Solution

ECE321/ECE595 Exam 4
Spring 2012

Notes: **You must show work for credit.**

This exam has 4 problems and 11 pages.

Note that problems 2 and 4 have different specifications depending on if you are in ECE321 or ECE595.

Handy Facts

$$\mu_0 = 4\pi 10^{-7} \text{ H/m}$$

Table A-1 Trigonometric Identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\cos A \cos B = \tfrac{1}{2}[\cos(A+B) + \cos(A-B)]$$
$$\sin A \sin B = \tfrac{1}{2}[\cos(A-B) - \cos(A+B)]$$
$$\sin A \cos B = \tfrac{1}{2}[\sin(A+B) + \sin(A-B)]$$
$$\sin A + \sin B = 2 \sin \tfrac{1}{2}(A+B) \cos \tfrac{1}{2}(A-B)$$
$$\sin A - \sin B = 2 \sin \tfrac{1}{2}(A-B) \cos \tfrac{1}{2}(A+B)$$
$$\cos A + \cos B = 2 \cos \tfrac{1}{2}(A+B) \cos \tfrac{1}{2}(A-B)$$
$$\cos A - \cos B = -2 \sin \tfrac{1}{2}(A+B) \sin \tfrac{1}{2}(A-B)$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \cos^2 A - \sin^2 A$$
$$\sin \tfrac{1}{2}A = \sqrt{\tfrac{1}{2}(1-\cos A)} \qquad \cos \tfrac{1}{2}A = \sqrt{\tfrac{1}{2}(1+\cos A)}$$
$$\sin^2 A = \tfrac{1}{2}(1-\cos 2A) \qquad \cos^2 A = \tfrac{1}{2}(1+\cos 2A)$$
$$\sin x = \frac{e^{jx} - e^{-jx}}{2j} \qquad \cos x = \frac{e^{jx} + e^{-jx}}{2} \qquad e^{jx} = \cos x + j \sin x$$
$$A \cos(\omega t + \phi_1) + B \cos(\omega t + \phi_2) = C \cos(\omega t + \phi_3)$$

where

$$C = \sqrt{A^2 + B^2 - 2AB\cos(\phi_2 - \phi_1)}$$
$$\phi_3 = \tan^{-1}\left[\frac{A \sin \phi_1 + B \sin \phi_2}{A \cos \phi_1 + B \cos \phi_2}\right]$$
$$\sin(\omega t + \phi) = \cos\left(\omega t + \phi - \frac{\pi}{2}\right)$$

Taken from, *Continuous and Discrete Signal and Systems Analysis, 2nd Edition,* by McGillem & Cooper, 1984, CBS College Publishing, and one heck of a good book.

1) 25 pts. The stator and rotor winding functions of a two-phase machine are given by

$$w_{as} = 50\cos(4\phi_{sm})$$
$$w_{bs} = 50\sin(4\phi_{sm})$$
$$w_{ar} = 50\sin(4\phi_{rm})$$
$$w_{br} = 50\cos(4\phi_{rm})$$

The currents given by

$$i_{as} = 4\cos(100t)$$
$$i_{bs} = -4\sin(100t)$$
$$i_{ar} = 4\sin(40t)$$
$$i_{br} = 4\cos(40t)$$

where t is time. What is the mechanical speed and direction of the rotor?

$$F_s = W_{as} \cdot i_{as} + W_{bs}\, i_{bs}$$

$$= 200\left(\cos(4\phi_{sm})\cos(100t) - \sin(4\phi_{sm})\sin(100t)\right)$$

$$= 200\cos(4\phi_{sm} + 100t)$$

$$\Rightarrow \frac{d\phi_{sm}}{dt} = -25 \text{ rad/s CCW}$$

$$F_r = W_{ar}\, i_{ar} + w_{br}\, i_{br}$$

$$= 200\left(\sin(4\phi_{rm})\sin(40t) + \cos(4\phi_{rm})\cos(40t)\right)$$

$$= 200\cos(4\phi_{rm} - 40t)$$

$$\phi_{sm} = \phi_{rm} + \theta_{rm}$$

$$F_r = 200\cos(4\phi_{sm} - 4\phi_{rm} - 40t)$$

$$\frac{d\phi_{sm}}{dt} = \omega_{rm} + 10 = -25$$

$$\therefore \omega_r = -35 \text{ rad/s CCW}$$
or 35 rad/s CW

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2) 25 pts. ECE321: Consider a single-phase induction machine with one winding on the rotor and one winding on the stator. The mutual inductance between the a-phase stator winding and the a-phase rotor windings is $5\cos(4\theta_{rm})$. Derive an expression for electromagnetic torque in terms of i_{as}, i_{ar}, θ_{rm} and constants and mathematical functions as needed. With suitable ac current, could this device produce constant torque?

