Midterm 2

MA353 - Spring 2015

All vector spaces appearing in this exam are finite-dimensional and all bases appearing are ordered bases. You will have 10 points for each correct answer of part I and 8 points for each correct answer of part II. Total: 230 points.

I. Choose the unique correct answer.

1. Let's consider $T_{\frac{\pi}{2}} : \mathbb{R}^2 \to \mathbb{R}^2$ (the counterclockwise rotation of angle $\frac{\pi}{2}$) and the following assertions (α = canonical basis):

1) $[T^{-1}_{\frac{\pi}{2}}]_\alpha = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

2) The characteristic polynomial of $[T_{\frac{\pi}{2}}]_\alpha$ splits over $\mathbb{R}$.

3) There exists a diagonal matrix D (of order 2) having the same characteristic polynomial as $[T_{\frac{\pi}{2}}]_\alpha$.

Then

(a) All assertions are true.
(b) Only 1) and 2) are true.
(c) Only 1) is true.
(d) Only 1) and 3) are true.

2. Let's consider the linear operator $T : \mathbb{R}^2 \to \mathbb{R}^2$, $T(a,b) = (3a - b, a + 3b)$, $\alpha = \{(2,4),(3,1)\}$, $\beta = \{(1,1),(1,-1)\}$, and the following assertions:

1) $[T]_\alpha = \begin{pmatrix} 4 & 1 \\ -2 & 2 \end{pmatrix}$.

2) $T(a,b) \neq \lambda(a,b)$ for all $\lambda \in \mathbb{R}$ and all nonzero (a,b).

3) The characteristic polynomial of $[T]_\beta$ splits over $\mathbb{R}$.

Then

(a) All assertions are true.
(b) Only 1) and 2) are true.
(c) Only 1) and 3) are true.
(d) Only 1) is true.

3. Let's consider a linear operator $T : V \to V$ and the set of all possible matrices $[T]_\alpha$, where α is a basis for V. Let's consider the assertions:

1) All matrices $[T]_\alpha$ are similar.
2) All matrices $[T]_\alpha$ have the same characteristic polynomial.
3) All matrices $[T]_\alpha$ have the same eigenvalues.
4) All matrices $[T]_\alpha$ have the same eigenvectors.
5) If one of the matrices $[T]_\alpha$ is diagonalizable, then they are all diagonalizable.

Then

(a) All assertions are true.
(b) Only the first four assertions are true.
(c) Only the assertion four is [redacted] untrue.
(d) Only the assertions 1, 2 and 5 are true.

4. Using the fact that there exist a diagonal matrix D and an invertible matrix Q s.t. $A = QDQ^{-1}$, find A^n for $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$. ($n$ is an integer ≥ 1) Show your work and box your answer.

(a) $\begin{pmatrix} -5^n + 2\cdot(-1)^n & 5^n + (-1)^{n+1} \\ -5^n + 2\cdot(-1)^n & 5^n + 2\cdot(-1)^{n+1} \end{pmatrix}$.

(b) $\begin{pmatrix} 5^n + 2\cdot(-1)^n & 2\cdot 5^n + (-1)^{n+1} \\ 5^n + (-1)^n & 5^n + (-1)^{n+1} \end{pmatrix}$.

(c) $\begin{pmatrix} -5^n + 2\cdot(-1)^n & 2\cdot 5^n + 2\cdot(-1)^{n+1} \\ -5^n + (-1)^n & 2\cdot 5^n + (-1)^{n+1} \end{pmatrix}$.

(d) $\begin{pmatrix} -5^n + (-1)^n & 5^n + (-1)^{n+1} \\ 5^n - (-1)^n & 5^n - (-1)^{n+1} \end{pmatrix}$.

5. The dimension of the solution set of the homogeneous system

$$\begin{aligned} 2x_1 + x_2 - x_3 &= 0 \\ x_1 - x_2 + x_3 &= 0 \\ x_1 + 2x_2 - 2x_3 &= 0 \end{aligned}$$

is:

(a) 0
(b) 1
(c) 2
(d) 3

6. How many of the properties 1), 2), 3) and 4) are used in the proof of the theorem stating that the linear system $Ax = b$ is consistent iff $rank(A) = rank(A \mid b)$?

