

**EE321 Exam 5**  
**Spring 2011**

**Notes: You must show work for credit on Problems 1-3.**

**Good luck!**

## Handy Facts

$$\mu_0 = 4\pi 10^{-7} \text{ H/m}$$

**Table A-1** Trigonometric Identities

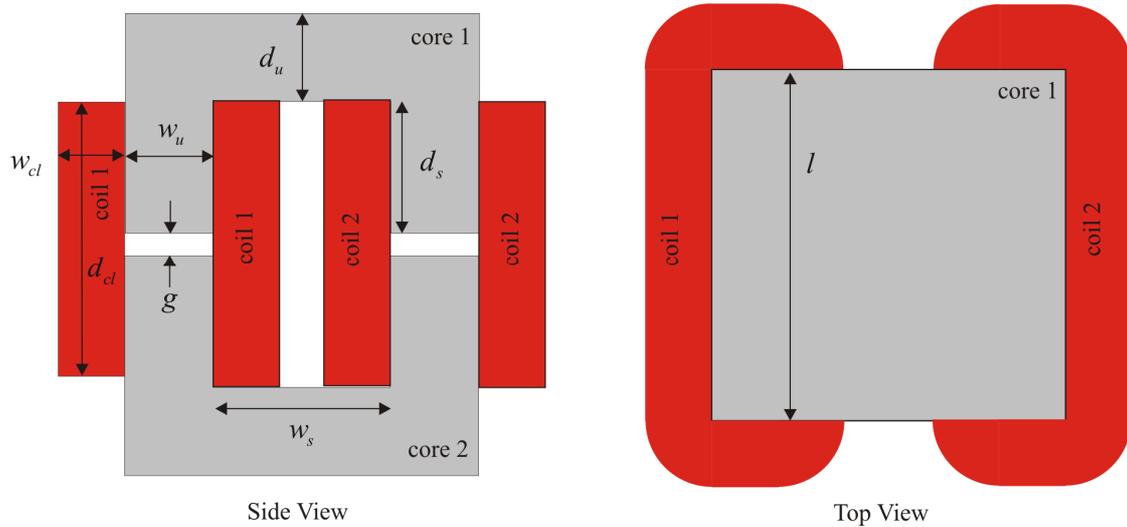
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$$\begin{aligned} \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \cos A \cos B &= \frac{1}{2}[\cos(A+B) + \cos(A-B)] \\ \sin A \sin B &= \frac{1}{2}[\cos(A-B) - \cos(A+B)] \\ \sin A \cos B &= \frac{1}{2}[\sin(A+B) + \sin(A-B)] \\ \sin A + \sin B &= 2 \sin \frac{1}{2}(A+B) \cos \frac{1}{2}(A-B) \\ \sin A - \sin B &= 2 \sin \frac{1}{2}(A-B) \cos \frac{1}{2}(A+B) \\ \cos A + \cos B &= 2 \cos \frac{1}{2}(A+B) \cos \frac{1}{2}(A-B) \\ \cos A - \cos B &= -2 \sin \frac{1}{2}(A+B) \sin \frac{1}{2}(A-B) \\ \sin 2A &= 2 \sin A \cos A \\ \cos 2A &= 2 \cos^2 A - 1 = 1 - 2 \sin^2 A = \cos^2 A - \sin^2 A \\ \sin \frac{1}{2}A &= \sqrt{\frac{1}{2}(1 - \cos A)} \quad \cos \frac{1}{2}A = \sqrt{\frac{1}{2}(1 + \cos A)} \\ \sin^2 A &= \frac{1}{2}(1 - \cos 2A) \quad \cos^2 A = \frac{1}{2}(1 + \cos 2A) \\ \sin x &= \frac{e^{jx} - e^{-jx}}{2j} \quad \cos x = \frac{e^{jx} + e^{-jx}}{2} \quad e^{jx} = \cos x + j \sin x \\ A \cos(\omega t + \phi_1) + B \cos(\omega t + \phi_2) &= C \cos(\omega t + \phi_3) \\ \text{where} \\ C &= \sqrt{A^2 + B^2 - 2AB \cos(\phi_2 - \phi_1)} \\ \phi_3 &= \tan^{-1} \left[ \frac{A \sin \phi_1 + B \sin \phi_2}{A \cos \phi_1 + B \cos \phi_2} \right] \\ \sin(\omega t + \phi) &= \cos \left( \omega t + \phi - \frac{\pi}{2} \right) \end{aligned}$$


