Solution

321 - 75
595 - 80

ECE321/ECE595 Exam 1
Spring 2013

Notes: **You must show work for credit.**

This exam has 5 problems and 12 pages.

Note that problems 1, 4, and 5 have different specifications depending on if you are in ECE321 or ECE595

Good luck!

Solution

1) 20 pts. Consider a B-field given by

$\mathbf{B} = \sin(10t)\mathbf{a}_x + 0.2\mathbf{a}_y$ [ECE321 Students Use This]

$\mathbf{B} = 0.2yz\sin(10t)\mathbf{a}_x + 5e^z\cos(10t)\mathbf{a}_y + 0.1\mathbf{a}_z$ [ECE595 Students Use This]

where $\mathbf{a}_x$, $\mathbf{a}_y$, and $\mathbf{a}_z$ are unit vectors in the x-, y-, and z- directions, respectively. Consider a surface whose normal direction is aligned with the x-axis. The surface is a rectangle, whose corners are at (x=1,y=1,z=1) and (x=1, y=2, z=4). Four turns of wire are wrapped around the periphery of the rectangular surface. There is no current in the wire. Express the voltage that would be measured across the coil as a function of time (the direction of the coil is the same as the surface described).

$$V = \frac{d\lambda}{dt}$$

$$\lambda = N\Phi$$

$$\Phi = \int B \cdot ds$$

321

$$\Phi = \int_1^2 \int_1^4 (\sin(10t)a_x + 0.2a_y)a_x \, dz \, dy$$

$$= \int_1^2 \int_1^4 \sin(10t) \, dz \, dy$$

$$= 3\sin(10t)$$

10

$$\lambda = N\Phi = 12\sin(10t)$$

$$V = 120\cos(10t)$$

10

~~595~~

$$\cancel{\Phi = \int_1^2 \int_1^4}$$

595

$$\Phi = \int_1^2 \int_1^4 (0.2zy\sin 10t \, a_x + 5e^z\cos(10t)a_y + 0.1a_z)a_x \, dz \, dy$$

$$= \int_1^2 \int_1^4 0.2zy\sin(10t) \, dz \, dy$$

$$= \int_1^2 0.1yz^2\sin(10t) \Big|_1^4 \, dy$$

$$= \int_1^2 0.1y(16-1)\sin 10t \, dy$$

$$\Phi = \int_1^2 1.5\, y \sin(10t)\, dy$$

$$= 0.75\, y^2 \sin(10t) \Big|_1^2$$

$$= 0.75(4-1) \sin(10t)$$

$$= 2.25 \sin(10t)$$

$$\lambda = N\Phi$$

$$= 9 \sin(10t)$$

$$v = 90 \cos(10t)$$

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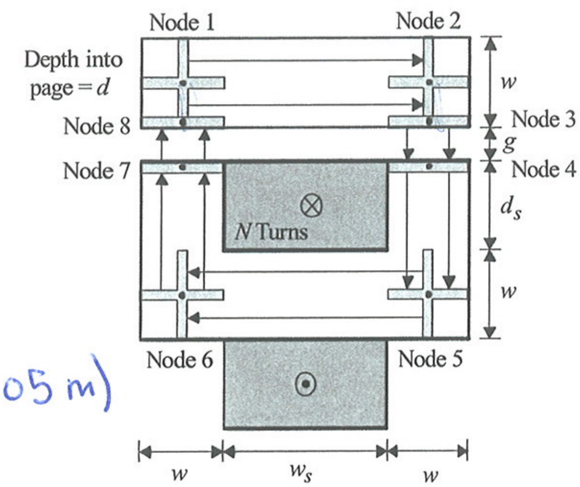
2.) 20 pts. Consider the UI core below. Consider the following parameters: $w = 1$ cm; $w_s = 5$ cm; $d_s = 2$ cm; $d = 5$ cm; $N = 100$. Suppose the material used is such that for a flux density less than 1.5 T (the saturation point), the magnetic material is linear and has a permeability 2000 times that of free space (i.e. a relative permeability of 2000). The air gap g is set such that a current of 50 A produces a flux density of 1.5 T. For this air gap, compute the inductance of the core. *Do not neglect the MMF drop across the magnetic material when working this problem.*



$F = (50\,A)(100)$

$F = 5000$ A

$B = 1.5$ T

$\Phi = B w d$

$= (1.5T)(0.01m)(0.05\,m)$

$= 0.75$ mWb

$$R_{mm} = \frac{\frac{w}{2}2 + 2(w_s + w) + 2(d_s + \frac{w}{2})}{w d \mu_r \mu_0} = \frac{2w_s + 2d_s + 4w}{w d \mu_r \mu_0}$$

would also accept ↓

$= 1.432 \times 10^5\ H^{-1}$ (or $1.353 \times 10^5\ H^{-1}$)

