EE321 Exam 3
Spring 2009

Notes**: There are five questions.**

You must show work for credit, except for problem 1.

Some problems have more information than is needed.

The last two pages have trigonometric identities. You can tear these off if more convenient.

A score of 60% on this exam satisfies ABET objectives 1,3, and 4.

Good luck !

The following questions should be answered true or false. They are worth 2 pts. each. You do not need to show work.

1a) Single phase machines have forward and backward rotating MMF components.

1b) The stator MMF in a balanced 2-phase machine is qualitatively the same as for a 3- phase machine.

1c) Setting $\phi_v = \text{atan}(\omega_r L_{ss} / r_s)$ yields the optimal efficiency.

1d) Setting the d-axis voltage to zero yields zero d-axis current.

1e) A disadvantage of brushless dc machines is that you need to know or estimate the rotor position.

1f) An oscilloscope trace of the open-circuit line-to-line voltage is sufficient to find λ_m'.

1g) One of the advantages of 3-phase power over 2-phase power involves the uniformity of the slot fill.

1h) In order to create a two-phase machine capable of producing a MMF wave with a single forward moving component, the a- and b-phase conductor density functions must have the same amplitude.

1i) The qd0 transformation of variables requires two distinct approximations.

1j) There is nothing as beautiful or as elegant as a rotating magnetic field (Hint: the answer that will get you points is based on the instructor's opinion)

On the remaining problems, show your work!

2.) 25 pts. The conductor density of the a-phase of a machine is given by

$$n_{as} = 100\cos(4\phi_{sm})$$
$$n_{bs} = 100\sin(4\phi_{sm})$$

The currents are given by

$$i_{as} = 5\sin(50t)$$
$$i_{bs} = 5\cos(50t)$$

If the machine has a radius of 10 cm, a length of 15 cm, and a uniform airgap of 1 mm, and there are no currents or magnets on the rotor, what is (a) the peak flux density in the air gap, and (b) the speed and direction of the flux density wave. (All angles are CCW as we discussed in class). Recall $\mu_0 = 4\pi 10^{-7}$ H/m.

3.) 25 pts. A brushless dc machine has the following parameters: $r_s = 0.1$ Ohm, $L_{ss} = 10$ mH, $\lambda_m^{'} = 0.05$ Vs, $P = 4$. It is desired to provide 10 Nm of torque at a mechanical speed of 5000 RPM. If the d-axis current is set to zero, find (a) the needed q-axis current, and the resulting (b) the rms phase voltage, (c) input and output powers, and (d) efficiency.

4.) 15 pts. The conductor density of the a-phase of a machine is given by

$$n_{as} = 100\sin(4\phi_{sm})$$

If the a-phase of the machine is open-circuited, and the radial component of the flux density is given by

$$B = 2\cos(4\phi_{rm})$$

at the stator radius which is 1 m. The machine is 2 m long. The rotor position is given by

$$\theta_{rm} = 100t$$

What is the a-phase voltage at $t = 0.01s$? Note: you should get a very large number.

5.) 15 pts. The flux linkage equations for a certain PMSM may be expressed

$$\lambda_{abcs} = \begin{bmatrix} L_{ss} & 0 & 0 \\ 0 & L_{ss} & 0 \\ 0 & 0 & L_{ss} \end{bmatrix} i_{abcs} + \lambda_m \begin{bmatrix} \cos\theta_r \\ \cos(\theta_r - 2\pi/3) \\ \cos(\theta_r + 2\pi/3) \end{bmatrix} - \lambda_{m3} \begin{bmatrix} \cos(3\theta_r) \\ \cos(3\theta_r) \\ \cos(3\theta_r) \end{bmatrix}$$

where L_{ss}, λ_m, and λ_{m3} are constants. The machine is wye-connected. Express the q- and d-axis flux linkage equations for the machine. Your answer should involve the q- and d-axis flux linkages, the q- and d-axis currents, and the parameters L_{ss}, λ_m, and λ_{m3}. Recall

$$\mathbf{K}_s^r = \frac{2}{3} \begin{bmatrix} \cos(\theta_r) & \cos(\theta_r - 2\pi/3) & \cos(\theta_r + 2\pi/3) \\ \sin(\theta_r) & \sin(\theta_r - 2\pi/3) & \sin(\theta_r + 2\pi/3) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Trigonometric Identities

Table A-1 Trigonometric Identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\cos A \cos B = \tfrac{1}{2}[\cos(A+B) + \cos(A-B)]$$
$$\sin A \sin B = \tfrac{1}{2}[\cos(A-B) - \cos(A+B)]$$
$$\sin A \cos B = \tfrac{1}{2}[\sin(A+B) + \sin(A-B)]$$
$$\sin A + \sin B = 2\sin\tfrac{1}{2}(A+B)\cos\tfrac{1}{2}(A-B)$$
$$\sin A - \sin B = 2\sin\tfrac{1}{2}(A-B)\cos\tfrac{1}{2}(A+B)$$
$$\cos A + \cos B = 2\cos\tfrac{1}{2}(A+B)\cos\tfrac{1}{2}(A-B)$$
$$\cos A - \cos B = -2\sin\tfrac{1}{2}(A+B)\sin\tfrac{1}{2}(A-B)$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \cos^2 A - \sin^2 A$$
$$\sin\tfrac{1}{2}A = \sqrt{\tfrac{1}{2}(1-\cos A)} \qquad \cos\tfrac{1}{2}A = \sqrt{\tfrac{1}{2}(1+\cos A)}$$
$$\sin^2 A = \tfrac{1}{2}(1-\cos 2A) \qquad \cos^2 A = \tfrac{1}{2}(1+\cos 2A)$$
$$\sin x = \frac{e^{jx} - e^{-jx}}{2j} \qquad \cos x = \frac{e^{jx} + e^{-jx}}{2} \qquad e^{jx} = \cos x + j\sin x$$
$$A\cos(\omega t + \phi_1) + B\cos(\omega t + \phi_2) = C\cos(\omega t + \phi_3)$$

