EE321 Exam 3
Spring 2010

Notes: **You must show work for credit.**

Trigonometric identities are towards the back.

The last page of exam is blank for extra paper if needed.

1) 18 pts. Consider the a-phase of a distributed winding machine. The number of conductors in each slot is given by

$$N_{as} = \begin{bmatrix} 0 & 4 & 4 & 0 & -4 & -4 & 0 & 4 & 4 & 0 & -4 & -4 \end{bmatrix}$$

The total flux through each of the teeth is given by

$$\Phi = [-1 \quad -2 \quad -1 \quad 1 \quad 2 \quad 1 \quad -1 \quad -2 \quad -1 \quad 1 \quad 2 \quad 1]$$

where the first element is the flux through the first tooth and so forth. What is the winding function (12 pts)? What is the flux linking the a-phase winding (i.e. what is λ_{as}) (6 pts)?

In Nas 044 0-4-4 pattern repeats twice

$\Rightarrow \frac{P}{2} = 2 \Rightarrow P = 4$

$W_{as,1} = \frac{1}{2}(0+4+4) = 4$

$W_{as,2} = W_{as,1} - N_{as,1} = 4$ so on.

$\Rightarrow W_{as} = [4\,4\,0\,-4\,-4\,0\;\;4\,4\,0\,-4\,-4\,0]$

$\lambda_{as} = W_{as}^T \phi$

$= -4-8+0-4-8+0\;-4-8+0\;-4-8+0$

$\lambda_{as} = -48$ Vs

2) 18 pts. The a- and b-phase winding functions of a two-phase machine may be expressed

$$w_{as} = 50\sin(4\phi_{sm})$$
$$w_{bs} = 50\cos(4\phi_{sm})$$

where ϕ_{sm} is measured in the counterclockwise direction (as is our custom). The a- and b-phase currents may be expressed

$$i_{as} = 100\sin(50t)$$
$$i_{bs} = -100\cos(50t)$$

The machine has a uniform airgap of π mm, a stator radius of 2 cm, and a length of $50/\pi$ cm, and there are no circuits on the rotor, PM materials, or other complications.

Part A. Find an expression for the radial component of the B-field in terms of ϕ_{sm} and t. (9pts)

Part B. What is the speed of the B field traveling wave in the counterclockwise direction? (3 pts)

Part C. Neglecting the resistance of the a-phase winding, what is the voltage across the a-phase in terms of time. (6 pts)

Part A $F = W_{as} i_{as} + W_{bs} i_{bs}$

$= 5000\,(\sin 4\phi_{sm} \sin 50t - \cos 4\phi_{sm} \cos 50t)$

Using Trig ID $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$F = -5000 \cos(4\phi_{sm} + 50t)$ or $5000 \cos(4\phi_{sm} + 50t + \pi)$ A

PART B

$B = \mu_0 H = \mu_0 \frac{F}{g} = \frac{4\pi \times 10^{-7} \times 5000}{\pi \times 10^{-3}} \cos(4\phi_{sm} + 50t + \pi)$

$= 2\cos(4\phi_{sm} + 50t + \pi)$ T

differentiate $4\phi_{sm} + 50t + \pi = K$ w.r.t. time

$\Rightarrow \frac{d\phi_{sm}}{dt} = -\frac{50}{4} = -12.5$ rad/s

in Counter Clockwise direction

(extra paper for problem 2)

Part c

$$\lambda_{as} = rL \int_0^{2\pi} B\, W_{as}\, d\phi_{sm}$$

$$= 2\times10^{-2} \times \frac{50}{\pi}\times10^{-2}\times 2\times 5000 \int_0^{2\pi} \cos(50t+4\phi_{sm}+\pi)\sin 4\phi_{sm}\, d\phi_{sm}$$

Using Trig ID $\sin A\cos B = \frac{1}{2}(\sin(A+B)+\sin(A-B))$

$$= \frac{1}{\pi}\cdot\frac{1}{2}\int_0^{2\pi} \overset{0}{\cancel{\sin(50t+4\phi_{sm}+4\phi_{sm}+\pi)}} + \sin(-50t-\pi)\, d\phi_{sm}$$

$$= \frac{-1}{2\pi}\int_0^{2\pi}\sin(50t+\pi)\, d\phi_{sm} = \frac{-\sin(50t+\pi)}{2\pi}\, \phi_{sm}\Big]_0^{2\pi}$$

$$= -\sin(50t+\pi)\ V_s$$

$$V_{as} = \frac{d\lambda_{as}}{dt} = -50\cos(50t+\pi)\ V$$

$$V_{as} = 50\cos 50t\ V$$

3.) 18 pts. A two-phase permanent magnet synchronous machine has the following flux linkage equation. Take it to be correct even though it doesn't make complete sense.

