Draft Solution

ECE321/ECE595 Exam 4
Spring 2013

Notes: You must show work for credit.

This exam has 4 problems and 10 pages.

The last page has handy trig facts. You may remove this from the exam.

Note that problem 1 and problem 4 have different specifications depending on if you are in ECE321 or ECE595

Good luck!

1.) 25 pts. Consider a ~~machine~~ single phase transformer. The primary side resistance and leakage inductance are 1 Ω and 2 mH, respectively. The (referred) secondary resistance and leakage inductance are 2 Ω and 0.5 mH. The primary to secondary turns ratio is 10. The primary voltage is

$$v_p(t) = \sqrt{2}100\cos(400t)$$

Compute a time domain expression for the primary and secondary current if the secondary is short circuited. [ECE321: You may neglect the effects of the magnetizing inductance. ECE595: You may not neglect the effects of the magnetizing inductance. Assume a value of 20 mH]

<u>321</u>

$\omega_e = 400$

$V_p = 100\angle 0$

$r_p = 1\Omega \quad L_{lp} = 2\,mH \quad r_s' = 2\Omega \quad L_{ls}' = 0.5\,mH$

$Z = (r_p + r_s') + j\omega_e(L_{lp} + L_{ls}')$

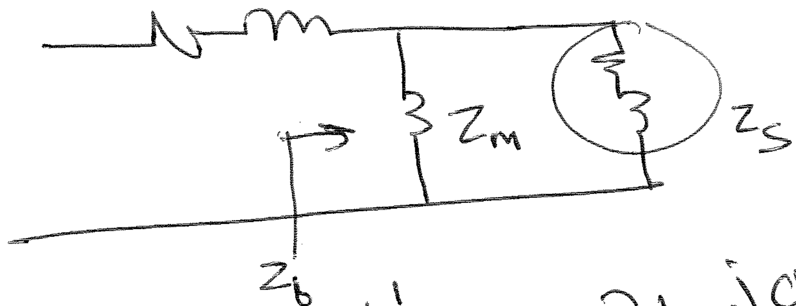
(15) $Z = 3 + j = 3.16\angle 18.4^\circ = 3.16\angle .32$

$\tilde{I}_p = \dfrac{\tilde{V}_p}{2} = 30 - j10 = 31.6\angle -18.4^\circ$

(10) $i_p = \underbrace{\sqrt{2}\,31.6}_{44.68}\cos(400t - .322)$ $\angle$ or 18.4°

$i_s = -\underbrace{\sqrt{2}\,316}_{446.89}\cos(400t - 0.322)$

595



$$Z_s = r_s' + j\omega_e L_{ep}' = 2 + j0.2$$

$$Z_m = j\omega_e L_m = 8j$$

(10) $$Z_b = \frac{Z_m Z_s}{Z_s + Z_m} = 1.797 + j0.6334$$

$$Z = r_p + j\omega_e L_{lp} + Z_b \quad = 3.14\angle .473$$

$$= 2.797 + j1.433$$

rad (-27.1°)

$$\tilde{I}_p = \frac{\tilde{V}_p}{Z} = 31.82\angle -0.4736$$

(10) $$i_p = \sqrt{2}\, 31.82 \cos(400t - 0.474)$$

rad

$$\tilde{I}_s' = -\tilde{I}_p \frac{Z_m}{Z_m + Z_s} = 30.16\angle 2.907$$

$$i_s = \sqrt{2}\, 30.2 \cos(400t + 2.907)$$

rad.

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2.) 25 pts. Consider an induction machine. All angles are defined to be positive in the counterclockwise direction. The winding function of the rotor of a machine are given by

$$w_{ar} = 10\sin(2\phi_{rm})$$
$$w_{br} = 10\cos(2\phi_{rm})$$

The rotor currents are given by

$$i_{ar} = 5\sin(10t)$$
$$i_{br} = 5\cos(10t)$$

If the rotor is moving at a mechanical speed of 100 rad/s in the clockwise direction, what is the mechanical speed and direction of the stator MMF relative to an observer on the stator? What is the absolute value of the frequency of the stator currents in rad/s?

$$F_r = W_{ar} i_{ar} + W_{br} i_{br}$$

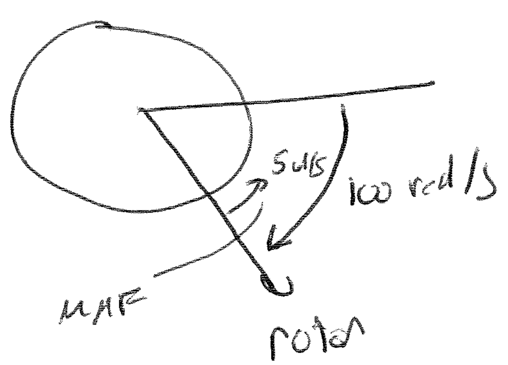
$$= 50\left[\sin(2\phi_{rm})\sin(10t) + \cos(2\phi_{rm})\cos(10t)\right]$$

$$= 50\cos(2\phi_{rm} - 10t)$$

$$\Rightarrow \frac{d\phi_{rm}}{dt} = 5 \text{ rad/s}$$

(15)

→ The rotor MMF is moving at 95 rad/s CW relative to the stator



⇒ the stator MMF is moving at 95 rad/s CW viewed from stator.

