ECE321. Fall 2009.

Exam 3 Solution Outline

Problem 1

1a) True
1b) True
1c) False
1d) False
1e) True
1f) True
1g) True
1h) False
1i) False
1j) True

Problem 2

$n_{as} = 100\cos(4\varphi_{sm})$ $\quad$ $i_{as} = 5\cdot\sin(50\cdot t)$ $\quad$ $g := 1\cdot 10^{-3}$

$n_{bs} = 100\sin(4\varphi_{sm})$ $\quad$ $i_{bs} = 5\cdot\cos(50\cdot t)$ $\quad$ $\mu_0 := 4\cdot 10^{-6}\cdot\pi$

Step 1: Find the winding functions (5 pts)

$$w_{as} = -25\cdot\sin(4\cdot\varphi_{sm})$$

$$w_{bs} = 25\cdot\cos(4\cdot\varphi_{sm})$$

Step 2: Find the stator MMF (5 pts)

$$F_s = 125\cdot(-\sin(4\cdot\varphi_{sm})\cdot\sin(50\cdot t) + \cos(4\cdot\varphi_{sm})\cdot\cos(50\cdot t))$$

$$F_s = 125\cdot\cos(4\cdot\varphi_{sm} + 50t)$$

Step 3: Find the B field (5 pts)

$$B = \frac{\mu_0\cdot F_s}{g}$$

$$B = \frac{125\cdot\mu_0}{g}\cdot\cos(4\cdot\varphi_{sm} + 50t)$$

Step 4: find the peak B-field (5 pts)

$$B_{pk} := \frac{125 \cdot \mu_0}{g} \qquad B_{pk} = 1.5708$$

Step 5: find the speed and direction (5 pts)

Holding $4 \cdot \varphi_{sm} + 50 \cdot t = \text{constant}$

The speed is clearly -12.5 rad/s in the CCW direction or 12.5 rad/s in CW direction

Problem 3

$r_s := 0.1$ $\lambda_m := 0.05$ $T_L := 10$ $i_d := 0$

$L_{ss} := 10 \cdot 10^{-3}$ $P := 4$ $\omega_{rm\_RPM} := 5000$

Step 1: find the mechanical and electrical speed in rad/s (5 pts)

$$\omega_{rm} := 5000 \cdot \left(\frac{2 \cdot \pi}{60}\right)$$

$$\omega_{rm} = 523.59878$$

$$\omega_r := \frac{P}{2} \cdot \omega_{rm}$$

$$\omega_r = 1.0472 \times 10^3$$

Step 2: find the q-axis current (5 pts)

$$i_q := \frac{T_L}{\frac{3}{2} \cdot \frac{P}{2} \cdot \lambda_m} \qquad i_q = 66.66667$$

Step 3: find the q- and d-axis voltage (6 pts)

$$v_q := r_s \cdot i_q + \omega_r \cdot L_{ss} \cdot i_d + \omega_r \cdot \lambda_m$$

$$v_q = 59.02654$$

$$v_d := r_s \cdot i_d - \omega_r \cdot L_{ss} \cdot i_q$$

$$v_d = -698.1317$$

$$v_s := \frac{1}{\sqrt{2}} \cdot \sqrt{v_q^{\,2} + v_d^{\,2}}$$

$$v_s = 495.41498$$

Step 4: find the powers (6 pts)

$$P_{in} := \frac{3}{2} \cdot \left(v_q \cdot i_q + v_d \cdot i_d\right)$$

$$P_{in} = 5.90265 \times 10^3$$

$$P_{out} := \omega_{rm} \cdot T_L$$

$$P_{out} = 5.23599 \times 10^3$$

Step 5: find the efficiency (3 pts)

$$\eta := \frac{P_{out}}{P_{in}} \cdot 100$$

$$\eta = 88.70565$$

Problem 4

$r := 1$

$l := 2$

Step 1 - find the winding function (2 pts)

$$w_{as} = 25 \cdot \cos(4 \cdot \varphi_{sm})$$

Step 2 - find the flux linkage (9 pts)

$$\lambda_{as} = r \cdot l \cdot \int_0^{2 \cdot \pi} w_{as} \cdot B \, d\varphi_{sm}$$

$$\lambda_{as} = r \cdot l \cdot \int_0^{2 \cdot \pi} 50 \cdot \cos(4 \cdot \varphi_{rm}) \cdot \cos(4 \cdot \varphi_{sm}) \, d\varphi_{sm}$$

Now

$$\varphi_{rm} = \varphi_{sm} - \theta_{rm}$$

So

$$\lambda_{as} = r \cdot l \cdot \int^{2 \cdot \pi} 50 \cdot \cos\left[4 \cdot \left(\varphi_{sm} - \theta_{rm}\right)\right] \cdot \cos(4 \cdot \varphi_{sm}) \, d\varphi_{sm}$$

$$\lambda_{as} = 25 \cdot r \cdot l \cdot \int_0^{2 \cdot \pi} \left(\cos(8 \cdot \varphi_{sm} - 4 \cdot \theta_{rm}) + \cos(4 \cdot \theta_{rm})\right) \, d\varphi_{sm}$$

$$\lambda_{as} = 50 \cdot \pi \cdot r \cdot l \cdot \cos(4 \cdot \theta_{rm})$$

Step 3: Compute the voltage (4 pts)

$$v_{as} = -200 \cdot \pi \cdot r \cdot l \cdot \sin(4 \cdot \theta_{rm}) \cdot \omega_{rm}$$

$\omega_{rm} := 100$

$\theta_{rm} := 100 \cdot 0.01$

$$v_{as} := -200\cdot\pi\cdot r\cdot l\cdot \sin(4\cdot\theta_{rm})\cdot\omega_{rm}$$

$$v_{as} = 9.51026 \times 10^{4}$$

Problem 5

$$\lambda_{qd0s} = K_s\cdot L_{ss}\cdot I_3\cdot (K_s)^{-1}\cdot i_{qd0s} + K_s\cdot\lambda_m\cdot\begin{pmatrix}\cos(\theta_r)\\ \cos\left(\theta_r - \frac{2\cdot\pi}{3}\right)\\ \cos\left(\theta_r + \frac{2\cdot\pi}{3}\right)\end{pmatrix} - K_s\cdot\lambda_{m3}\cdot\begin{pmatrix}\cos(3\theta_r)\\ \cos(3\theta_r)\\ \cos(3\theta_r)\end{pmatrix}$$

After simplifying

$$\lambda_{qds} = L_{ss}\cdot i_{qds} + \begin{pmatrix}\lambda_m\\ 0\end{pmatrix}$$

Grading: 5 pts for each of 3 correct terms