 1pt. Name Key

Closed book and notes. 60 minutes.

1. True or false. (for each, 2 points if correct, 1 point if left blank.)

2 pts each

(a) T F If $P(B) = 0$, then $P(B \mid A)$ is undefined.

(b) T F Given a sample space S, the empty set, ϕ, is always an event and always $P(\phi) = 0$.

(c) T F If A is an event in the sample space S, then A and its complement A' always partition S.

(d) T F If events A and B are independent, then $P(A \mid B) = P(B)$.

(e) T F Two events are independent if they have no outcomes in common.

(f) T F For all events A and B, $P(A \mid B) \le P(A \cap B)$. because $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$ and $P(B) \le 1$.

2. Fill in the blanks.

3 pts each

(a) A sample space S is the set of all possible outcomes of an experiment.

(b) An event is a subset of S.

(c) The experiment produces exactly one outcome.

(d) The event B "occurs" if the experiment's outcome is in the set B.

(e) The probability of B is a numerical measure of how likely B is to occur.

(f) In terms of the sample space S, the empty set Φ can be written S'

3. Result: $P(\Phi) = 0$. Provide a reason for each line of the proof.

3 points each

$P(S) = P(S \cup \Phi)$ set theory

$= P(S) + P(\Phi)$ axiom 3 (mutually exclusive)

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4. A large industrial firm uses three local motels to provide overnight accommodations for its clients. From past experience it is known that 20% of the clients are assigned rooms at the Ramada Inn, 50% at the Sheraton, and 30% at the Lakeview Motor Lodge. The plumbing is faulty in 5% of the rooms at the Ramada Inn, in 4% of the rooms at the Sheraton, and in 8% of the rooms at the Lakeview Motor Lodge.

4 pts. (a) Define event notation for this problem.

R = "assigned to Ramada Inn"
S = " - - - Sheraton"
L = " —— Lakeview"
F = "plumbing is faulty"

4 pts (b) Write the given probabilities in your notation.

$P(R) = .20$ $\quad P(F|R) = .05$
$P(S) = .50$ $\quad P(F|S) = .04$
$P(L) = .30$ $\quad P(F|L) = .08$

5 pts (c) What is the probability that a client will be assigned a room with faulty plumbing? (Work the problem carefully, using yours answers to parts (a) and (b).)

$$P(F) = P(F|R)P(R) + P(F|S)P(S) + P(F|L)P(L)$$
$$= (.05)(.20) + (.04)(.50) + (.08)(.30)$$
$$= .01 + .02 + .024$$
$$= .054$$

5 pts (d) What is the probability that a client with a room having faulty plumbing was assigned accommodations at the Lakeview Motor Lodge? (Work the problem carefully, using yours answers to parts (a) and (b), and possibly part (c).)

$$P(L|F) = \frac{P(F|L)P(L)}{P(F)}$$
$$= \frac{(.08)(.30)}{.054}$$
$$= 4/9$$

5. A regional telephone company operates three identical relay stations at different locations. During a one-year period, the number of malfunctions reported by each station and the causes are shown below.

	Stations			
	X	Y	Z	
(D_1) Problems with electricity supplied	2	1	4	7
(D_2) Computer malfunction	4	3	5	12
(D_3) Malfunctioning electrical equipment	5	4	2	11
(D_4) Caused by other human error	7	7	5	19
	18	15	16	49

Consider a randomly chosen reported malfunction. Let E_1 denote the event that the malfunction is at station X, E_2 denote the event that the malfunction is at station Y, E_3 denote the event that the malfunction is at station Z. Let D_1, D_2, D_3, and D_4 denote the events corresponding to the four causes, as shown in the table.

4 pts (a) Find the probability that the random malfunction is caused by "other human error."

$$P(D_4) = \frac{19}{49} \longleftarrow$$

4 pts (b) Find the probability that the random malfunction is at station Y.

$$P(E_2) = \frac{15}{49} \longleftarrow$$

4 pts (c) If the random malfunction is at station Y, use the notation to write the probability that the it is caused by "other human error". (Wait for Part (d) to begin solving for the numerical value.)

$$P(D_4 | E_2)$$

4 pts (d) Find the numerical value for part (c).

$$P(D_4 | E_2) = \frac{P(D_4 \cap E_2)}{P(E_2)} = \frac{7/49}{15/49} = \frac{7}{15} \longleftarrow$$

6. Recall the Monte Hall problem. There are three doors: 1, 2, 3. A grand prize lies behind one door; nothing lies behind the other two doors. The contestant wins the prize by choosing the correct door. The game begins with the contestant choosing an initial door. Monte Hall then opens some one other door, showing that nothing lies behind it, and asks the contestant whether she wants to keep the original door, or to choose the other unopened door. Suppose that this contestant decides to change doors.

Define two events: C = "the contestant's initial choice is correct" and W = "the contestant wins the prize".

3 pts (a) What is the value of P(C)? → 1/3 assuming no cheating

3 pts (b) What is the value of P(W | C)? → $0 = \frac{P(C|W)P(W)}{P(C)} = \frac{0 \cdot P(W)}{P(C)}$

3 pts (c) Are W and C mutually exclusive? yes no

3 pts (d) Are W and C independent? yes no

7. Consider a sample of data $x_1, x_2, \ldots, x_n$. Write the following formulas.

3 pts (a) The sample mean.

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

3 pts (b) The sample standard deviation.

$$s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}} \overset{\text{or}}{=} \sqrt{\frac{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2}{n-1}}$$

3 pts (c) The sample mean for recoded data $y_1, y_2, \ldots, y_n$, where $y_i = 2+3x_i$ for $i = 1, 2, \ldots, n$.

$$\bar{y} = 2 + 3\bar{x}$$

3 pts (d) The textbook discussion of histograms mentions both *frequency* and *relative frequency*. For a sample of size n, what is the relationship between the two?

$$\text{relative frequency} = \frac{\text{frequency}}{n}$$

(chapter 2)