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Taken from, *Continuous and Discrete Signal and Systems Analysis, 2<sup>nd</sup> Edition*, by McGillem & Cooper, 1984, CBS College Publishing, and one heck of a good book.

- 1.) 25 pts. Consider the UU core inductor below. This inductor has two coils which slide on the cores. Each coil has  $N$  turns and is wound such that positive current causes positive flux in the clockwise direction. The permeability of air is denoted  $\mu_0$ , the permeability of the magnetic material is  $\mu_r \mu_0$ , where  $\mu_r$  is the relative permeability of the material. Derive an expression for the electromagnetic force between the U-cores in terms of the geometrical parameters,  $N$ ,  $\mu_0$ ,  $\mu_r$  and the coil currents  $i_1$  and  $i_2$ . You may assume magnetic linearity, and neglect fringing and leakage flux components. You may also assume that the dimensions are such that the coils are not being crushed by the cores ( $2d_s + g > d_{cl}$ ) even though in the picture  $2d_s + g = d_{cl}$ .



(more paper next page)

This page is paper for problem 1.

2.) 25 pts. Consider a dc machine with the following parameters:  $k_v = 0.2$  Vs and  $r_a = 100$  m $\Omega$ . It is fed by a single quadrant dc/dc converter (a buck converter, as we studied in class) with  $v_{fd} = 2$  V,  $v_{fsw} = 2.4$  V, and  $v_{dc} = 150$  V. If the duty cycle is 0.5 and the speed is 2000 rpm, find the average armature current, the average switch current, and the system efficiency. Assume continuous operation.

3.) 25 pts. Consider the voltage equations of a two-phase system

$$\mathbf{v}_{abs} = r_s \mathbf{i}_{abs} + p \boldsymbol{\lambda}_{abs}$$

Now consider the transformation to the time reference frame given by

$$\mathbf{f}_{qds}^t = \mathbf{K}_s^t \mathbf{f}_{abs}$$
$$\mathbf{K}_s^t = \begin{bmatrix} t & 1 \\ 1 & 0 \end{bmatrix}$$

where  $t$  is time. Express the voltage equations for the two-phase system in the time reference frame. Note that it may or not be the case that  $t$  appears in your answer.

(more paper next page)

This is extra paper for problem 3

- 4.) 2 pts. What is the definition of flux?
- 5.) 2 pts. What is a Pareto-optimal front?
- 6.) 2 pts. Is  $\lambda = \frac{1}{1+x^2}i$  magnetically linear?
- 7.) 2 pts. Does the field-energy approach to finding force take into account magnetic hysteresis?
- 8.) 2 pts. For a brushless dc machine, in general does setting  $\phi_v = \text{atan}(\omega_r L_{ss} / r_s)$  yield the most torque for a given voltage, the most efficient operation for a given torque, or both?
- 9.) 2 pts. What is a simple physical interpretation of the winding function?
- 10.) 2 pts. Why do we refer variables in machine and transformer analysis?
- 11.) 2 pts. Why do we use qd0 transformations in machine analysis?
- 12.) 3 pts. Name three uses for the winding function.
- 13.) 2 pts. Of the machines we studied in class, which can be used from a 3-phase ac source w/o any power electronics or converters?
- 14.) 2 pts. Does discontinuous mode of a buck converter fed dc machine tend to occur under light loads or heavy loads?
- 15.) 2 pts. Give an example for excitation sequence of a 3-phase variable reluctance stepper motor in half-step mode.