ECE595: Consider an induction machine with two windings on the rotor and one winding on the stator. The mutual inductance between the a-phase stator winding and the a-phase rotor windings is $5\cos(4\theta_{rm})$. The mutual inductance between the a-phase stator winding and the b-phase rotor windings is $5\sin(4\theta_{rm})$. Derive an expression for electromagnetic torque in terms of i_{as}, i_{ar}, i_{br}, θ_{rm} and constants and mathematical functions as needed. With suitable ac currents, could this device produce constant torque?

595:

$$W_c = \frac{1}{2}\begin{bmatrix} I_{as} \\ I_{ar} \\ I_{br} \end{bmatrix}^T \begin{bmatrix} \star & 5\cos(4\theta_{rm}) & \sin(4\theta_{rm}) \\ 5\cos(4\theta_{rm}) & \star & \star \\ 5\sin(4\theta_{rm}) & \star & \star \end{bmatrix} \begin{bmatrix} I_{as} \\ I_{ar} \\ I_{br} \end{bmatrix}$$

$\star$ = Constant.

$$T_e = \frac{\partial W_c}{\partial \theta_{rm}} = 10\begin{bmatrix} I_{as} \\ I_{ar} \\ I_{br} \end{bmatrix}^T \begin{bmatrix} 0 & -\sin 4\theta_{rm} & \cos 4\theta_{rm} \\ -\sin 4\theta_{rm} & 0 & 0 \\ \cos 4\theta_{rm} & 0 & 0 \end{bmatrix} \begin{bmatrix} I_{as} \\ I_{ar} \\ I_{br} \end{bmatrix}$$

$$= 20\left[- i_{as} i_{ar} \sin 4\theta_{rm} + I_{as} I_{br} \cos 4\theta_{rm} \right]$$

321:

$$W_c = \frac{1}{2}\begin{bmatrix} I_{as} \\ I_{ar} \end{bmatrix}^T \begin{bmatrix} \star & 5\cos 4\theta_{rm} \\ 5\cos 4\theta_{rm} & \star \end{bmatrix} \begin{bmatrix} I_{as} \\ I_{ar} \end{bmatrix}$$

$\star$ = Constant.

$$T_e = \frac{\partial W_c}{\partial \theta_{rm}} = 10\begin{bmatrix} I_{as} \\ I_{ar} \end{bmatrix}^T \begin{bmatrix} 0 & -\sin 4\theta_{rm} \\ -\sin 4\theta_{rm} & 0 \end{bmatrix} \begin{bmatrix} I_{as} \\ I_{ar} \end{bmatrix}$$

$$= -20 I_{ar} I_{as} \sin 4\theta_{rm}$$

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3.) 25 pts. Recall that

$$v_{qr}^{'s} = r_r^{'} i_{qr}^{'s} - \omega_r \lambda_{dr}^{'s} + p\lambda_{qr}^{'s}$$
$$\lambda_{qr}^{'s} = L_{lr}^{'} i_{qr}^{'s} + L_{ms}\left(i_{qs}^{s} + i_{qr}^{'s}\right)$$
$$s = \frac{\omega_e - \omega_r}{\omega_e}$$

Based on these two equations, and the knowledge of phasor relationships and transformation theory, derive the phasor model rotor voltage equation. You must show work for credit.