$F = \Phi (R_{mm} + 2R_g)$

$R_g = \frac{1}{2}\left(\frac{F}{\Phi} - R_{mm}\right) = 3.262 \times 10^6$ H (or 3.26×10^6)

$R_g = \frac{g}{\mu_0 w d}$ $\quad g = 2.049$ mm (or 2.052 mm)

$L = \frac{N^2}{2R_g + R_m} = \cancel{2.818}$ 1.5 mH

Easier method

$\lambda = N\Phi$

$= 0.075$

$L = \frac{\lambda}{i} = 1.5$ mH

3) 20 pts. Suppose

$$W_f = (2+x)(\lambda_1 + 2\lambda_2)^2$$

Now consider a trajectory where

$$x = 2t$$
$$\lambda_1 = t$$
$$\lambda_2 = t^2$$

where t is time. What is the field energy at 1 s? How much of this energy was contributed by the mechanical system? How much of this energy was contributed by the electrical system? For conservative fields such as W_c and W_f, the energy stored is a function of state and not how you arrived at that state. Is this true from W_e and W_m?

5

At $t = 1s$

$X = 2$

$\lambda_1 = 1$

$\lambda_2 = 1$

$W_f = (2+2)(1+2)^2 = 4 \cdot 9 = 36 J$

To break down components

7

$f_e = -\frac{\partial W_f}{\partial x} = -(\lambda_1 + 2\lambda_2)^2$

$W_m = -\int f_e dx = -\int_0^1 f_e \frac{dx}{dt} dt$

$\frac{dx}{dt} = 2$

$W_m = -\int_0^1 -(\lambda_1 + 2\lambda_2)^2 \, 2 \, dt$

$= \int_0^1 (t + 2t^2)^2 2 \, dt = \int_0^1 2(t^2 + 4t^3 + 4t^4) dt$

$= 2\left[\frac{1}{3}t^3 + t^4 + \frac{4}{5}t^5\right]\Big|_0^1$

$= 2\left[\frac{1}{3} + 1 + \frac{4}{5}\right] = 4.267 J$

5

$W_e + W_m = W_F$

$W_e = W_F - W_m = 31.73 J$

3

W_e and W_m are a function of path

4) 20 pts. Consider an electromechanical device with the flux linkage equations given below. Compute an expression for torque T_e in terms of i_1, i_2, and θ_{rm}.

$$\lambda_1 = 4i_1 + (5+4\cos(4\theta_{rm}))(i_1+2i_2)^3$$
$$\lambda_2 = i_2 + 2(5+4\cos(4\theta_{rm}))(i_1+2i_2)^3$$

[Use if in ECE321]

$$\lambda_1 = 4i_1 + (5+4\cos(4\theta_{rm}))\frac{i_1+2i_2}{\sqrt{1+(i_1+2i_2)^2}}$$
$$\lambda_2 = i_2 + 2(5+4\cos(4\theta_{rm}))\frac{i_1+2i_2}{\sqrt{1+(i_1+2i_2)^2}}$$

[Use if in ECE595]

321

Step 1 (brings up i_1)

$$W_c = \int_0^{i_{1f}} \lambda_1\Big|_{i_2=0} di_1 + \cancel{\int \lambda_2 \, di_2}^{\,0}$$

7

$$= \int_0^{i_{1f}} 4i_1 + (5+4\cos(4\theta_{rm}))\, i_1^3 \, di_1$$

$$= 2i_1^2 + (5+4\cos(4\theta_{rm}))\frac{i_1^4}{4}\Big|_0^{i_{1f}}$$

$$= 2i_{1f}^2 + \frac{i_{1f}^4}{4}(5+4\cos(4\theta_{rm}))$$

Step 2 (bring up i_2)

$$W_c = \cancel{\int_{i_{1f}}^{i_{1f}} \lambda_1 \, di_1}^{\,0} + \int_0^{i_{2f}} \lambda_2\Big|_{i_1=i_{1f}} di_2$$

$$= \int_0^{i_{2f}} i_2 + 2(5+4\cos(4\theta_{rm}))(i_{1f}+2i_2)^4 \, di_2$$

7

$$= \frac{1}{2}i_2^2 + \frac{2}{4\cdot 2}(5+4\cos(4\theta_{rm}))(i_{1f}+2i_2)^4\Big|_0^{i_{2f}}$$

$$= \frac{1}{2}i_{2f}^2 + \frac{1}{4}(5+4\cos(4\theta_{rm}))\left[(i_{1f}+2i_{2f})^4 - i_{1f}^4\right]$$

