Solution

ECE321/ECE595 Exam 3
Spring 2014

Notes: You must show work for credit.

This exam has 5 problems and 11 pages.

Note that problem 4 has different depending on if you are in ECE321 or ECE595.

Page 11 contains trigonometric identities. You may remove this page from the back of the exam if you wish. You need not turn in the identity sheet with the exam.

Don't forget to write your name.

Good luck!

1) 20 pts. Consider a buck converter (the single quadrant dc/dc converter we primarily discussed in class) connected to a permanent magnet dc machine. Derive an expression for the amount of time the diode conducts per cycle, denoted t_d, in terms of the maximum armature current, i_{mx}, the armature inductance L_{AA}, the armature resistance r_a, the diode forward diode drop v_{fd}, the back emf coefficient k_v, and the rotor speed ω_r.

Consider the period when the diode is on

$$-v_{fd} = r_a i_a + k_v \omega_r + L_{AA} \frac{di_a}{dt}$$

We approximate this as

$$-v_{fd} = r_a \frac{i_{mx}}{2} + k_v \omega_r + L_{AA} \frac{di_a}{dt}$$

Thus

$$L_{AA} \frac{di_a}{dt} = -v_{fd} - r_a \frac{i_{mx}}{2} - k_v \omega_r$$

So

$$L_{AA} \left(-\frac{i_{mx}}{t_d}\right) = -v_{fd} - r_a \frac{i_{mx}}{2} - k_v \omega_r$$

$$t_d = \frac{L_{AA} i_{mx}}{v_{fd} + \frac{r_a}{2} i_{mx} + k_v \omega_r}$$

2.) 20 pts. The flux density at the inner stator radius of a device due to a permanent magnet is given as

$$B = B_{pm} \sin(2\phi_{rm})$$

The conductor density of the a-phase winding on the stator of that device may be expressed

$$n_{as} = N_1 \sin(2\phi_{sm} - \pi / 4)$$

Find an expression for the flux linking the a-phase due to the permanent magnet in terms of B_{pm}, N_1, the inner stator radius r, the length of the machine l, and mechanical rotor position θ_{rm}.

Since B and n_{as} repeat twice per cycle, we conclude $P = 4$

$$W_{as} = \frac{1}{2}\int_0^{\pi/2} n_{as}\, d\phi_{sm} - \int_0^{\phi_{sm}} n_{as}\, d\phi_{sm}$$

$$\int n_{as}\, d\phi_{sm} = -\frac{N_1}{2}\cos(2\phi_{sm} - \pi/4)$$

$$W_{as} = -\frac{N_1}{4}\cos(2\phi_{sm} - \pi/4)\Big|_0^{\pi/2} + \frac{N_1}{2}\cos(2\phi_{sm} - \pi/4)\Big|_0^{\phi_{sm}}$$

$$= -\frac{N_1}{4}\left[\cos\left(\tfrac{3\pi}{4}\right) - \cos\left(-\tfrac{\pi}{4}\right)\right] + \frac{N_1}{2}\left[\cos(2\phi_{sm} - \pi/4) - \cos(\pi/4)\right]$$

$$= \cancel{\frac{N_1}{2}\cos\left(\tfrac{\pi}{4}\right)} + \frac{N_1}{2}\left[\cos(2\phi_{sm} - \pi/4) - \cancel{\cos\left(\tfrac{\pi}{4}\right)}\right]$$

$$= \frac{N_1}{2}\cos(2\phi_{sm} - \pi/4)$$

Next

$$\lambda_{as} = rl\int_0^{2\pi} W_{as} B\, d\phi_{sm}$$

$$= rl\frac{N_1}{2}B_{pm}\int_0^{2\pi}\cos(2\phi_{sm} - \pi/4)\sin(2\phi_{rm})\, d\phi_{sm}$$

$$= \frac{rlN_1B_{pm}}{4}\int_0^{2\pi}\sin(2(\phi_{sm} + \phi_{rm}) - \pi/4) + \sin(2(\phi_{rm} - \phi_{sm}) + \pi/4)\, d\phi_{sm}$$

Now $\theta_{rm} + \phi_{rm} = \phi_{sm}$

$$\phi_{rm} = \phi_{sm} - \theta_{rm}$$

$\int$ to zero

$$\lambda_{as} = \frac{r \ell N_1 B_{pm}}{4} \int_0^{2\pi} \cancel{\sin(4\phi_{sm} - 2\theta_{rm} - \pi/4)} + \sin(-2\theta_{rm} + \pi/4)\, d\phi_{sm}$$

$$\lambda_{as} = -\frac{r \ell N_1 B_{pm} \pi}{2} \sin(2\theta_{rm} - \pi/4)$$

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3.) 20 pts. The conductors of the b-phase of a machine (positive out of the machine as in class) may be expressed:

$$N_{bs} = [2 \ \ 4 \ \ 2 \ \ -2 \ \ -4 \ \ -2 \ \ 2 \ \ 4 \ \ 2 \ \ -2 \ \ -4 \ \ -2]$$

The flux in each tooth of the machine (going from rotor to stator) may be expressed

$$\Phi_{bs} = [0 \ \ 0 \ \ 2 \ \ 0 \ \ 0 \ \ -2 \ \ 0 \ \ 0 \ \ 2 \ \ 0 \ \ 0 \ \ -2] \cdot 10^{-3} \text{ Wb}$$

What is the flux linking the b-phase (i.e. λ_{bs})?