$$\tilde{f}_{qr}^{'s} = \tilde{f}_{ar}^{'}$$
$$\tilde{f}_{dr}^{'s} = -\tilde{f}_{br}^{'}$$
$$\tilde{f}_{qr}^{'s} = -j\tilde{f}_{dr}^{'s} \Rightarrow \tilde{f}_{dr}^{'s} = j\tilde{f}_{qr}^{'s}$$

$$V_{qr}^{'s} = r_r' i_{qr}^{'s} - \omega_r \lambda_{dr}^{s} + p\lambda_{qr}^{'s}$$
$$= r_r' i_{qr}^{'s} - \omega_r j\lambda_{qr}^{'s} + j\omega_e \lambda_{qr}^{'s}$$
$$= r_r' i_{qr}^{'s} + j(\omega_e - \omega_r)\left[L_{lr}' i_{qr}^{'s} + L_{ms}(i_{qs}^{s} + i_{qr}^{'s})\right]$$
$$= r_r' i_{qr}^{'s} + js\cdot\omega_e\left[L_{lr}' i_{qr}^{'s} + L_{ms}(i_{qs}^{s} + i_{qr}^{'s})\right]$$

$$\frac{V_{qr}'}{s} = \frac{r_r'}{s} i_{qr}^{'s} + j\omega_e L_{lr}' i_{qr}^{'s} + j\omega_e L_{ms}(i_{qs}^{s} + i_{qr}^{'s})$$

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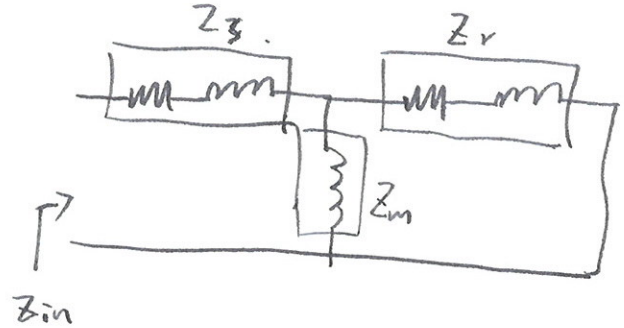
4) 25 pts. Consider a 3-phase machine with the following parameters: $r_s = 72.5\, m\Omega$, $L_{ls} = L_{lr}^{'} = 1.32$ mH, $L_m = 20.1$ mH, $r_r^{'} = 41.3\, m\Omega$, and $P = 4$. A balanced 3-phase voltage source with of $v_{as} = 376\cos(377t)$ is applied to the machine.

Part A: Suppose the rotor is blocked. What is the steady-state a-phase current expressed in the time domain? What is the rms rotor current (referred)? What is the efficiency? ECE595: What is the input power? How much torque is produced?

Part B: Suppose the machine is not loaded (i.e. the load torque is zero). What is the no-load stator current expressed in the time doman? What is the rms rotor current (referred) ? What is the efficiency? ECE595: What is the input power?

Part A



$w_r = 0 \Rightarrow S = 1$

$Z_s = 72.5m + jW_e 1.32m$

$Z_m = jW_e 20.1m$

$Z_r = 41.3m + jW_e 1.32m$

$Z_{in} = Z_s + Z_m // Z_r = 0.1089 + j0.9647$

$\tilde{I}_{as} = \frac{376\angle 0}{\sqrt{2}} \frac{1}{Z_{in}} = 30.704 - j272.108$

$I_{as} = 387.26 \cos(377t - 1.4584)$

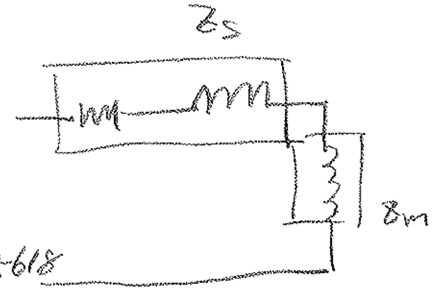
$\tilde{I}_{ar} = -I_{as} \frac{Z_m}{Z_m + Z_r} = 256.96\angle 1.6883$

$I_{ar} = 363.39 \cos(377t + 1.6883)$

$P_{in} = 3\, Re\,(V_{as} I_{as}^{*}) = 24.49kW$ $\eta = 0$ as $P_{out} = 0$

$T_e = 3 \frac{P}{2} Re[j I_{as}^{*} I_{ar}] L_m = 43.3989\, N\cdot m$

Part B:



$$I_{as} = \frac{V_{as}}{Z_s + Z_m} = 32.9226\angle -1.5618$$

$$\therefore i_{as} = 45.56\cos(377t - 1.56)$$

$I'_{ar} = 0$ as $T_e = 0$

$\eta = 0$ as $P_{out} = 0$

$P_{in} = 3\,\mathrm{Re}(V_{as} I_{as}^*) = 236W$.

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