ECE321. Spring 2009.

Exam 5 Solution Outline

Problem 1

$w := 0.01$ $\quad g := 0.001$

$w_s := 0.05$ $\quad N := 100$

$d_s := 0.02$ $\quad \mu_r := 1000$

$d := 0.05$ $\quad \mu_0 := 4\cdot\pi\cdot10^{-7}$

Step 1: Lets compute how much flux we are talking about (5 pts)

$B := 1.2$

$\Phi := B\cdot w\cdot d$

$\Phi = 6\times10^{-4}$

Step 2: Lets find the reluctance of the MEC (10 pts)

$$R_{gap} := \frac{g}{d\cdot w\cdot\mu_0} \qquad R_{gap} = 1.59155\times10^{6}$$

$$R_I := \frac{w_s + \frac{w}{2} + \frac{w}{2}}{d\cdot w\cdot\mu_0\cdot\mu_r} \qquad R_I = 9.5493\times10^{4}$$

$$R_U := \frac{2\cdot\left(d_s + \frac{w}{2}\right) + w_s + w}{d\cdot w\cdot\mu_0\cdot\mu_r} \qquad R_U = 1.7507\times10^{5}$$

$$R_{total} := R_U + R_I + 2\cdot R_{gap} \qquad R_{total} = 3.45366\times10^{6}$$

Step 3: Let's find the current (5 pts)

$$\Phi = \frac{N \cdot i}{R_{total}}$$

$$i := \frac{\Phi \cdot R_{total}}{N}$$

$$i = 20.72197$$

Problem 2

Step 1: Let's find the co-energy. (10 pts)
Step 1a: Let's bring up i1 with i2=0

$$W_{c1} = \int_0^{i_1} 2 \cdot i_1 + \frac{5}{3 + x^2} \cdot \left(1 - e^{-2 \cdot i_1}\right) di_1$$

$$W_{c1} = i_1^2 + \frac{5}{3 + x^2} \cdot \left(i_1 + \frac{1}{2} \cdot e^{-2i_1}\right)$$ evaluated between 0 and i1

$$W_{c1} = i_1^2 + \frac{5}{3 + x^2} \cdot \left[i_1 + \frac{1}{2}\left(e^{-2 \cdot i_1} - 1\right)\right]$$

Step 1b: Let's bring up i2 with i1 fixed

$$W_{c2} = \int_0^{i_2} 4 \cdot i_2 + \frac{10}{3 + x^2} \cdot \left[1 - e^{-\left(2 \cdot i_1 + 4 \cdot i_2\right)}\right] di_1$$

$$W_{c2} = 2 \cdot i_2^2 + \frac{10}{3 + x^2} \cdot \left[i_2 + \frac{1}{4} \cdot e^{-\left(2i_1 + 4 \cdot i_2\right)}\right]$$ evaluated between 0 and i2

$$W_{c2} = 2\cdot i_2^{\,2} + \frac{10}{3+x^2}\cdot\left[i_2 + \frac{1}{4}\cdot e^{-\left(2\cdot i_1+4\cdot i_2\right)} - \frac{1}{4}\cdot e^{-2\cdot i_1}\right]$$

Step 1c (adding the terms together)

$$W_c = i_1^{\,2} + 2\cdot i_2^{\,2} + \frac{5}{3+x^2}\left(i_1 + 2\cdot i_2 + \frac{1}{2}\cdot e^{-2\cdot i_1-4\cdot i_2} - \frac{1}{2}\right)$$

Step 2 (find force) (5pts)

$$f_e = \frac{-10x}{\left(3+x^2\right)}\cdot\left(i_1 + 2\cdot i_2 + \frac{1}{2}\cdot e^{-2\cdot i_1-4\cdot i_2} - \frac{1}{2}\right)$$

Step 3 (now for the numbers)

$i_1 := 2$ $\quad i_2 := 3$ $\quad x := 1$

$$f_e := \frac{-10x}{\left(3+x^2\right)}\cdot\left(i_1 + 2\cdot i_2 + \frac{1}{2}\cdot e^{-2\cdot i_1-4\cdot i_2} - \frac{1}{2}\right)$$

$f_e = -18.75$

Problem 3 [See comments on Exam on web page. This problem discounted.]

$v_{dc} := 100$ $\quad v_{fd} := 1$ $\quad L_{aa} := 3\cdot 10^{-3}$ $\quad d := 0.7$

$v_{fsw} := 2$ $\quad r_a := 0.1$ $\quad k_v := 0.1$ $\quad f_{sw} := 5000$

We know that

$$i_{abar} = \frac{d\cdot(v_{dc} - v_{fsw}) - (1-d)\cdot v_{fsw} - k_v\cdot\omega_r}{r_a}$$

and that

$$\Delta i = \frac{1}{f_{sw}\cdot L_{aa}}(v_{dc} - v_{fsw} + v_{fd})\cdot d\cdot(1-d)$$