1) If $T : V \to W$ is a linear transformation, then $Im(T) = span\{T(\vec{v}_1), \ldots, T(\vec{v}_n)\}$, where $\{\vec{v}_1, \ldots, \vec{v}_n\}$ is a basis of V.
2) The system $Ax = b$ is consistent iff $b \in Im(L_A)$, where L_A is the left-multiplication transformation by the matrix A.
3) $rank(A) = rank(A^t)$.
4) $rank(A) = dim$(Column space of A).

(a) All of them.
(b) All but 4).
(c) All but 3).
(d) All but 1).

7. How many of the definitions 1), 2) and 3) are used in the proof of the theorem stating that $rank(A) = dim$(Column space A).?

1) The definition of the rank of a matrix A.
2) The definition of the rank of a linear transformation.
3) The definition [redacted] of the left-multiplication transformation with a matrix A.

(a) All of them.
(b) All but 3).
(c) All but 2).
(d) All but 1).

II. Specify for each of the following statements if it is true or false.

8. A linear operator $T : V \to V$ is invertible iff $ker(T) = \vec{0}_V$.

(a) True.
(b) False.

9. If $T : V \to W$ is an isomorphism and if $\{T(\vec{v}_1), \ldots, T(\vec{v}_n)\}$ is a basis for W, then $\{\vec{v}_1, \ldots, \vec{v}_n\}$ is a basis for V.

(a) True.
(b) False.

10. Let $T: V \to W$ be a linear transformation. Then $dim(V) = dim(W)$ iff $rank(T) = dim(V)$.
(a) True.
(b) False.

11. Let's consider the identity transformation $1_V : V \to V$, as well as a departure basis α and an arrival basis β. Then $[1_V]_\alpha^\beta = I_n$, where $n = dim(V)$.
(a) True.
(b) False.

12. Let $T: V \to V$ be a linear operator and α, β bases for V. If $[T]_\beta = Q[T]_\alpha Q^{-1}$, then $Q = [1_V]_\alpha^\beta$.
(a) True.
(b) False.

13. Let $T: V \to V$ be a linear operator and α a basis for V. Suppose $dim(V) = n$. If there exists a square matrix A s.t. $[T]_\alpha A = I_n$, then T is invertible.
(a) True.
(b) False.

14. Let V be a vector space of dimension n and $\alpha = \{\vec{v}_1, \ldots, \vec{v}_n\}$, $\beta = \{\vec{w}_1, \ldots, \vec{w}_n\}$ two bases in V. If Q is the matrix that changes α-coordinates into β-coordinates, then the j-th column of Q is $[\vec{w}_j]_\beta$.
(a) True.
(b) False.

15. If Q is the above defined matrix, then for any $\vec{v} \in V$, we have $[\vec{v}]_\alpha = Q[\vec{v}]_\beta$.
(a) True.
(b) False.

16. If Q is the above defined matrix, then for any linear operator T on V we have $[T]_\alpha = Q[T]_\beta Q^{-1}$.
(a) True.
(b) False.

17. If the matrix $A \in M_4(\mathbb{R})$ has the distinct eigenvalues 2 and 7 and $dim(E_2) = 3$, then A is diagonalizable.
(a) True.
(b) False.

18. Let $T: V \to V$ be a linear operator and α a basis for V. Then $[T]_\alpha$ is a diagonal matrix iff α consists only in eigenvectors of T.
(a) True.
(b) False.

19. Let A be an invertible matrix. Then A is diagonalizable iff A^{-1} is diagonalizable.
(a) True.
(b) False.

20. If the coefficient matrix of a system of m linear equations in n unknowns has rank m, then the system has a solution.
(a) True.
(b) False.

21. The matrix $\begin{pmatrix} 4 & 1 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{pmatrix}$ is diagonalizable.
(a) True.

(b) False.

22. The operator $T : P_2(\mathbb{R}) \to P_2(\mathbb{R})$, $T(f) = f(0) + f(1)(x + x^2)$ is diagonalizable.

(a) True.

(b) False.

23. The rank of a diagonalizable matrix of order n is equal to n.

(a) True.

(b) False.

24. For any matrix A, $dim(\text{Column space } A) = dim(\text{Row space } A)$.

(a) True.

