl + \int_D^A H \cdot dl$$

$$\int_A^B H \cdot dl = \int_0^{10} 10x\, dx = 5x^2\Big|_0^{10} = 500$$

$$\int_B^C H \cdot dl = \int_0^{10} 10\underset{\substack{\uparrow \\ x=10}}{x}y\, dy = 5\underset{\substack{\uparrow \\ 10}}{x}y^2\Big|_0^{10} = 5000$$

$$\int_C^D H \cdot dl = \int_{10}^{0} 10x\, dx = 5x^2\Big|_{10}^{0} = -500$$

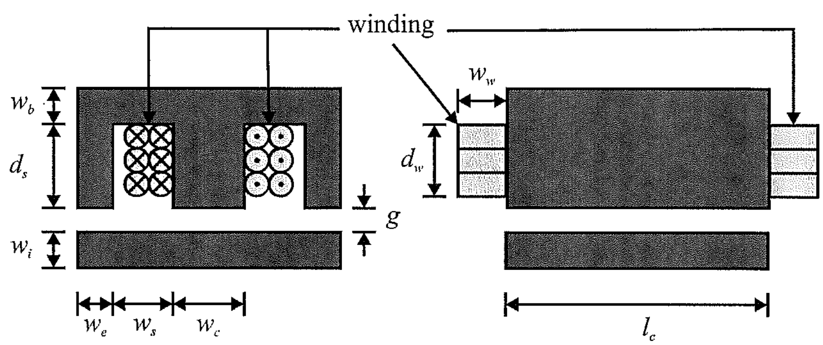
$$\int_D^A H \cdot dl = \int_{10}^{0} 10\underset{\substack{\uparrow \\ x=0}}{x}y\, dy = 0$$

$$\therefore \oint H \cdot dl = 500 + 5000 - 500 = 5000\text{ A} = I_{enc}$$

This is out of page since our path is CCW.

2.) 22 pts. Consider the EI core below. Assuming the permeability of the magnetic material is infinite, and neglecting leakage and fringing flux as we did in class, derive an expression for the inductance of the coil in terms of the parameters in the figure (you'll not need all of them) and the number of turns N. Note that the winding does not completely fill the slot.

ECE595 Only. In addition to the above, derive an expression for the resistance of the coil in terms of the parameters listed in the figure (you'll not need all of them), the number of turns N, and the packing factor k_{pf}.



(a) cross section (b) side view

$$R_{eff} = R_{g1} + \tfrac{1}{2} R_{g2}$$

Now $\quad R_{g1} = \dfrac{g}{w_c l_c \mu_0} \qquad R_{g2} = \dfrac{g}{w_e l_c \mu_0}$

so $\quad R_{eff} = \dfrac{g}{l_c \mu_0}\left[\dfrac{1}{w_c} + \dfrac{1}{2w_e}\right] = \dfrac{g}{l_c \mu_0}\left[\dfrac{2w_e + w_c}{2w_e w_c}\right]$

$$L = \frac{N^2}{R_{eff}} = \frac{N^2 l_c \mu_0 \, 2 w_e w_c}{g\,(2w_e + w_c)}$$

Resistance on next page

$$V_{cd} = \left[W_w d_w (2l_c + 2W_c) + d_w \pi W_w^2\right] K_{pf}$$

$$= W_w d_w \left[2l_c + 2W_c + \pi W_w\right] K_{pf}$$

$$l_{cd} = \frac{V_{cd}}{a_c}$$

$$N a_c = W_w d_w K_{pf}$$

$$\therefore \quad a_c = \frac{W_w d_w K_{pf}}{N}$$

$$R = \frac{l_{cd}}{a_{cd} G} = \frac{V_{cd}}{a_{cd}^2 G}$$

$$= \frac{W_w d_w \left[2l_c + 2W_c + \pi W_w\right] K_{pf}}{\left[\frac{W_w d_w K_{pf}}{N}\right]^2 G}$$

$$= \frac{N^2 \left[2l_c + 2W_c + \pi W_w\right]}{W_w d_w K_{pf} G}$$

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3) 24 pts. Suppose

$$\lambda_1 = \frac{10i_1 + 10\left(1 - e^{-(i_1+2i_2)}\right)}{4+x}$$

$$\lambda_2 = \frac{4i_2 + 20\left(1 - e^{-(i_1+2i_2)}\right)}{4+x}$$

The currents are positive. Is this system magnetically linear? Show that the system is conservative. Derive an expression for force in terms of i_1, i_2, and x.

