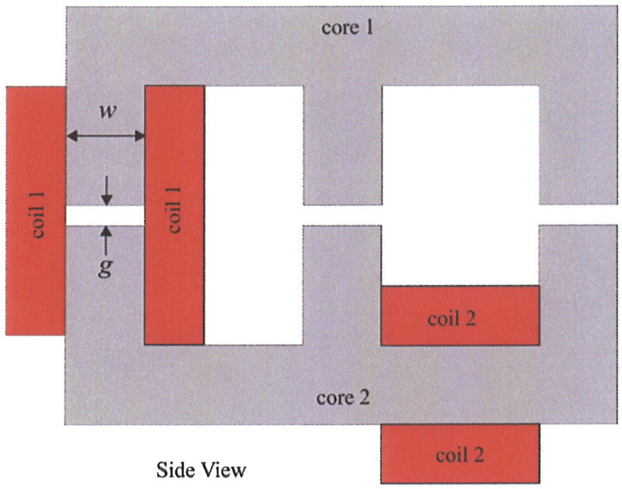
EE321 Exam 2
Spring 2011

Notes: **You must show work for credit on Problems 1-4.**
The last page of exam has a piece of extra paper if needed.

Good luck!

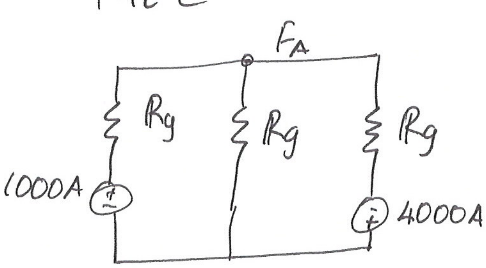
1.) 20 pts. Consider the magnetic system below. The permeability of the cores is infinite, and you may neglect fringing and leakage flux. The air gap $g = 3.14159$ mm, $w = 2.5$ cm, and the depth into the page is 10 cm. Coil 1 has 100 turns (with a direction such that positive current tends to cause flux to go up in the leftmost leg) and a current of 10 A. Coil 2 has 100 turns (with a direction such that positive current tends to cause positive flux to travel to the left through the coil) and a current of 40 A. What is the total flux linkage for winding 2 (i.e. λ_2)?



$$A = wd = 2.5 \times 10 \times 10^{-4} = 2.5 \times 10^{-3} \, m^2$$

$$\mathcal{R}_g = \frac{g}{\mu_0 A} = \frac{\pi \times 10^{-3}}{4\pi \times 10^{-7} \times 2.5 \times 10^{-3}} = 10^6 \, H^{-1}$$

MEC



KCL at Node A

$$\frac{F_A - 1000}{\mathcal{R}_g} + \frac{F_A}{\mathcal{R}_g} + \frac{F_A + 4000}{\mathcal{R}_g} = 0$$

$$F_A = \frac{-3000/\mathcal{R}_g}{3/\mathcal{R}_g} = -1000 \, A$$

$$\phi_2 = \frac{-1000 + 4000}{\mathcal{R}_g} = 3 \times 10^{-3} \, Wb$$

$$\lambda_2 = N_2 \phi_2 = 0.3 \, Vs$$

2.) 20 pts. Consider a four-phase multi-stack stepper motor. All phase currents are zero except for the ds-phase. The ds-phase inductance is given by

$$L_{ds} = 0.005 - 0.003\cos(4\theta_r - \pi/12)$$

The machine is at a equilibrium point at $\theta_r = 0.5$ rad under a load torque of 0.1 Nm. What is the ds-phase current (you may assume it is positive)? Is this equilibrium point stable or unstable?

At equilibrium points

$$T_e = T_L = 0.1\,Nm$$

$$W_c = \frac{1}{2} L_{ds} i_{ds}^2 \qquad T_e = \frac{\partial W_c}{\partial \theta_r}$$

$$= \frac{4}{2} 0.003 \sin(4\theta_r - \pi/12)\, i_{ds}^2$$

$$\Rightarrow i_{ds}^2 = \frac{0.1}{0.006 \sin(2-\pi/12)} \qquad \Rightarrow i_{ds} = 4.1113\ A$$

$$\frac{\partial T_e}{\partial \theta_r} = 4(0.006)\, i_{ds}^2 \cos(2-\pi/12) \quad < 0$$

