Neat Solution

ECE321/ECE595 Exam 2
Spring 2013

Notes: **You must show work for credit.**

This exam has 5 problems and 12 pages.

Note that problems 2, 4, and 5 have different specifications depending on if you are in ECE321 or ECE595

Good luck!

1) 20 pts. Consider a 4-phase multistack VR stepper motor with 8 rotor teeth being fed from a two transistor per stack (phase) circuit which we discussed in class. If the dc voltage is 12 V, the transistor drop is 1 V, and the diode drop is 2 V, sketch the a-phase voltage waveform if the machine is traveling at 50π/16 rad/s (average speed). Quantitatively label the maximum voltage, the minimum voltage, the time duration of the maximum voltage, and the period of the a-phase voltage waveform.

(5)

$$SL = \frac{2\pi}{RT \cdot N} = \frac{2\pi}{8 \cdot 4} = \frac{\pi}{16} = .196 \text{ rad}$$

(5)

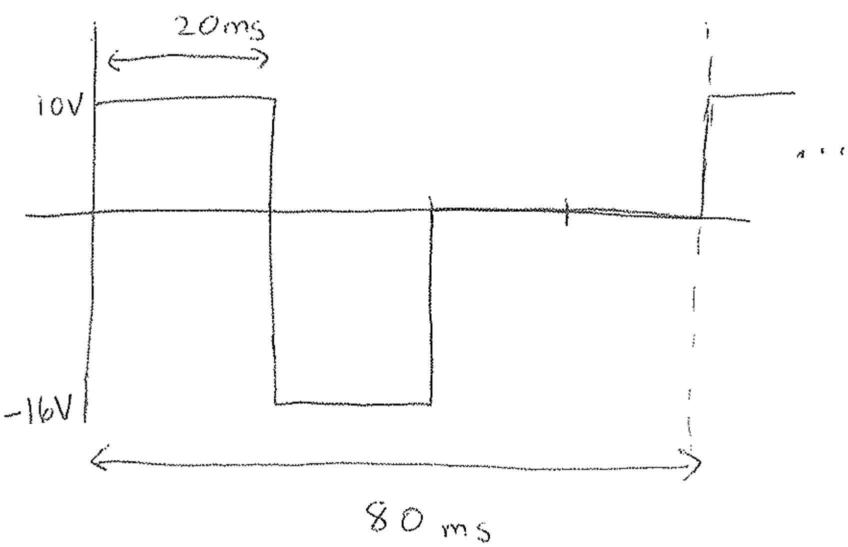
$$SL \cdot f = \frac{50}{16}\pi$$

↑ frequency (steps per second)

$$f = \frac{\frac{50\pi}{16}}{\frac{\pi}{16}} = 50 \text{ Hz}$$

$$T = \frac{1}{f} = 20 \text{ ms}$$

(10)



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2.) 20 pts. An 'effective' armature winding of a separately excited dc machine has a voltage and flux linkage equation of

$$v_{ae} = r_a i_{ae} + p\lambda_{ae}$$
$$\lambda_{ae} = (L_\alpha - L_\beta \cos 2\theta_r) i_{ae} - L_\gamma \sin\theta_r i_f$$

where the 'e' denotes effective winding. This effective winding is switched in at a rotor position of zero. Using a derivational approach similar to the one we used in class, derive the armature voltage equation in terms of armature current i_a, field current i_f, the time derivatives of these currents, rotor speed ω_r, and inductances L_α, L_β, and L_γ. [ECE595: Derive an expression for torque in terms of the inductances, field current, and armature current].

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$$\frac{d\lambda_{ae}}{dt} = (L_\alpha - L_\beta \cos 2\theta_r)\frac{d i_{ae}}{dt} + (L_\beta \sin 2\theta_r) 2\omega_r i_{ae} - L_\gamma \sin\theta_r \frac{d i_f}{dt} - L_\gamma \cos\theta_r \omega_r i_f$$

(10)

setting $\theta_r = 0$

$$\frac{d\lambda_{ae}}{dt} = (L_\alpha - L_\beta)\frac{d i_{ae}}{dt} - L_\gamma \omega_r i_f$$

