EE321 Exam 2
Spring 2010

Notes: **You must show work for credit.**
The last page of exam is blank for extra paper if needed.

Good luck, and have a safe and relaxing spring break!

1) 34 pts. Measuring the a-phase impedance looking into a stepper motor, it is found that the inductance varies between 4 mH and 20 mH, and has 8 maxima as it is rotated over one revolution, and that the resistive part of the impedance is constant at 4 Ω. If the machine is fed from a drive with a 12 V dc source, and neglecting forward semiconductor voltage drops, what is the maximum load torque that can be applied if the machine is to step forward correctly using single step excitation? What is the minimum load torque (i.e. the most negative value of load torque) which can be applied if the machine is to step forward correctly using single step excitation?

$R_T = 8$

$L_A + L_B = 20\,mH$

$L_A - L_B = 4\,mH$

$L_A = 12\,mH \quad L_B = 8\,mH$

$L_{as}(\theta_r) = 12 + 8\cos 8\theta_r\ mH$

$V_{dc} = 12\,V \quad r = 4\Omega$

$I_{dc} = \frac{V_{dc}}{r} = \frac{12}{4} = 3\,A$

$T_e = -\frac{1}{2} L_B R_T i_{as}^2 \sin R_T \theta_{rm}$ $\qquad i_{as} = i_{dc}$

$T_e = -\frac{1}{2} \times 8 \times 8 \times 9 \ \sin 8\theta_{rm}\ 10^{-3}\ Nm$

$T_e = -0.288 \sin 8\theta_{rm}\ Nm$

$T_{adv} = \frac{1}{2} T_{e\,max} = \frac{1}{2}\ 0.288 = 0.144\ Nm$

Advancement Torque

$T_{e\,min} = T_{e\,min} = -0.288\ Nm$

2) 22 pts. A separately excited dc machine has a armature resistance of 0.2 Ohms, a field resistance of 10 Ohms, L_{af} of 150 mH. We wish to determine the currents (armature and field) in such a way as to achieve a torque of 2 Nm, while minimizing the loss. What should the armature and field current be ? What will the machine loss be at this point ? Note: you may assume you are not against any voltage limits.

Resistive losses occur in armature and field resistances

$$\Rightarrow \quad P_{Loss} = I_a^2 r_a + I_f^2 r_f$$

for separately excited DC machine

$$T_e = L_{af} I_a I_f \quad \Rightarrow \quad I_a I_f = \frac{T_e}{L_{af}} \quad \text{or} \quad I_f = \frac{T_e}{L_{af} I_a}$$

Rewrite P_{loss} in terms of I_a (can also be rewritten in terms of I_f)

$$P_{Loss} = r_a I_a^2 + \left(\frac{T_e}{L_{af} I_a}\right)^2 r_f$$

Minimize loss $\quad \frac{d P_{Loss}}{d I_a} = 0$

$$\frac{d P_{loss}}{d I_a} = 2 r_a I_a - 2 \left(\frac{T_e}{L_{af}}\right)^2 \frac{r_f}{I_a^3} = 0$$

$$\Rightarrow \quad I_a^4 = \frac{r_f}{r_a} \frac{T_e^2}{L_{af}}$$

$$I_a = \left(\frac{r_f}{r_a} \frac{T_e^2}{L_{af}}\right)^{1/4} = \left(\frac{10}{0.2} \frac{4}{150 \times 10^{-3}}\right)^{1/4} = 9.710 \text{ A}$$

$$\Rightarrow \quad I_f = 1.373 \text{ A}$$

Use these values in P_{Loss} equation

$$P_{Loss} = 37.71 \text{ W}$$

3.) 22 pts. A permanent magnet dc machine is being fed from a single quadrant chopper, as we discussed in class. The armature resistance is 100 mΩ, and the back emf constant is 0.2 Vs, and the armature inductance is 0.25 mH. The chopper is supplied from a 25 V dc source, has a 1 V forward transistor drop, and a 0.75 V forward diode drop. At a speed of 100 rad/s, the chopper is in discontinuous mode, and has a peak current of 5 A. How long is the transistor on? How long does the diode conduct (i.e. what is the duration of non-zero current in the diode within a cycle)?

When transistor is ON the DC machine voltage equation is

$$V_{dc} = \hat{I}_a r_a + V_{fsw} + L_a \frac{dI_a}{dt} + K_v \omega_r \quad -(1)$$

$$\hat{I}_a = \frac{I_{MX}}{2} \quad \& \quad \frac{dI_a}{dt} = \frac{I_{MX}}{T_{on}}$$

from (1)

$$\Rightarrow \frac{I_{MX}}{T_{on}} = \frac{V_{dc} - V_{fsw} - \hat{I}_a r_a - K_v \omega_r}{L_{aa}}$$

$$T_o = \frac{L_{aa} I_{MX}}{V_{dc} - V_{fsw} - \hat{I}_a r_a - K_v \omega_r} = \frac{0.25 \times 5 \times 10^{-3}}{3.75} = 0.333\,ms$$

Similarly when transistor is off

$$-\frac{I_{MX}}{t_d} = \frac{-V_d - \hat{I}_a r_a - K_v \omega_r}{L_{aa}}$$

$$\Rightarrow t_d = \frac{I_{MX} L_{aa}}{V_d + \hat{I}_a r_a + K_v \omega_r} = \frac{5 \times 0.25 \times 10^{-3}}{21} = 0.059\,ms$$

4.) 22 pts. A permanent magnet dc machine is supplied by a four-quadrant converter with a hysteresis control. The machine has no armature resistance, and the converter has no forward semiconductor drops (wouldn't it be nice). The desired armature current is i_a^* and the hysteresis level is denoted h. When the armature current falls below $i_a^* - h$, the converter is switched such that the armature voltage is v_{dc}. When the armature current goes above $i_a^* + h$, the converter is switched such that the armature voltage is $-v_{dc}$. Derive an expression for the switching frequency in terms of h, v_{dc}, L_{aa}, the voltage constant k_v, and the rotor speed ω_r.

$\cancel{i_a < i_a^* +}$ $\quad i_a < i_a^* - h$

Transistor on

$$V_{dc} = K_v \omega_r + L_{aa} \frac{d i_a}{dt}$$

$$\frac{d i_a}{dt} = \frac{2h}{T_{ON}}$$

$$\Rightarrow \quad L_{aa} \frac{2h}{T_{ON}} = V_{dc} - K_v \omega_r \quad \text{or} \quad T_{ON} = \frac{2h L_{aa}}{V_{dc} - K_v \omega_r}$$

$i_a > i_a^* + h$

Transistor off

$$-V_{dc} = K_v \omega_r + L_{aa} \frac{d i_a}{dt}$$

$$\frac{d i_a}{dt} = \frac{-2h}{T_{off}}$$

$$\Rightarrow \quad L_{aa} \frac{2h}{T_{off}} = V_{dc} + K_v \omega_r \quad \text{or} \quad T_{off} = \frac{2h L_{aa}}{V_{dc} + K_v \omega_r}$$

$$T_{sw} = T_{ON} + T_{off} = 2h L_{aa} \frac{2 V_{dc}}{V_{dc}^2 - (K_v \omega_r)^2}$$

$$f_{sw} = \frac{1}{T_{sw}} = \frac{V_{dc}^2 - (K_v \omega_r)^2}{4 h L_{aa} V_{dc}} \quad \text{or} \quad \frac{(V_{dc} - K_v \omega_r)(V_{dc} + K_v \omega_r)}{2h L_{aa} V_{dc}}$$