Adding the terms

$$W_c = 2 i_{1f}^2 + \frac{1}{2} i_{2f}^2 + \frac{1}{4}(5+4\cos(4\theta_{rm}))(i_{1f}+2i_{2f})^4$$ 6

or

$$W_c = 2 i_1^2 + \frac{1}{2} i_2^2 + \frac{1}{4}(5+4\cos(4\theta_{rm}))(i_1+2i_2)^4$$

$$T_e = \frac{\partial W_c}{\partial \theta_{rm}} = -4\sin(4\theta_{rm})(i_1+2i_2)^4$$

595

$T = 5+4\cos 4\theta_{rm}$

Step 1

$$W_c = \int_0^{i_{1f}} \lambda_1\Big|_{i_2=0} di_1 + \cancel{\int_0^0 \lambda_2 di_2} = \int_0^{i_{1f}} 4 i_1 + T \underbrace{\frac{i_1}{\sqrt{1+i_1^2}}}_{i_1(1+i_1^2)^{-\frac{1}{2}}} di_1$$

$$= 2 i_1^2 + T (1+i_1^2)^{\frac{1}{2}} \Big|_0^{i_{1f}}$$

$$= 2 i_{1f}^2 + T\left[(1+i_{1f}^2)^{\frac{1}{2}} - 1\right]$$

note

$$\int u u'(1+u^2)^{-\frac{1}{2}} dx = (1+u^2)^{\frac{1}{2}}$$

Step 2

$$W_c = \cancel{\int_0^0 \lambda_1 di_1} + \int_0^{i_{2f}} \lambda_2\Big|_{i_1=i_{1f}} di_2$$

$$= \int_0^{i_{2f}} i_2 + \cancel{2(5+4\cos} \; T\, 2\underbrace{\frac{i_{1f}+2i_2}{\sqrt{1+(i_{1f}+2i_2)^2}}}_{2(i_{1f}+2i_2)(1+(i_{1f}+2i_2)^2)^{-\frac{1}{2}}} di_2$$

$$= \frac{1}{2} i_2^2 + T(1+(i_{1f}+2i_2)^2)^{\frac{1}{2}} \Big|_0^{i_{2f}}$$

$$= \frac{1}{2} i_{2f}^2 + T\left[(1+(i_{1f}+2i_{2f})^2)^{\frac{1}{2}} - (1+i_{1f}^2)^{\frac{1}{2}}\right]$$

Adding terms

$$W_c = 2 i_{1f}^2 + \frac{1}{2} i_{2f}^2 + T\left[(1+(i_{1f}+2i_{2f})^2)^{\frac{1}{2}} - 1\right]$$

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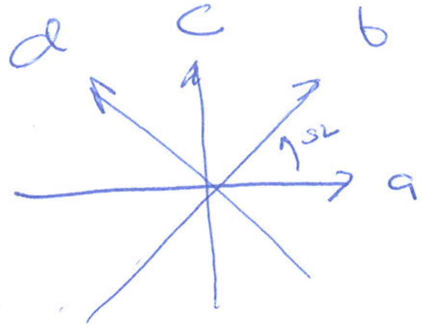
or

$$W_c = 2 i_1^2 + \frac{1}{2} i_2^2 + (5+4\cos 4\theta_{rm})\left[\sqrt{1+(i_1+2i_2)^2} - 1\right]$$

$$T_e = \frac{\partial W_c}{\partial \theta_{rm}} = -16\sin 4\theta_{rm} (\qquad)$$

5) 20 pts. Consider a 4-phase variable reluctance stepper motor with 2 rotor teeth. The 4 is not a typo. Express inductances for the 4-phases (as, bs, cs, and ds) in terms of L_A, L_B, and θ_{rm}. These equations should be written such that an a,b,c,d excitation sequence produces counterclockwise rotation. [ECE595 Only: generalize your expressions to an arbitrary number or rotor teeth]

Take θ_{rm} to be CCW as normal



$$\cancel{4\,TP = SL}$$

$$\cancel{4\left(\frac{2\pi}{RT}\right) = SL}$$

$$4\,SL = TP$$

$$4\,SL = \frac{2\pi}{RT}$$

$$SL = \frac{\pi}{2RT}$$

$$L_{asas} = L_A + L_B \cos(RT\,\theta_{rm})$$

$$L_{bsbs} = L_A + L_B \cos(RT(\theta_{rm} - SL))$$

$$= L_A + L_B \cos\left(RT\left(\theta_{rm} - \frac{\pi}{2RT}\right)\right)$$

$$= L_A + L_B \cos\left(RT\,\theta_{rm} - \frac{\pi}{2}\right)$$

$$L_{cscs} = L_A + L_B \cos(RT(\theta_{rm} - 2SL))$$

$$= L_A + L_B \cos(RT\,\theta_{rm} - \pi)$$

$$L_{dsds} = L_A + L_B \cos(RT(\theta_{rm} - 3SL))$$

$$= L_A + L_B \cos\left(RT\,\theta_{rm} - \frac{3\pi}{2}\right)$$

or $\pi/2$

for 321 replace RT with 2.