EE321 Exam 3
Spring 2011

Notes: **You must show work for credit on Problems 1-4.**

Good luck!

1.) 25 pts. Consider a machine with stator conductor distributions given by

$$n_{as} = 100\cos(4\phi_{sm})$$
$$n_{bs} = 50\sin(4\phi_{sm})$$

Suppose the a- and b-phase stator currents are given by

$$i_{as} = 4\sin(100t)$$
$$i_{bs} = 8\cos(100t)$$

The machine has a round rotor, and the airgap is 0.1 mm. Express the radial flux density in the machine as a function of ϕ_{sm} and t. What is the speed of the MMF wave ($d\phi_{sm}/dt$) of the peak of the traveling wave.

$P = 8$

$$W_{as} = \frac{1}{2}\int_0^{\pi/4} 100\cos(4\phi_{sm})\,d\phi_{sm} - \int_0^{\phi_{sm}} 100\cos 4\phi_{sm}'\,d\phi_{sm}'$$

$$= 0 - \frac{100}{4}\sin(4\phi_{sm}')\Big]_0^{\phi_{sm}} = -25\sin(4\phi_{sm})$$

$$W_{bs} = \frac{1}{2}\int_0^{\pi/4} 50\sin(4\phi_{sm})\,d\phi_{sm} - \int_0^{\phi_{sm}} 50\sin(4\phi_{sm}')\,d\phi_{sm}'$$

$$= \frac{50}{8}\left(-\cos 4\phi_{sm}\Big]_0^{\pi/4}\right) - \frac{50}{4}\left(-\cos 4\phi_{sm}'\Big]_0^{\phi_{sm}}\right)$$

$$= \frac{50}{4} + \frac{50}{4}\cos 4\phi_{sm} - \frac{50}{4} = 12.5\cos(4\phi_{sm})$$

$$MMF = W_{as}i_{as} + W_{bs}i_{bs} = -100\sin(4\phi_{sm})\sin(100t) + 100\cos(4\phi_s)\cos(100t)$$

$$= 100\cos(4\phi_{sm} + 100t)\ A$$

$$\Rightarrow \frac{d\phi_{sm}}{dt} = \frac{-100}{4} = -25\ rad/s \quad \text{(Speed of MMF wave)}$$

$$B = \frac{\mu_0 F}{g} = \frac{4\pi\times10^{-7}\times +100}{1\times10^{-4}} = +1.2566\cos(100t + 4\phi_{sm})\ T$$

2.) 25 pts. Consider a 4-phase machine. The winding functions of the a- and b-phases are given by

$$w_{as} = 100\cos(2\phi_{sm})$$
$$w_{bs} = 100\cos(2\phi_{sm} - \pi/4)$$

The machine stator has a radius of 0.05 m and a length of 0.1 m. The airgap is 0.1 mm and the rotor is round and made of steel. What is the mutual inductance between the a- and b-phases? The answer is not zero.

$$L_{ab} = \frac{\mu_0 r L}{g} \int_0^{2\pi} w_{as} w_{bs} \, d\phi_{sm}$$

$$= \frac{4\pi \times 10^{-7} \times 0.05 \times 0.1 \times 10^4}{1 \times 10^{-4}} \int_0^{2\pi} \cos 2\phi_{sm} \cos(2\phi_{sm} - \pi/4) \, d\phi_{sm}$$

$$= \frac{0.6283}{2} \int_0^{2\pi} \xcancel{\cos(2\phi_{sm} + 2\phi_{sm} - \pi/4)}^{\,0} + \cos(\pi/4) \, d\phi_{sm}$$

$$= 0.3141 \quad \phi_{sm} \cos(\pi/4) \Big]_0^{2\pi}$$

$$= 0.3141 \times 2\pi \times \cos(\pi/4)$$

$$L_{ab} = 1.3955 \text{ H}$$

3.) 25 pts. A three-phase brushless DC machine has the following parameters: $r_s = 0\,\Omega$, $L_{ss} = 10$ mH, $\lambda_m = 0.17$ Vs, $P = 4$. The inverter that drives the machine is limited to producing a voltage $v_s < 100$ V. The machine/inverter is operated so as to produce torque with the minimum possible rms current i_s (this would also produce the minimum loss for non-zero stator resistance). If it is desired to produce a torque of 10 Nm, what is the highest speed (mechanical speed, in rpm) at which this machine can be operated with this strategy. In other words, at what speed will we run into the voltage constraint.

$$T_e = 10 = \frac{3}{2} \times \frac{4}{2} \times 0.17 \times i_{qs}^r$$

$$\Rightarrow i_{qs}^r = 19.6078 \text{ A}$$

Set $i_d = 0$

$$V_{qs}^r = \cancel{r_s i_{qs}^r}^{\to 0} + \cancel{\omega_r L_{ss} i_{ds}^r}^{\to 0} + \omega_r \lambda_m' = 0.17\omega_r$$

$$V_{ds}^r = \cancel{r_s i_{ds}^r}^{\to 0} - \omega_r L_{ss} i_{qs}^r = -0.1961\omega_r$$

$$V_s = \frac{1}{\sqrt{2}} \sqrt{V_{qs}^{r\,2} + V_{ds}^{r\,2}} < 100$$

$$\Rightarrow \frac{\omega_r^2}{2} (0.17^2 + 0.1961^2) < 100$$

$$\omega_r < 544.9158 \text{ rad/s}$$

$$\omega_{r\,max} = 544.9158 \text{ rad/s}$$

$$\omega_{rM,mx} = \frac{\omega_{r\,max}}{P/2} \times \frac{60}{2\pi} \text{ RPM}$$

$$\omega_{rM,mx} = 2601.7813 \text{ RPM}$$

4.) 25 pts. A three-phase wye-connected brushless dc machine is open-circuited, and is spun using a dynamometer. The a- to b-phase voltage measured from an oscilloscope if found to be sinusoidal. The waveform has a zero-to-peak voltage of 100 V, and a frequency of 41 Hz. What is λ_m ?

Open circuit $\Rightarrow$ $i_{abcs} = 0$

$$V_{abcs} = \cancel{r_s i_{abcs}}^{\,0} + \frac{d\lambda_{abcs}}{dt}$$

$$\lambda_{as} = \lambda_m \sin\theta_r$$

$$\lambda_{bs} = \lambda_m \sin(\theta_r - 2\pi/3)$$

$$V_{as} = \frac{d\lambda_{as}}{dt} = \omega_r \lambda_m \cos\theta_r$$

$$V_{bs} = \frac{d\lambda_{bs}}{dt} = \omega_r \lambda_m \cos(\theta_r - 2\pi/3)$$

$$V_{ab} = V_{as} - V_{bs} = \sqrt{3}\, \omega_r \lambda_m \cos(\theta_r + \pi/6)$$

Compare magnitudes

$\Rightarrow \omega_r = \omega_e = 2\pi\, 21$ rad/s

$$100 = \sqrt{3}\, \omega_r \lambda_m = \sqrt{3}\ 82\pi\ \lambda_m$$

$$\Rightarrow \lambda_m = \frac{100}{\sqrt{3} \times 82\pi} = 0.2241\ Vs$$