(10)

$$V_a = r_a i_a + (L_\alpha - L_\beta)\frac{d i_a}{dt} - L_\gamma \omega_r i_f$$

595

$$W_c = \frac{1}{2}\begin{bmatrix} i_{ae} & i_f \end{bmatrix}\begin{bmatrix} L_\alpha - L_\beta \cos 2\theta_r & -L_\gamma \sin\theta_r \\ -L_\gamma \sin\theta_r & L_\gamma \end{bmatrix}\begin{bmatrix} i_{ae} \\ i_f \end{bmatrix}$$

$$T_e = \frac{1}{2}\begin{bmatrix} i_{ae} & i_f \end{bmatrix}\begin{bmatrix} 2L_\beta \overset{0}{\cancel{\sin 2\theta_r}} & -L_\gamma \overset{1}{\cancel{\cos\theta_r}} \\ -L_\gamma \overset{1}{\cancel{\cos\theta_r}} & 0 \end{bmatrix}\begin{bmatrix} i_{ae} \\ i_f \end{bmatrix}$$

$$= -\frac{1}{2} L_\gamma \begin{bmatrix} i_{ae} & i_f \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} i_{ae} \\ i_f \end{bmatrix}$$

$$= -L_\gamma i_{ae} i_f$$

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3) 20 pts. A separately excited dc machine has a armature resistance of 0.1 Ohms, a field resistance of 100 Ohms, and L_{af} of 250 mH. The field current limit is 5 A, the armature current limit is 100 A, and the armature voltage limit is 200 V. What is the maximum torque that could be produced at a speed of 2000 rpm? What is the highest efficiency that can be obtained if the desired torque is 40 Nm at 2000 rpm?

Max torque

with $I_f = I_{f mx}$, $I_a = I_{a,mx}$

$$T_e = L_{AF} I_f I_a = 125 \text{ Nm}$$

$$V_a = r_a I_{amx} + \omega_r L_{AF} I_f = 271.8 \text{ V} \Rightarrow \text{can't achieve}$$

set

(10)

$$I_a = I_{a,mx}$$

$$I_f = \frac{V_{a,mx} - r_a I_{amx}}{L_{AF}\omega_r} = 3.629 \text{ A}$$

$$T_e = L_{AF} I_a I_f = 90.7 \text{ Nm}$$

Highest Efficiency

$$P_{loss} = V_a I_a + V_f I_f - T_e \omega_r$$

$$= [r_a I_a + \omega_r L_{AF} I_f] I_a + r_f I_f^2 - T_e \omega_r$$

$$= r_a I_a^2 + r_f I_f^2$$

(10)

$$= r_a I_a^2 + r_f \left(\frac{T_e}{I_a}\right)^2$$

$$\frac{\partial P_{loss}}{\partial I_a} = 2 r_a I_a - \frac{2 r_f T_e^2}{I_a^3} = 0$$

$$r_a I_a^4 = r_f T_e^2$$

$$I_a = \sqrt[4]{\frac{r_f}{r_a} T_e^2} = 35.6 \text{ A}$$

$$I_f = \frac{T_e}{I_a} = 4.5 \text{ A}$$

$$P_{in} = I_a \left[r_a I_a + L_{AF} \omega_r I_f \right] + r_f I_f^2 = 10.5 \text{ kW}$$

$$P_{out} = T_e \omega_r = 8.38 \text{ kW}$$

$$\eta = \frac{P_{out}}{P_{in}} = 79.6 \%$$

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4) A PM DC machine has a back emf constant of 0.15 Vs, and an armature resistance of 300 mΩ. The armature voltage is 25 V, and the load torque may be expressed

$$T_l = \begin{cases} 2\left(\dfrac{\omega_r}{400}\right) & ECE321 \\ 2\left(\dfrac{\omega_r}{400}\right)^2 & ECE595 \end{cases}$$

Find the rotor speed, the armature current, and the efficiency.

$\frac{2}{400}$

321

$$K_v\left[\frac{V_a - K_v w_r}{r_a}\right] = b w_r$$

$$V_a - K_v w_r = \frac{r_a b}{K_v} w_r$$

$$V_a = \left[K_v + \frac{r_a b}{K_v}\right] w_r$$

(10)

$$w_r = \left[\frac{V_a}{K_v + \frac{r_a b}{K_v}}\right]$$

$$w_r = 156.25 \text{ rad/s}$$

$$T_L = b \cdot w_r = 0.781 \text{ Nm}$$

(5)

$$P_{out} = 122 \text{ W}$$

$$I_a = \frac{V_a - K_v w_r}{r_a} = 5.2 \text{ A}$$

$$P_{in} = 130.2 \text{ W}$$

(5)

$$\eta = \frac{P_{out}}{P_{in}} = 93.7\%$$

595

$$K_v \left[\frac{V_a - k_v \omega_r}{r_a} \right] = \Gamma \omega_r^2 \qquad \text{where } \Gamma = \frac{2}{400^2}$$

$$V_a - K_v \omega_r = \frac{\Gamma r_a}{K_v} \omega_r^2$$

$$\underbrace{\frac{\Gamma r_a}{K_v}}_{a} \omega_r^2 + \underbrace{K_v}_{b} \omega_r \underbrace{- V_a}_{c} = 0$$

$$\omega_r = \frac{-b + \sqrt{b^2 - 4ac}}{2a} = 162.3 \text{ rad/s}$$

$$I_a = \frac{V_a - K_v \omega_r}{r_a} = 2.195 \text{ A}$$

$$P_{in} = V_a I_a = \cancel{Ma} \ 54.9 \text{ W}$$

$$P_{out} = \Gamma \omega_r^3 = 53.4 \text{ W}$$

$$\eta = \frac{P_{out}}{P_{in}} = 97.4\%$$

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5) 20 pts. Consider the buck converter we studied in class. Assuming operation is in continuous mode, derive expressions to answer the questions below in terms of v_{dc}, v_{fsw}, v_{fd}, L_{AF}, L_{AA}, L_{FF}, r_a, r_f, d, and f_{sw}.