① The system is not magnetically linear.

② $$\frac{\partial \lambda_1}{\partial i_2} = \frac{20\, e^{-(i_1+2i_2)}}{4+x}$$

$$\frac{\partial \lambda_2}{\partial i_1} = \frac{20\, e^{-(i_1+2i_2)}}{4+x}$$

Since $\dfrac{\partial \lambda_1}{\partial i_2} = \dfrac{\partial \lambda_2}{\partial i_1}$ the system is conservative.

③ Let's find the co-energy

Bringing up i_1

$$W_c = \int_0^{i_1} \lambda_1\, di_1 + \int_0^{0} \lambda_2\, di_2$$

$$= \int_0^{i_1} \frac{10i_1 + 10(1 - e^{-i_1})}{4+x}\, di_1$$

$$= \left.\frac{5i_1^2 + 10i_1 + 10e^{-i_1}}{4+x}\right|_0^{i_1}$$

$$= \frac{5i_1^2 + 10i_1 + 10(e^{-i_1} - 1)}{4+x}$$

Let's bring up i_2

$$W_{c2} = \cancel{\int_{i_1}^{i_1} \lambda_1 \, di_1}^{\,0} + \int_0^{i_2} \lambda_2 \, di_2$$

$$= \int_0^{i_2} \frac{4i_2 + 20(1 - e^{-(i_1+2i_2)})}{4+x} \, di_2$$

$$= \left. \frac{\cancel{\$} \; 2i_2^2 + 20i_2 + 10e^{-(i_1+2i_2)}}{4+x} \right|_0^{i_2}$$

$$= \frac{2i_2^2 + 20i_2 + 10\left(e^{-(i_1+2i_2)} - e^{-i_1}\right)}{4+x}$$

Adding the two terms together

$$W_c = \frac{5i_1^2 + 10i_1 + 2i_2^2 + 20i_2 + 10\left(e^{-(i_1+2i_2)} - 1\right)}{4+x}$$

We can now find force

$$f_e = \frac{\partial W_c}{\partial x} = - \frac{5i_1^2 + 10i_1 + 2i_2^2 + 20i_2 + 10\left(e^{-(i_1+2i_2)} - 1\right)}{(4+x)^2}$$

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4) 22 pts. Perform the requested derivation.

ECE321: Prove that for a linear magnetic system wherein

$$\lambda_1 = L_{11}i_1 + L_{12}i_2$$
$$\lambda_2 = L_{12}i_1 + L_{22}i_2$$

and where the inductances are not functions of current that

$$W_c = \frac{1}{2}\begin{bmatrix} i_1 & i_2 \end{bmatrix}\begin{bmatrix} L_{11} & L_{12} \\ L_{12} & L_{11} \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

This should have been L_{22}

Hint: You may want to expand/simplify the above expression for W_c as part of your derivation.

ECE595: Prove that for a linear magnetic system of the form

$$\boldsymbol{\lambda} = \mathbf{L}\mathbf{i}$$
$$\mathbf{L} = \mathbf{L}^T$$

where $\boldsymbol{\lambda}$ and $\mathbf{i}$ are vectors, and $\mathbf{L}$ is a matrix of suitable dimension and is not a function of current or flux linkage, that

$$W_c = \frac{1}{2}\mathbf{i}^T\mathbf{L}\mathbf{i}$$

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Bring up i_1 with $i_2 = 0$

$$W_{c1} = \int_0^{i_1} \lambda_1 di_1 + \cancel{\int_0^0 \lambda_2 di_2} = \int_0^{i_1} L_{11} i_1 = \frac{1}{2}L_{11}i_1^2$$