The stator MMF will be of a form

$$F_s = X \cos\left(2\phi_{sm} \pm \omega_e t + \phi\right)$$

(10) $\therefore \; |\omega_e| = \left|2\frac{d\phi_{sm}}{dt}\right|$

$$= 2 \cdot 95 = 190 \text{ rad/s.}$$

$\omega = 2\pi f$, $f = 30.23$

$\frac{190}{2\pi}$

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q-axis

3.) 25 pts. Starting with the ~~qd~~ rotor voltage equation

$$v'^{s}_{qr} = r'_r i'^{s}_{qr} - \omega_r \lambda'^{s}_{dr} + p\lambda'^{s}_{qr}$$

$$\cancel{v'^{s}_{dr} = r'_r i'^{s}_{dr} + \omega_r \lambda'^{s}_{qr} + p\lambda'^{s}_{dr}}$$

and that

$$\tilde{F}'^{s}_{qr} = -j\tilde{F}'^{s}_{dr}$$

show that

$$\frac{\tilde{v}'^{s}_{qr}}{s} = \frac{r'_r}{s}\tilde{i}'^{s}_{qr} + j\omega_e \tilde{\lambda}'^{s}_{qr}$$

and derive the expression for s in terms of ω_e and ω_r.

5 $$\tilde{V}'^{s}_{qr} = r'_r \tilde{I}'^{s}_{qr} - j\omega_r \tilde{\lambda}'^{s}_{dr} + j\omega_e \tilde{\lambda}'^{s}_{qr}$$

5 $$= r'_r \tilde{I}'^{s}_{qr} + j(\omega_e - \omega_r)\tilde{\lambda}'^{s}_{qr}$$

5 $$\frac{\omega_e}{\omega_e - \omega_r}\tilde{V}'^{s}_{qr} = r'_r \frac{\omega_e}{\omega_e - \omega_r}\tilde{I}'^{s}_{qr} + j\omega_e \tilde{\lambda}'^{s}_{qr}$$

define $s = \left(\frac{\omega_e - \omega_r}{\omega_e}\right)$

5

5 $$\frac{\tilde{V}'^{s}_{qr}}{s} = \frac{r'_r}{s}\tilde{I}'^{s}_{qr} + j\omega_e \tilde{\lambda}'^{s}_{qr}$$

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4. 25 pts. Consider a 3-phase machine with the following parameters: $r_s = 0$, $L_{ls} = L'_{lr} = 0$ mH, $L_m = 20.1$ mH, $r'_r = 41.3\ m\Omega$, and $P = 4$. A balanced 3-phase voltage source with of $460/\sqrt{3}$ V rms amplitude and 60 Hz frequency is applied to the machine. Suppose the speed is 1750 RPM. Taking the a-phase voltage to be of zero phase reference, find the a-phase stator current (magnitude and angle), electromagnetic torque, the input power, the output power, and the efficiency. [ECE595. What is the best efficiency you could obtain if you could change the electrical frequency and stator voltage magnitude as desired].

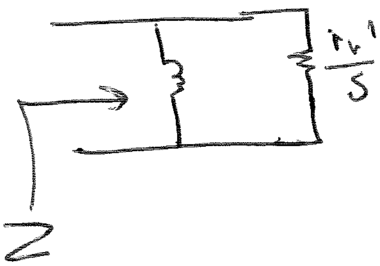
$$\omega_{rm} = 1750 \frac{rev}{min}\left(\frac{2\pi}{rev}\right)\left(\frac{min}{60s}\right) = 183.3$$

$$\omega_r = 366.5$$

$$S = \frac{\omega_e - \omega_r}{\omega_e} = 0.0278$$

$$Z_m = j\omega_e L_m = 7.57$$

$$Z_r = \frac{r_r'}{S} = 1.48$$



$$Z = \frac{Z_m Z_r}{Z_m + Z_r} = 1.43 + j0.281$$

(5) $$\tilde{I}_{as} = \frac{\overbrace{460/\sqrt{3}}^{\tilde{V}_{as}}}{Z} = 182\angle{-11.1^\circ}$$

$$\tilde{I}_{qr}^{s} = -\frac{\tilde{V}_{as}}{\frac{r_r'}{S}} = -178.62$$

(5) $$T_e = 3\frac{P}{2} L_m \text{Re}\left[j \tilde{I}_{as}^{*} \tilde{I}_{qr}^{s}\right] = 755 \text{ Nm}$$