One the boundary,

$$i_{abar} = \frac{1}{2}\cdot\Delta i$$

Thus

$$\frac{d\cdot(v_{dc} - v_{fsw}) - (1-d)\cdot v_{fsw} - k_v\cdot\omega_r}{r_a} = \frac{1}{2}\cdot\left[\frac{1}{f_{sw}\cdot L_{aa}}(v_{dc} - v_{fsw} + v_{fd})\cdot d\cdot(1-d)\right]$$

which can be solved for

$$\omega_r = \frac{r_a\cdot\left[\frac{d\cdot(v_{dc} - v_{fsw}) + v_{fsw}\cdot(d-1)}{r_a} + \frac{d\cdot(d-1)\cdot(v_{dc} + v_{fd} - v_{fsw})}{2\cdot L_{aa}\cdot f_{sw}}\right]}{k_v}$$

Putting in the numbers

$$\omega_r := \frac{r_a\cdot\left[\frac{d\cdot(v_{dc} - v_{fsw}) + v_{fsw}\cdot(d-1)}{r_a} + \frac{d\cdot(d-1)\cdot(v_{dc} + v_{fd} - v_{fsw})}{2\cdot L_{aa}\cdot f_{sw}}\right]}{k_v}$$

$\omega_r = 679.307$ this is in rads/

Problem 4

$$\lambda_{abr} = (L_{lr} + L_{ms}) \cdot i_{abr} + L_{ms} \cdot \begin{pmatrix} \cos(\theta_r) & \sin(\theta_r) \\ -\sin(\theta_r) & \cos(\theta_r) \end{pmatrix} \cdot i_{abs} + \lambda_m \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$K_{rr} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$K_{sr} = \begin{pmatrix} \sin(\theta_r) & -\cos(\theta_r) \\ \cos(\theta_r) & \sin(\theta_r) \end{pmatrix}$$

$$\lambda_{qdr} = K_{rr} \cdot \left[(L_{lr} + L_{ms}) \cdot i_{abr} + L_{ms} \cdot \begin{pmatrix} \cos(\theta_r) & \sin(\theta_r) \\ -\sin(\theta_r) & \cos(\theta_r) \end{pmatrix} \cdot i_{abs} + \lambda_m \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right]$$

$$\lambda_{qdr} = K_{rr} \cdot \left[(L_{lr} + L_{ms}) \cdot K_{rr}^{-1} \cdot i_{qdr} + L_{ms} \cdot \begin{pmatrix} \cos(\theta_r) & \sin(\theta_r) \\ -\sin(\theta_r) & \cos(\theta_r) \end{pmatrix} \cdot K_{sr}^{-1} \cdot i_{qds} + \lambda_m \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right]$$

$$\lambda_{qdr} = (L_{lr} + L_{ms}) \cdot i_{qdr} + L_{ms} \cdot K_{rr} \cdot \begin{pmatrix} \cos(\theta_r) & \sin(\theta_r) \\ -\sin(\theta_r) & \cos(\theta_r) \end{pmatrix} \cdot K_{sr}^{-1} \cdot i_{qds} + \lambda_m \cdot \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \cdot \begin{pmatrix} \cos(\theta_r) & \sin(\theta_r) \\ -\sin(\theta_r) & \cos(\theta_r) \end{pmatrix} \cdot \begin{pmatrix} \sin(\theta_r) & -\cos(\theta_r) \\ \cos(\theta_r) & \sin(\theta_r) \end{pmatrix}^{-1} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\lambda_{qdr} = (L_{lr} + L_{mr}) i_{qdr} - L_{ms} \cdot i_{qds} - \lambda_m \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Problem 5.

$P := 4$ $\qquad L_{ss} := 2 \cdot 10^{-3}$

$\lambda_m := 0.05$ $\qquad T_e := 20$

$r_s := 0.02$ $\qquad \omega_{rm\_rpm} := 5000$

Step 1 - let's get the speed (5 pts)

$$\omega_{rm} := \omega_{rm\_rpm} \cdot \frac{2 \cdot \pi}{60}$$

$$\omega_r := \frac{P}{2} \cdot \omega_{rm}$$

$$\omega_r = 1.0472 \times 10^3$$

Step 2 - q- and d-axis currents (5 pts)

$$i_q := \frac{T_e}{\frac{3}{2} \cdot \frac{P}{2} \cdot \lambda_m} \qquad i_q = 133.33333$$

$$i_d := 0$$

Step 3 - voltages (5 pts)

$$v_q := r_s \cdot i_q + \omega_r \cdot L_{ss} \cdot i_d + \omega_r \cdot \lambda_m$$

$$v_q = 55.02654$$

$$v_d := r_s \cdot i_d - \omega_r \cdot L_{ss} \cdot i_q$$

$$v_d = -279.25268$$

Step 4 - input and output power and efficiency (5 pts)

$$P_{in} := \frac{3}{2} \cdot \left(v_q \cdot i_q + v_d \cdot i_d\right)$$

$$P_{in} = 1.10053 \times 10^4$$

$$P_{out} := \omega_{rm} \cdot T_e$$

$$P_{out} = 1.0472 \times 10^4$$

$$\eta := \frac{P_{out}}{P_{in}}$$

$$\eta \cdot 100 = 95.15385$$