EE321 Exam 5
Spring 2010

Notes**: You must show work for credit.**

Trigonometric identities are towards the back.

The last page of exam is blank for extra paper if needed.

1) 20 pts. A winding has a resistance of 0.1 Ohms. The flux linkage of the device obeys the relationship

$$\lambda = \frac{1 - e^{-3i}}{5 + x^2}$$

Suppose the current is held constant at 0.2 A, while the position is moved in accordance with

$$x = 10t$$

Compute the electromagnetic force, winding voltage, and power into the device at $t = 0.1$ s.

2) 20 pts. The conductor density of the a-phase of a reluctance machine may be expressed

$$n_{as} = N_s \cos\left(\frac{P}{2}\phi_{sm}\right)$$

where N_s is the peak conductor density, P is the number of poles, and ϕ_{sm} is position measured relative to the stator, as usual. The machine has an airgap which may be expressed

$$g = \frac{1}{A - B\cos\left(P\phi_{rm}\right)}$$

where A and B are constants, and ϕ_{rm} is position measured relative to the rotor, as is our custom. Find an expression for the a-phase self-inductance in terms of mechanical rotor position, θ_{rm} (as defined, again, in our usual way) and the constants N_s P, A, B, the stator radius, r_s, and the machine length, l.

(extra paper for problem 2)

3.) 20 pts. A permanent magnet dc machine has $k_v = 0.2$ Vs and $r_a = 100\,\mathrm{m\Omega}$. It is fed from a converter with an input voltage of 100 V, a forward diode drop of 2 V, and a forward switch drop of 2.4 V. Suppose the duty cycle is 0.7, and the speed is 300 rad/s. Find the average armature current, the average switch current, the converter efficiency, the motor efficiency, and the system efficiency. You may assume continuous operation.

4.) 20 pts. A three phase brushless DC machine has the following parameters: $r_s = 0.3\Omega$, $L_{ss} = 10$ mH, $\lambda_m = 0.17$ Vs, $P = 4$. The machine is running happily at 4000 RPM. Then, horrific tragedy strikes. The inverter fails, and the machine terminals effectively become short-circuited. Assuming the speed stays constant (for a while, anyway) find (i) the rms phase current, (ii) the electromagnetic torque, (iii) the power being dissipated by the machines windings. In making your calculations, you may assume steady-state operation.

5.) 20 pts (2 pts each). Short Answer

a.) Name a physical effect that reduces the accuracy of the co-energy approach to finding force and torque.

b) Which machine would you use if you wished to have open-loop position control?

c) What is the best method to control a brushless dc machine, if you want the device to look like a torque transducer?

d) Describe the effect of rotor resistance on the torque-speed curve of an induction machine fed from a constant voltage constant frequency source.

e) A fundamental tradeoff in electromechanical devices is between what and what?

f) What type of machine dominates world power consumption?

g) Referring to a stepper motor’s torque-position characteristic, what is the difference between a stable and unstable equilibrium?

h) What assumption in the analysis of single-stack stepper motors allows them to be treated (for analysis purposes) like multi-stack machines.

i) Of the converter topologies we studied for dc machines, which would be most appropriate for a hybrid vehicle (assume a single, fixed gear ratio with no reverse gear)?

j) Why is it more difficult to control an induction machine than a brushless dc machine?

Trigonometric Identities

Table A-1 Trigonometric Identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\cos A \cos B = \tfrac{1}{2}[\cos(A+B) + \cos(A-B)]$$
$$\sin A \sin B = \tfrac{1}{2}[\cos(A-B) - \cos(A+B)]$$
$$\sin A \cos B = \tfrac{1}{2}[\sin(A+B) + \sin(A-B)]$$
$$\sin A + \sin B = 2\sin\tfrac{1}{2}(A+B)\cos\tfrac{1}{2}(A-B)$$
$$\sin A - \sin B = 2\sin\tfrac{1}{2}(A-B)\cos\tfrac{1}{2}(A+B)$$
$$\cos A + \cos B = 2\cos\tfrac{1}{2}(A+B)\cos\tfrac{1}{2}(A-B)$$
$$\cos A - \cos B = -2\sin\tfrac{1}{2}(A+B)\sin\tfrac{1}{2}(A-B)$$
$$\sin 2A = 2\sin A\cos A$$
$$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \cos^2 A - \sin^2 A$$
$$\sin\tfrac{1}{2}A = \sqrt{\tfrac{1}{2}(1-\cos A)} \qquad \cos\tfrac{1}{2}A = \sqrt{\tfrac{1}{2}(1+\cos A)}$$
$$\sin^2 A = \tfrac{1}{2}(1-\cos 2A) \qquad \cos^2 A = \tfrac{1}{2}(1+\cos 2A)$$
$$\sin x = \frac{e^{jx} - e^{-jx}}{2j} \qquad \cos x = \frac{e^{jx} + e^{-jx}}{2} \qquad e^{jx} = \cos x + j\sin x$$
$$A\cos(\omega t + \phi_1) + B\cos(\omega t + \phi_2) = C\cos(\omega t + \phi_3)$$