(5) $$P_{in} = 3\,\text{Re}(\tilde{I}_{as}^{*} \tilde{V}_{as}) = 142.3 \text{ kW}$$

(5) $$P_{out} = T_e\,\omega_{rm} = 138.4 \text{ kW}$$

(5) $$\eta = \frac{P_{out}}{P_{in}} = 97.2\%$$

$\underline{S95}$

$\omega_e = \frac{r_r'}{L_m} \overset{+\omega_r}{=} 368.6$ rad/s (58.7 hz)

~~[illegible]~~ $\omega_s = \frac{r_r'}{L_m} = 2.0547$

$$I_s = \sqrt{\frac{2|T_e|(r_r'^2 + (\omega_s L_m)^2)}{3 P \omega_s L_m^2 r_r'}}$$

$= \cancel{397.3}\ 111.9$

so $I_{as} = \cancel{397.3}\ \underset{111.9}{} \angle 0$

~~V_{as}~~ $s = \frac{\omega_e - \omega_r}{\omega_e} = 0.0057748$

$Z_m = j\omega_e L_m = j7.408$

~~[illegible]~~ $Z_r = \frac{r_r'}{s} = 7.408$

$V_{as} = \tilde{I}_{as} \frac{Z_m Z_r}{Z_m + Z_r} = \cancel{2081.3 \angle 45^0}\ 586.2 \angle 45^0$

$P_{in} = 3\, Re(I_{as}^* V_{as}) = 139.1$ kW

$\eta = \frac{P_{in}}{P_{out}} = 99.4\%$

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Handy Facts

$$\mu_0 = 4\pi 10^{-7} \text{ H/m}$$

Basic Trigonometric Relationships

$$e^{jx} = \cos x + j\sin x$$
$$a\cos x + b\sin x = \sqrt{a^2+b^2}\cos(x+\phi) \quad \phi = \text{angle}(a - jb)$$
$$\cos^2 x + \sin^2 x = 1$$
$$\sin 2x = 2\sin x \cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$
$$2\cos x \cos y = \cos(x+y) + \cos(x-y)$$
$$2\sin x \sin y = \cos(x-y) - \cos(x+y)$$
$$2\sin x \cos y = \sin(x+y) + \sin(x-y)$$
$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$
$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

Three Phase Trigonometric Relationships

$$\cos x + \cos\left(x - \frac{2\pi}{3}\right) + \cos\left(x + \frac{2\pi}{3}\right) = 0$$
$$\sin x + \sin\left(x - \frac{2\pi}{3}\right) + \sin\left(x + \frac{2\pi}{3}\right) = 0$$
$$\cos^2 x + \cos^2\left(x - \frac{2\pi}{3}\right) + \cos^2\left(x + \frac{2\pi}{3}\right) = \frac{3}{2}$$
$$\sin^2 x + \sin^2\left(x - \frac{2\pi}{3}\right) + \sin^2\left(x + \frac{2\pi}{3}\right) = \frac{3}{2}$$
$$\sin x\cos x + \sin\left(x - \frac{2\pi}{3}\right)\cos\left(x - \frac{2\pi}{3}\right) + \sin\left(x + \frac{2\pi}{3}\right)\cos\left(x + \frac{2\pi}{3}\right) = 0$$
$$\sin x\cos y + \sin\left(x - \frac{2\pi}{3}\right)\cos\left(y - \frac{2\pi}{3}\right) + \sin\left(x + \frac{2\pi}{3}\right)\cos\left(y + \frac{2\pi}{3}\right) = \frac{3}{2}\sin(x-y)$$
$$\sin x\sin y + \sin\left(x - \frac{2\pi}{3}\right)\sin\left(y - \frac{2\pi}{3}\right) + \sin\left(x + \frac{2\pi}{3}\right)\sin\left(y + \frac{2\pi}{3}\right) = \frac{3}{2}\cos(x-y)$$
$$\cos x\cos y + \cos\left(x - \frac{2\pi}{3}\right)\cos\left(y - \frac{2\pi}{3}\right) + \cos\left(x + \frac{2\pi}{3}\right)\cos\left(y + \frac{2\pi}{3}\right) = \frac{3}{2}\cos(x-y)$$