where

$$C = \sqrt{A^2 + B^2 - 2AB\cos(\phi_2 - \phi_1)}$$
$$\phi_3 = \tan^{-1}\left[\frac{A\sin\phi_1 + B\sin\phi_2}{A\cos\phi_1 + B\cos\phi_2}\right]$$
$$\sin(\omega t + \phi) = \cos\left(\omega t + \phi - \frac{\pi}{2}\right)$$

Taken from, *Continuous and Discrete Signal and Systems Analysis, 2nd Edition*, by McGillem & Cooper, 1984, CBS College Publishing, and one heck of a good book.

Even More Trigonometric Identities

TRIGONOMETRIC RELATIONS, CONSTANTS AND CONVERSION FACTORS, AND ABBREVIATIONS

TRIGONOMETRIC RELATIONS

$$\cos^2 x + \cos^2\left(x - \frac{2\pi}{3}\right) + \cos^2\left(x + \frac{2\pi}{3}\right) = \frac{3}{2}$$

$$\sin^2 x + \sin^2\left(x - \frac{2\pi}{3}\right) + \sin^2\left(x + \frac{2\pi}{3}\right) = \frac{3}{2}$$

$$\sin x \cos x + \sin\left(x - \frac{2\pi}{3}\right)\cos\left(x - \frac{2\pi}{3}\right) + \sin\left(x + \frac{2\pi}{3}\right)\cos\left(x + \frac{2\pi}{3}\right) = 0$$

$$\cos x + \cos\left(x - \frac{2\pi}{3}\right) + \cos\left(x + \frac{2\pi}{3}\right) = 0$$

$$\sin x + \sin\left(x - \frac{2\pi}{3}\right) + \sin\left(x + \frac{2\pi}{3}\right) = 0$$

$$\sin x \cos y + \sin\left(x - \frac{2\pi}{3}\right)\cos\left(y - \frac{2\pi}{3}\right) + \sin\left(x + \frac{2\pi}{3}\right)\cos\left(x + \frac{2\pi}{3}\right) = \frac{3}{2}\sin(x - y)$$

$$\sin x \sin y + \sin\left(x - \frac{2\pi}{3}\right)\sin\left(y - \frac{2\pi}{3}\right) + \sin\left(x + \frac{2\pi}{3}\right)\sin\left(y + \frac{2\pi}{3}\right) = \frac{3}{2}\cos(x - y)$$

$$\cos x \sin y + \cos\left(x - \frac{2\pi}{3}\right)\sin\left(y - \frac{2\pi}{3}\right) + \cos\left(x + \frac{2\pi}{3}\right)\sin\left(y + \frac{2\pi}{3}\right) = -\frac{3}{2}\sin(x - y)$$

$$\cos x \cos y + \cos\left(x - \frac{2\pi}{3}\right)\cos\left(y - \frac{2\pi}{3}\right) + \cos\left(x + \frac{2\pi}{3}\right)\cos\left(y + \frac{2\pi}{3}\right) = \frac{3}{2}\cos(x - y)$$

$$\sin x \cos y + \sin\left(x + \frac{2\pi}{3}\right)\cos\left(y - \frac{2\pi}{3}\right) + \sin\left(x - \frac{2\pi}{3}\right)\cos\left(y + \frac{2\pi}{3}\right) = \frac{3}{2}\sin(x + y)$$

$$\sin x \sin y + \sin\left(x + \frac{2\pi}{3}\right)\sin\left(y - \frac{2\pi}{3}\right) + \sin\left(x - \frac{2\pi}{3}\right)\sin\left(y + \frac{2\pi}{3}\right) = -\frac{3}{2}\cos(x + y)$$

$$\cos x \sin y + \cos\left(x + \frac{2\pi}{3}\right)\sin\left(y - \frac{2\pi}{3}\right) + \cos\left(x - \frac{2\pi}{3}\right)\sin\left(y + \frac{2\pi}{3}\right) = \frac{3}{2}\sin(x + y)$$

$$\cos x \cos y + \cos\left(x + \frac{2\pi}{3}\right)\cos\left(y - \frac{2\pi}{3}\right) + \cos\left(x - \frac{2\pi}{3}\right)\cos\left(y + \frac{2\pi}{3}\right) = \frac{3}{2}\cos(x + y)$$

Taken from, *Analysis of Electric Machinery and Drive Systems*, 2nd Edition, by Krause, Wasyncuk, and Sudhoff, 2002, Wiley Press, and also a heck of a good book.