where

$$C = \sqrt{A^2 + B^2 - 2AB\cos(\phi_2 - \phi_1)}$$
$$\phi_3 = \tan^{-1}\left[\frac{A\sin\phi_1 + B\sin\phi_2}{A\cos\phi_1 + B\cos\phi_2}\right]$$
$$\sin(\omega t + \phi) = \cos\left(\omega t + \phi - \frac{\pi}{2}\right)$$

Taken from, *Continuous and Discrete Signal and Systems Analysis, 2nd Edition,* by McGillem & Cooper, 1984, CBS College Publishing, and one heck of a good book.

Even More Trigonometric Identities

TRIGONOMETRIC RELATIONS, CONSTANTS AND CONVERSION FACTORS, AND ABBREVIATIONS

TRIGONOMETRIC RELATIONS

$$\cos^2 x + \cos^2\left(x - \frac{2\pi}{3}\right) + \cos^2\left(x + \frac{2\pi}{3}\right) = \frac{3}{2}$$

$$\sin^2 x + \sin^2\left(x - \frac{2\pi}{3}\right) + \sin^2\left(x + \frac{2\pi}{3}\right) = \frac{3}{2}$$

$$\sin x \cos x + \sin\left(x - \frac{2\pi}{3}\right)\cos\left(x - \frac{2\pi}{3}\right) + \sin\left(x + \frac{2\pi}{3}\right)\cos\left(x + \frac{2\pi}{3}\right) = 0$$

$$\cos x + \cos\left(x - \frac{2\pi}{3}\right) + \cos\left(x + \frac{2\pi}{3}\right) = 0$$

$$\sin x + \sin\left(x - \frac{2\pi}{3}\right) + \sin\left(x + \frac{2\pi}{3}\right) = 0$$

$$\sin x \cos y + \sin\left(x - \frac{2\pi}{3}\right)\cos\left(y - \frac{2\pi}{3}\right) + \sin\left(x + \frac{2\pi}{3}\right)\cos\left(x + \frac{2\pi}{3}\right) = \frac{3}{2}\sin(x - y)$$

$$\sin x \sin y + \sin\left(x - \frac{2\pi}{3}\right)\sin\left(y - \frac{2\pi}{3}\right) + \sin\left(x + \frac{2\pi}{3}\right)\sin\left(y + \frac{2\pi}{3}\right) = \frac{3}{2}\cos(x - y)$$

$$\cos x \sin y + \cos\left(x - \frac{2\pi}{3}\right)\sin\left(y - \frac{2\pi}{3}\right) + \cos\left(x + \frac{2\pi}{3}\right)\sin\left(y + \frac{2\pi}{3}\right) = -\frac{3}{2}\sin(x - y)$$

$$\cos x \cos y + \cos\left(x - \frac{2\pi}{3}\right)\cos\left(y - \frac{2\pi}{3}\right) + \cos\left(x + \frac{2\pi}{3}\right)\cos\left(y + \frac{2\pi}{3}\right) = \frac{3}{2}\cos(x - y)$$

$$\sin x \cos y + \sin\left(x + \frac{2\pi}{3}\right)\cos\left(y - \frac{2\pi}{3}\right) + \sin\left(x - \frac{2\pi}{3}\right)\cos\left(y + \frac{2\pi}{3}\right) = \frac{3}{2}\sin(x + y)$$

$$\sin x \sin y + \sin\left(x + \frac{2\pi}{3}\right)\sin\left(y - \frac{2\pi}{3}\right) + \sin\left(x - \frac{2\pi}{3}\right)\sin\left(y + \frac{2\pi}{3}\right) = -\frac{3}{2}\cos(x + y)$$

$$\cos x \sin y + \cos\left(x + \frac{2\pi}{3}\right)\sin\left(y - \frac{2\pi}{3}\right) + \cos\left(x - \frac{2\pi}{3}\right)\sin\left(y + \frac{2\pi}{3}\right) = \frac{3}{2}\sin(x + y)$$

$$\cos x \cos y + \cos\left(x + \frac{2\pi}{3}\right)\cos\left(y - \frac{2\pi}{3}\right) + \cos\left(x - \frac{2\pi}{3}\right)\cos\left(y + \frac{2\pi}{3}\right) = \frac{3}{2}\cos(x + y)$$

Taken from, *Analysis of Electric Machinery and Drive Systems*, 2nd Edition, by Krause, Wasyncuk, and Sudhoff, 2002, Wiley Press, and also a heck of a good book.

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