(b) False.

25. For any square matrices A and B of the same order, $rank(AB) = rank(BA)$.

(a) True.

(b) False.

26. If A is diagonalizable and $A = QDQ^{-1}$, with D diagonal matrix and Q invertible matrix, then $rank(A) = rank(D)$.

(a) True.

(b) False.

27. If E is an elementary matrix of order n, then there exists a square matrix B of order n s.t. $I_n = EB$.

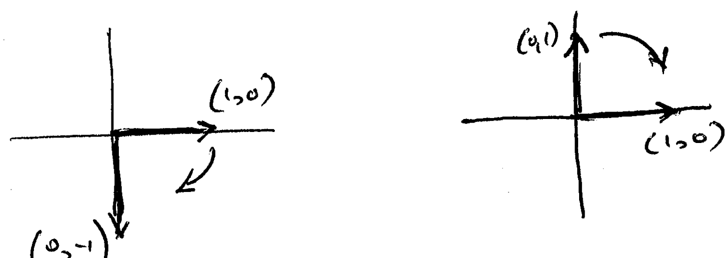
(a) True.

(b) False.

Solutions Midterm 2

(1) $T_{\frac{\pi}{2}}^{-1}$ is the clockwise rotation of angle $\frac{\pi}{2}$.

1) $[T_{\frac{\pi}{2}}^{-1}]_\alpha = \left([T_{\frac{\pi}{2}}^{-1}\begin{pmatrix}1\\0\end{pmatrix}]_\alpha \; [T_{\frac{\pi}{2}}^{-1}\begin{pmatrix}0\\1\end{pmatrix}]_\alpha \right) = \begin{pmatrix}0&1\\-1&0\end{pmatrix}$. True



$[T_{\frac{\pi}{2}}]_\alpha = \begin{pmatrix}0&-1\\1&0\end{pmatrix}$. Its characteristic polynomial is:

2) $p(\lambda) = \lambda^2 - T\cdot\lambda + D = \lambda^2 - 0\cdot\lambda + 1 = \lambda^2 + 1$ and does not split over $\mathbb{R}$.

False.

3) As $p(\lambda)$ does not split over $\mathbb{R}$, $[T_{\frac{\pi}{2}}]_\alpha$ is not diagonalizable, so $\nexists$ a diagonal matrix D having the same characteristic polynomial as it. False.

Correct answer: (c).

(2) 1) $[T]_\alpha = \left([T\begin{pmatrix}2\\4\end{pmatrix}]_\alpha \; [T\begin{pmatrix}3\\1\end{pmatrix}]_\alpha \right) = \left([\begin{pmatrix}2\\14\end{pmatrix}]_\alpha \; [\begin{pmatrix}8\\6\end{pmatrix}]_\alpha \right) = \begin{pmatrix}4&1\\-2&2\end{pmatrix}$.

True.

2) $p_{[T]_\alpha}(\lambda) = \lambda^2 - T \cdot \lambda + D = \lambda^2 - 6\lambda + 10$ does not split over $\mathbb{R}$ => $[T]_\alpha$ does not have real eigenvalues and consequently does not have any eigenvectors =>
=> same goes for the operator T => 2) is true.

3) As $p_{[T]_\alpha}(\lambda)$ does not split and $p_{[T]_\alpha}(\lambda) = p_{[T]_\beta}(\lambda)$, $p_{[T]_\beta}(\lambda)$ also will not split.
False.

Correct answer: 6.

3 When we defined $p_T(\lambda)$ we said all matrices $[T]_\alpha$ (which we know from 2.5 to be similar) have the same characteristic polynomial, so it makes sense to define $p_T(\lambda)$ as $p_{[T]_\alpha}(\lambda)$ for an arbitrarily chosen α.

So 1) and 2) are true. Having the same characteristic polynomial, they have the same eigenvalues, so 3) is true as well.

5) is true: If $[T]_\alpha = PDP^{-1}$ and $[T]_\beta \sim [T]_\alpha$, we'll have $[T]_\beta = Q[T]_\alpha Q^{-1}$, so $[T]_\beta = Q(PDP^{-1})Q^{-1} =$

$= (QP)D(QP)^{-1}$, which means $[T]_\beta$ is diagonalizable also.