$$\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \end{bmatrix} + \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

Now consider the following two-phase qd transformation to the 'weird' reference frame

$$\begin{bmatrix} f_{qs}^{w} \\ f_{ds}^{w} \end{bmatrix} = \mathbf{K}_s^w \begin{bmatrix} f_{as} \\ f_{bs} \end{bmatrix}$$

$$\mathbf{K}_s^w = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} \quad \Rightarrow K_s^{w\,-1} = \frac{1}{1} \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$$

Express the machine flux linkage equations in the weird reference frame.

$$\lambda_{qds}^{w} = K_s^w \, \bar{L} \, K_s^{w-1} \, i_{qds}^{w} + K_s^w \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 5 & -2 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} i_{qds}^{w} + \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 9 & -20 \\ -12 & 29 \end{bmatrix} i_{qds}^{w} + \begin{bmatrix} 31 \\ 13 \end{bmatrix}$$

$$\lambda_{qds}^{w} = \begin{bmatrix} 21 & -42 \\ 6 & -11 \end{bmatrix} i_{qds}^{w} + \begin{bmatrix} 31 \\ 13 \end{bmatrix}$$

4.) 18 pts. A three phase brushless DC machine has the following parameters: $r_s = 3\Omega$, $L_{ss} = 10$ mH, $\lambda_m = 0.17$ Vs, $P = 4$. At is desired to operate at obtain a torque of 2 Nm at a speed of 4000 RPM. What is the highest machine efficiency that can be obtained to achieve the desired torque at the desired speed.

for optimum efficiency set $i_{ds}^r = 0$

from $T_e = \frac{3}{2}\frac{P}{2}\lambda_m i_{qs}^r$

$$i_{qs}^r = \frac{2}{\frac{3}{2}\times\frac{4}{2}\times 0.17} = 3.921 \text{ A}$$

$$\omega_{rm} = 4000 \times \frac{2\pi}{60} = \frac{400\pi}{3} \text{ rad/s}$$

$$\omega_r = \frac{P}{2}\omega_{rm} = \frac{4}{2}\frac{400\pi}{3} = \frac{800\pi}{3}$$

$$V_{qs}^r = r_s i_{qs}^r + \cancel{\omega_r L_{ss} i_{ds}^r}^{\,0} + \omega_r \lambda_m$$

$$= 154.18 \text{ V}$$

$$P_{in} = \frac{3}{2}\left(V_{qs}^r i_{qs}^r + \cancel{V_{ds}^r}^{\,0} i_{ds}^r\right)$$

$$= 1.5\,(154.18 \times 3.92) = 906.82$$

$$P_{out} = T_e\,\omega_{rm}$$

$$= 2 \times \frac{400\pi}{3} = \frac{800\pi}{3} = 837.76 \text{ W}$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{837.76}{906.82} = 0.9238 \text{ or } 92.38\%$$

5.) 28 pts. Answer true or false. Grading on each is as follows: Correct(2pts), Incorrect(-1pt), No response (0 pts).

ANSWER (T-True, F - False)

T a.) Distributed windings are used to create a smoothly rotating MMF waves.

T b.) Continuous and discrete winding descriptions are both used to represent distributed windings.

F c.) In a distributed winding machine, the conductor density function serves a similar role to turns in our lumped winding analysis.

T d.) In many ac machines, the rotor moves at the same speed as the MMF wave.

T e.) The speed of the stator MMF wave is controlled by the frequency of the currents and the number of poles.

T f.) In a distributed winding machines with a uniform airgap and a round iron rotor (as we discussed in class), the radial flux density is a sinusoidal function in space (i.e. ϕ_{sm})

T g.) In a distributed winding machine with a uniform airgap and a round iron rotor (as we discussed in class), the radial flux density is a sinusoidal function of time.

T h.) In practice, "single-phase" machines are actually two-phase machines.

T i.) All other factors being as equal as possible, a three-phase machine fed from a three-phase bus will be more efficient than a single-phase machine.

T or F j.) Flux is your friend.

F k.) Obtaining a sinusoidal back emf in a brushless dc machine requires a specific magnet shape.

F l.) Setting $\phi_v = \text{atan}(\omega_r L_{ss} / r_s)$ yields maximum efficiency.

F m.) Voltage controlled brushless dc motor drives are, in general, more efficient than current-sourced drives.

T n.) Permanent magnets contribute to co-energy.