EE321: If the converter is connected to a series connected dc machine. Derive an approximate expression for the peak-to-peak current ripple

EE595: If the converter is connected to a shunt connected dc machine. Derive an approximate expression for the peak-to-peak torque ripple. Note, for this problem variation, it is acceptable to have the average armature current $\bar{i}_a$ and average field current $\bar{i}_f$ in your answer, in addition to those quantities listed above.

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$$V_a = r_a I_a + \omega_r L_{AF} I_f + L_{AA} \frac{dI_a}{dt}$$

$$V_f = r_f I_f + L_{FF} \frac{dI_f}{dt}$$

$$V_T = \underbrace{(r_a + r_f + \omega_r L_{AF})}_{r} I_T + \underbrace{(L_A + L_{AF})}_{L} \frac{dI_T}{dt}$$

(10) switch on

$$L \frac{dI_T}{dt} = (V_{dc} - V_{fsw}) - r\hat{I}_T$$

$$L\left(\frac{I_{mx} - I_{mn}}{dT}\right) = (V_{dc} - V_{fsw}) - r\hat{I}_T$$

switch off

$$L\left(\frac{I_{mx} - I_{mn}}{(1-d)T}\right) = V_{fd} + r\hat{I}_T$$

(10)

$$\frac{L}{T}(I_{mx} - I_m)\underbrace{\left[\frac{1}{d} + \frac{1}{1-d}\right]}_{\frac{1}{d(1-d)}} = V_{dc} - V_{fsw} + V_{fd}$$

$$I_{mx} - I_{mn} = \frac{d(1-d)(V_{dc} - V_{fsw} + V_{fd})}{f(L_{AA} + L_{FF})}$$

595 switch on

$$L_{FF}\frac{I_{fmx}-I_{fmn}}{dT} = V_{dc}-V_{fsw}-r_f\bar{I}_f$$

$$L_{AA}\frac{I_{a,mx}-I_{a,mn}}{(1-d)T} = V_{dc}-V_{fsw}-r_a\bar{I}_a$$

$$\Delta I_a = \frac{d(1-d)(V_{dc}-V_{fsw}+V_{fd})}{2L_{AA}f_{sw}}$$

(10) switch off

$$-L_{FF}\frac{I_{fmx}-I_{fmn}}{(1-d)T} = -V_{fd}-r_f\bar{I}_f$$

$$-L_{AA}\frac{I_{amx}-I_{amn}}{(1-d)T} = -V_{fd}-r_a\bar{I}_a$$

$$\Delta I_f = \frac{d(1-d)(V_{dc}-V_{fsw}+V_{fd})}{L_{FF}f_{sw}}$$

$$T_e = (I_{fmx}I_{amx} - I_{fmn}I_{amn})L_{AF}$$

(10)

$$= \left[\left(\bar{I}_f+\frac{\Delta I_f}{2}\right)\left(\bar{I}_a+\frac{\Delta I_a}{2}\right) - \left(\bar{I}_f-\frac{\Delta I_f}{2}\right)\left(\bar{I}_a-\frac{\Delta I_a}{2}\right)\right]L_{AF}$$

$$\approx \Big[\cancel{\bar{I}_f\bar{I}_a} + \frac{\Delta I_f}{2}\bar{I}_a - \bar{I}_f\frac{\Delta I_a}{2} + \cancel{\frac{\Delta I_f\Delta I_a}{4}}$$

$$- \left[\cancel{\bar{I}_f\bar{I}_a} - \frac{\Delta I_f}{2}\bar{I}_a - I_f\frac{\Delta I_a}{2} + \cancel{\frac{\Delta I_f\Delta I_a}{4}}\right]\Big]$$

$$= L_{AF}\left[\bar{I}_a\Delta I_f + \bar{I}_f\Delta I_a\right]$$

$$= L_{AF}\frac{d(1-d)(V_{dc}-V_{fsw}+V_{fd})}{f_{sw}}\left[\frac{1}{L_{FF}}\bar{I}_a + \frac{1}{L_{AA}}\bar{I}_f\right]$$

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