ECE321. Spring 2009.

Exam 4 Solution Outline

Problem 1
1a) False
1b) True
1c) False
1d) True
1e) True
1f) False
1g) False
1h) False
1i) False
1j) True

Problem 2

Step 1 - 5 pts

$$w_{as} = 25\cdot\sin\left(4\cdot\varphi_{sm}\right)$$

$$w_{bs} = 25\cdot\cos\left(4\cdot\varphi_{sm}\right)$$

Step 2 - 5 pts

$$F = w_{as}\cdot i_{as} + w_{bs}\cdot i_{bs}$$

$$F = 125\cdot\left(\sin\left(4\cdot\varphi_{sm}\right)\cdot\sin(50\cdot t) + \cos\left(4\cdot\varphi_{sm}\right)\cdot\cos(50\cdot t)\right)$$

$$F = 125\cdot\cos\left(4\cdot\varphi_{sm} - 50\cdot t\right)$$

Speed of MMF relative to stator is 12.5 rad/s CCW

Step 3 - 5 pts

Speed of MMF relative to rotor is thus 12.5 rad/s CCW - - 100 rad/s CCW = 112.5 rad/s
This is the mechanical speed relative to the rotor

Step 4 - 5pts

The frequency of the rotor currents will thus be 112.5 rad/s or

$$\frac{112.5}{2\cdot\pi} = 17.90493 \quad \text{Hz}$$

Problem 3

Step 1 - 10 pts

$$W_c = \frac{1}{2}\cdot i^T \cdot \begin{pmatrix} L_{ss} & L_{sr}\cdot\cos(\theta_r) \\ L_{sr}\cdot\cos(\theta_r) & L_{rr} \end{pmatrix} \cdot i$$

where

$$i = \begin{pmatrix} i_{as} \\ i_{ar} \end{pmatrix}$$

Step 2 - 10 pts

$$T_e = \frac{P}{2}\cdot\left(\frac{d}{d\theta_r} W_c\right)$$

$$T_e = \frac{P}{4}\cdot i^T \cdot \begin{pmatrix} 0 & -L_{sr}\cdot\sin(\theta_r) \\ -L_{sr}\cdot\sin(\theta_r) & 0 \end{pmatrix} \cdot i$$

$$T_e = \frac{-P}{2}\cdot\sin(\theta_r)\cdot L_{sr}\cdot i_{as}\cdot i_{ar}$$

Problem 4

$$L_M := 20.1\cdot 10^{-3}$$

$$r_r := 41.3\cdot 10^{-3}$$

$$P := 4$$

Step 1 - compute Vs, we, speed, and slip (5 pts)

$$V_s := \frac{460}{\sqrt{3}}$$

$$\omega_e := 2\cdot\pi\cdot 60$$

$$\omega_{rm} := 1795 \cdot \frac{2 \cdot \pi}{60}$$

$$\omega_{rm} = 187.97196$$

$$\omega_r := \frac{P}{2} \cdot \omega_{rm}$$

$$S := \frac{\omega_e - \omega_r}{\omega_e} \qquad S = 2.77778 \times 10^{-3}$$

Step 2: Find the stator current (5 pts)

$$Z := \frac{1}{\frac{1}{j \cdot L_M \cdot \omega_e} + \frac{1}{\frac{r_r}{S}}}$$

$$I_{as} := \frac{V_s}{Z}$$

$$\left| I_{as} \right| = 39.33794$$

$$\arg\left(I_{as}\right) \cdot \frac{180}{\pi} = -62.9942$$

Step 3: Find the rotor current (5 pts)

$$I_{ar} := \frac{-V_s}{\frac{r_r}{S}}$$

$$\left| I_{ar} \right| = 17.8626$$

$$\arg\left(I_{ar}\right) \cdot \frac{180}{\pi} = 180$$

Step 4: Find the torque (5 pts)

$$T_e := 3 \cdot \frac{P}{2} \cdot L_M \cdot \mathrm{Re}\left(j \cdot \overline{I_{as}} \cdot I_{ar}\right)$$

$$T_e = 75.50261$$

Problem 5

Step 1 - 10 pts

$$w_{as} = N_s \cdot \sin\left(\varphi_{sm}\right)$$

$$w_{ar} = \frac{N_r}{2} \cdot \sin\left(2\varphi_{rm}\right)$$

Step 2 - 10 pts

$$L_{asar} = \frac{r \cdot l \cdot \mu_0}{g} \cdot \int_0^{2\cdot\pi} N_s \cdot \sin\left(\varphi_{sm}\right) \cdot \left(\frac{N_r}{2} \cdot \sin\left(2 \cdot \varphi_{rm}\right)\right) d\varphi_{sm}$$

$$L_{asar} = \frac{r \cdot l \cdot \mu_0}{g} \cdot \int_0^{2\cdot\pi} N_s \cdot \sin\left(\varphi_{sm}\right) \cdot \left[\frac{N_r}{2} \cdot \sin\left[2\left(\varphi_{sm} - \theta_{rm}\right)\right]\right] d\varphi_{sm}$$

$$L_{asar} = 0$$

This is why the stator and rotor need to have the same number of poles !