$\Rightarrow$ The conductor pattern repeats twice, so $P = 4$

$\Rightarrow$ $S_s = 12$

$S_s / P = 3$

$$W_{bs1} = \frac{1}{2} \sum_{i=1}^{4} N_{bs,i} = \frac{1}{2}(2+4+2) = 4$$

$$W_{bs2} = W_{bs1} - N_{as1} = 4 - 2 = 2$$

$$W_{bs3} = 2 - 4 = -2$$

$$W_{bs4} = -2 - 2 = -4$$

$$W_{bs5} = -4 - -2 = -2$$

$$W_{bs6} = -2 - -4 = 2$$

$$W_{bs} = [4 \ \ 2 \ \ -2 \ \ -4 \ \ -2 \ \ 2 \ \ 4 \ \ 2 \ \ -2 \ \ -4 \ \ -2 \ \ 2]$$

$$\lambda_{bs} = \sum W_{bs,i} \Phi_{bs,i}$$

$$= [-2(2) + 2(-2) - 2(2) + 2(-2)] \cdot 10^{-3}$$

$$= -16 \cdot 10^{-3} = -0.016 \text{ Wb}$$

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4.) 20 pts. ECE321: Consider a machine with a two-phase stator winding. The winding functions may be expressed

$$w_{as} = 10\cos(2\phi_{sm})$$
$$w_{bs} = 2\sin(2\phi_{sm})$$

The a-phase current is given by

$$i_{as} = 2\cos(100t)$$

Find the b-phase current that will result in a single MMF wave traveling in the forward direction.

20 pts. ECE595: Consider a machine with a two-phase stator winding. The winding functions may be expressed

$$w_{as} = 10\cos(2\phi_{sm})$$
$$w_{bs} = 2\cos(2\phi_{sm} - \pi/4)$$

The a-phase current is given by

$$i_{as} = 2\cos(100t)$$

Find the b-phase current that will result in a single MMF wave traveling in the forward direction.

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$$F_s = W_{as} i_{as} + W_{bs} i_{bs}$$
$$= 20\cos(2\phi_{sm})\cos(100t) + 2\sin(2\phi_{sm})\, i_{bs}$$
$$= 10\cos(2\phi_{sm} + 100t) + 10\cos(2\phi_{sm} - 100t) + 2\sin(2\phi_{sm})\, i_{bs}$$

Let $i_{bs} = A\sin(100t + \phi)$

Then $F_s = 10\cos(2\phi_{sm} + 100t) + 10\cos(2\phi_{sm} - 100t) + A\cos(2\phi_{sm} - 100t - \phi) - A\cos(2\phi_{sm} + 100t + \phi)$

Choose $A = 10$ and $\phi = 0$ to yield

$$F_s = 20\cos(2\phi_{sm} - 100t)$$

which is a forward traveling wave.

$\therefore\ i_{bs} = 10\sin(100t)$

595

$$F_s = W_{as} i_{as} + W_{bs} i_{bs}$$

$$= 10\cos(2\phi_{sm} + 100t) + 10\cos(2\phi_{sm} - 100t)$$

$$+ 2\cos(2\phi_{sm} - \pi/4)\, i_{bs}$$

let $i_{bs} = A\cos(100t + \phi - \pi/4)$

so $F_s = 10\cos(2\phi_{sm} + 100t) + 10\cos(2\phi_{sm} - 100t)$

$$+ A\cos(2\phi_{sm} + 100t + \phi - \pi/2)$$

$$+ A\cos(2\phi_{sm} - 100t - \phi)$$

choose $A = 10$ and $\phi = \pi/2 + \pi = 3\pi/2$

so $F_s = 10\cos(2\phi_{sm} - 100t)$

$$+ 10\cos(2\phi_{sm} - 100t - 3\pi/2)$$

which can be expressed as a single forward wave since both components are forward waves.

$$\therefore\ i_{bs} = 10\cos\left(100t + \frac{3\pi}{2} - \frac{\pi}{4}\right)$$

$$= 10\cos\left(100t + \frac{5\pi}{4}\right)$$

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5) 20 pts. A three-phase brushless DC machine has the following parameters: $r_s = 1\Omega$, $L_{SS} = 10$ mH, $\lambda_m = 0.2$ Vs, $P = 4$. The machine is operating in the steady-state at 50 rad/s (mechanical) and producing 6 Nm of torque. The *d*-axis current is zero. What is the efficiency?

$$\omega_r = 50\left(\frac{P}{2}\right) = 100 \text{ rad/s}$$

$$T_e = \frac{3}{2}\frac{P}{2}\lambda_m i_{qs}^r$$

$$i_{qs}^r = \frac{2}{3}\frac{2}{P}\frac{T_e}{\lambda_m} = \frac{2}{3}\frac{2}{4}\frac{6}{0.2} = 10\text{A}$$

$$i_{ds}^r = 0$$

$$v_{qs}^r = r_s i_{qs}^r + \omega_r L_{ss} i_{ds}^r + \omega_r \lambda_m$$

$$= 1 \cdot 10 + (100)(0.01)(0) + 100(0.2) = 30\text{V}$$

$$v_{ds}^r = \cancel{r_s i_{ds}^r}^{\,0} - \omega_r L_{ss} i_{qs}^r$$

$$= -100(0.01)(2) = -2\text{V}$$

$$P_{in} = \frac{3}{2}\left(v_{qs}^r i_{qs}^r + \cancel{v_{ds}^r i_{ds}^r}^{\,0}\right)$$

$$= \frac{3}{2}(30)(10) = 450\text{W}$$

$$P_{out} = (50)(6) = 300\text{W}$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{300}{450} = \frac{2}{3}$$

$$\eta = 66.7\%$$