$\Rightarrow$ Stable equilibrium point

3.) 20 pts. Consider a permanent magnet dc machine being fed from the one quadrant chopper (buck converter) we discussed at length in class. The armature resistance is 100 mΩ, and $k_v = 0.1$ Vs. It is desired to operate the machine so at to satisfy a load of 1 Nm at a speed of 3000 rpm. You may assume that the switching frequency is such that operation is in continuous mode. If the dc voltage is 50 V, the forward switch voltage drop is 1.5 V, and the forward diode voltage drop is 1 V, compute the system efficiency of the buck converter / motor system at this operating point.

$$\omega_r = 3000 \frac{rev}{min} \frac{2\pi}{rev} \frac{min}{60} = 314 \text{ rad/s}$$

$$T_L = T_E = 1 Nm$$

$$\bar{I}_a = \frac{T_e}{k_v} = \frac{1}{0.1} = 10 A$$

$$\bar{V}_a = 10 \times 0.1 + 0.1 \times 100\pi = 32.416 V$$

$$d = \frac{\bar{V}_a + V_{fd}}{V_{dc} - V_{fsw} + V_{fd}} = 0.675$$

$$\bar{I}_s = d \bar{I}_a = 6.75 A$$

$$P_{out} = T_e \omega_{r2} \quad 1 \times 100\pi = 314.15 W$$

$$P_{IN} = V_{dc} \bar{I}_s = 50 \times 6.75 = 337.5 W$$

$$\eta = \frac{P_{out}}{P_{in}} = 93.084\%$$

4.) 10 pts. Consider a machine where

$$N_{as} = [0\,1\,1\,0\,0\,-1\,-1\,0\,0\,1\,1\,0\,0\,-1\,-1\,0\,0\,1\,1\,0\,0\,-1\,-1\,0]^T$$

1 1 2 1 3

Find the winding function.

Patter repeats 3 times $\Rightarrow \frac{P}{2} = 3 \quad P = 6$

$N_{slots} = 24$

$W_1 = \frac{1}{2} \sum_{1}^{4} N_i = \frac{1}{2}(0+1+1+0) = 1$

$W_2 = 1 - 0 = 1$

$W_3 = 1 - 1 = 0$

$W_4 = 0 - 1 = -1$

$W_5 = -1 - 0 = -1$

$W_6 = -1 - 0 = -1$

$W_7 = -1 + 1 = 0$

$W_8 = 0 + 1 = 1$

$\Rightarrow W_{as} = [1\,1\,0\,-1\,-1\,-1\,0\,1,\; 1\,1\,0\,-1\,-1\,-1\,0\,1,\; 1\,1\,0\,-1\,-1\,-1\,0\,1]$

5.) 3 pts. Suppose you are given a variable reluctance stepper motor. What technique can you use to increase its advance torque?

Half step operation

6.) 3 pts. What it the chief difference (it terms of the magnetics) between single- and multi-stack stepper motors (even though we tend to ignore it)?

Mutual coupling

7.) 3 pts. What is the chief advantage of the two-transistor per phase drive circuit over the one-transistor per phase drive circuit we looked at?

Lower losses

8.) 3 pts. For a dc machine operated from a buck converter, for a given switching frequency what duty cycle give the worst case current ripple?

$d = 0.5$

9.) 3 pts. In our derivations concerning discontinuous mode diode conduction time, what value did we use to approximate the armature current?

$\bar{I_a} = I_{mx}/2$

10.) 3 pts. What is a capability curve?

Region in T_e vs ω_r plot where a machine can be operated

11.) 3 pts. Which dc machine configuration is more apt to be suitable for very high-speed operation – a shunt connected dc machine or a series connected dc machine?

Series connected DC machine

12.) 3 pts. Consider the conductor distribution $n_{as} = 500\cos(6\phi_{sm})$. How many poles does this machine have?

$P/2 = 6 \Rightarrow P = 12$

13.) 3 pts. In distributed winding machines, what is the purpose of the stator teeth?

To conduct flux

14.) 3 pts. Consider a 4 pole machine. At an instant of time, the air gap flux density from the rotor to the stator has a value of 0.5 T at a position of 0.1 rad. At what position is the air gap flux density -0.5 T? (multiple correct answers are possible, you only need to name one of these)

$0.1 + \pi\left(\frac{2}{P}\right) = 0.1 + \frac{\pi}{2} = 1.67 \pm N\pi$ rad

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