Bring up i_2 with i_1 fixed

$$W_{c2} = \cancel{\int_{i_1}^{i_1} \lambda_1 di_1} + \int_0^{i_2} \lambda_2 di_2 = \int_0^{i_2} L_{12} i_1 + L_{22} i_2 \, di_2 = L_{12}i_1i_2 + \frac{1}{2}L_{22}i_2^2$$

$$\therefore W_c = \frac{1}{2}L_{11}i_1^2 + L_{12}i_1i_2 + \frac{1}{2}L_{22}i_2^2 \quad (1)$$

Now

$$W_c = \frac{1}{2}\begin{bmatrix} i_1 & i_2 \end{bmatrix}\begin{bmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} i_1 & i_2 \end{bmatrix}\begin{bmatrix} L_{11}i_1 + L_{12}i_2 \\ L_{12}i_1 + L_{22}i_2 \end{bmatrix}$$

$$= \frac{1}{2}\left[L_{11}i_1^2 + L_{12}i_2i_1 + L_{12}i_1i_2 + L_{22}i_2^2\right]$$

$$= \frac{1}{2}L_{11}i_1^2 + L_{12}i_1i_2 + L_{22}i_2^2 \quad (2)$$

Clearly, (1) = (2) and so the desired result is obtained

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let $i = \alpha i_f$ where i_f is a vector of final currents

$$W_c = \int \lambda_1 di_1 + \int \lambda_2 di_2 + \cdots$$

Now

$$di_1 = i_{f1}\, d\alpha \quad di_2 = i_{f2}\, d\alpha \quad \cdots$$

So

$$W_c = \int_0^1 \lambda_1 i_{f1}\, d\alpha + \int_0^1 \lambda_2 i_{f2}\, d\alpha + \cdots$$

$$= \int_0^1 \lambda_1 i_{f1} + \lambda_2 i_{f2} + \cdots \; d\alpha$$

$$= \int_0^1 \lambda^T i_f \, d\alpha$$

Next

$$\lambda = L i = L i_f \alpha$$

So

$$W_c = \int_0^1 (L i_f \alpha)^T i_f \, d\alpha$$

$$= \int_0^1 i_f^T L^T i_f \, \alpha \, d\alpha = i_f^T L^T i_f \int_0^1 \alpha \, d\alpha$$

$$= i_f^T L^T i_f \tfrac{1}{2}\alpha^2 \Big|_0^1 = \tfrac{1}{2} i_f^T L^T i_f$$

Since $L^T = L$ and i_f can be any current

$$W_c = \tfrac{1}{2} i^T L i$$

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5a) 1 pt. Name an advantage of ferromagnetic material over ferrimagnetic material.

It has a higher saturation flux density. (Alternate: higher permeability)

5b) 1 pt. Why do ferromagnetic materials saturate?

All domains align.

5c) 1 pt. Why is it societally important that inductance can change in time (for example, as position varies)?

So we can have electromechanical energy conversion (motors/generators)

5d) 1 pt. Why are transformers societally important?

Enables long distance power transmission

5e) 1 pt. Which of the following is more of a suggestion than a law: Ampere's Law, Faraday's Law, Ohm's Law.

5f) 1 pt. What effect which we studied is not consistent with a conservative magnetic field.

Hysteresis

5g) 2 pt. Why can we derive an expression for field energy without knowing anything about force?

Because the system is conservative we can find the field energy using any trajectory of state variables. We choose a trajectory where fe is zero when we position the mechanical system thus preventing an energy contribution through Wm.

5h) 1 pt. For a linear magnetic system with one electrical input, does

$$\frac{1}{2}\mathbf{i}^T\mathbf{L}\mathbf{i} = \frac{1}{2}\mathbf{L}\mathbf{i}^2$$

Yes!

5h) 1 pt. For a linear magnetic system with more than one electrical input, does

$$\frac{1}{2}\mathbf{i}^T\mathbf{L}\mathbf{i} = \frac{1}{2}\mathbf{L}\mathbf{i}^2$$

No!

(here **i** is the vector of currents)