3) is not true: Counterexample:

If $[T]_\alpha = \begin{pmatrix} -2 & 0 \\ 3 & 1 \end{pmatrix}$ and $[T]_\beta = \begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix}$, then $[T]_\alpha \sim [T]_\beta$ (see ex. 11 practice exam), but the eigenvectors of $[T]_\alpha$ are $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ while the eigenvectors of $[T]_\beta$ are $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

Correct answer: c

4 $p(\lambda) = \lambda^2 - T\cdot\lambda + D = \lambda^2 - 4\lambda - 5 = (\lambda - 5)(\lambda + 1)$

5 $\quad -1$

For $\lambda_1 = 5$:

$$\begin{pmatrix} 1-5 & 4 \\ 2 & 3-5 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \iff x_1 = x_2 \longrightarrow \text{eigenvector} \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

For $\lambda_2 = -1$:

$$\begin{pmatrix} 1-(-1) & 4 \\ 2 & 3-(-1) \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \iff x_1 = -2x_2 \longrightarrow \text{eigenvector} \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

So A diagonalizable, with $D = \begin{pmatrix} 5 & 0 \\ 0 & -1 \end{pmatrix}$ and $Q = \begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}$.

Consequently $A^n = QD^nQ^{-1} = \begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 5^n & 0 \\ 0 & (-1)^n \end{pmatrix}\begin{pmatrix} \frac{1}{3} & \frac{2}{3} \\ -\frac{1}{3} & \frac{1}{3} \end{pmatrix} =$

$$= \boxed{\frac{1}{3}\begin{pmatrix} 5^n + 2(-1)^n & 2[5^n + (-1)^{n+1}] \\ 5^n + (-1)^{n+1} & 2\cdot 5^n + (-1)^n \end{pmatrix}}$$

(4)

(5) $A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & -1 & 1 \\ 1 & 2 & -2 \end{pmatrix}$

Third row is row 1 - row 2, so third row disappears.
The 2 remaining rows are independent so $\operatorname{rank}(A) = 2$.
The dim. of the solution set will be $n - \operatorname{rank}(A) = 3 - 2 = 1$.
Correct answer: (b)

(6) Review the pf. of th. 3.11 from the book (page 174-175)
or lecture notes from 3.3 and check that properties 2) and 1)
have been used in the beginning of the proof and
property 4) towards the end of the proof. The only property
we did not use is 3), so the correct answer is (c).

(7) Review the pf. of the theorem 3.5 from the book
(page 153-154) or lecture notes from 3.2 and check
that all definitions 1), 2), 3) have been used (exactly
in this order: 1), 2) and 3)). So correct answer: (a).

(8) True.
$$\text{Invertible} \Longleftrightarrow \text{Bijective} \underset{\substack{\text{left}\\\text{and}\\\text{right}\\\text{are equal}}}{\overset{\text{dim}}{\Longleftrightarrow}} \text{Injective} \Longleftrightarrow \ker(T) = \vec{0}_V.$$

9) True.

It is one of the basic properties of an isomorphism to move bases into bases. And here it is applied for T^{-1}.

10) False.

Counterexample:

$$T : \mathbb{R}^2 \to \mathbb{R}^2, \quad T\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \text{(the zero transformation).}$$

We have $\dim(V) = \dim(W)$, still $\operatorname{rank}(T) = 0 \neq \dim(V)$

11) False. Counterexample:

Let $1_{\mathbb{R}^2} : \mathbb{R}^2 \to \mathbb{R}^2$ and $\alpha = \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$, $\beta = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$.

Then
$$[1_{\mathbb{R}^2}]_\alpha^\beta = \left(\left[1_{\mathbb{R}^2}\begin{pmatrix} 1 \\ 2 \end{pmatrix}\right]_\beta \; \left[1_{\mathbb{R}^2}\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right]_\beta \right) = \left(\left[\begin{pmatrix} 1 \\ 2 \end{pmatrix}\right]_\beta \; \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right]_\beta \right) =$$

$$= \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} \neq I_2.$$

12) False. Counterexample:

Let $1_{\mathbb{R}^2} : \mathbb{R}^2 \to \mathbb{R}^2$, $\alpha = \left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 5 \\ 3 \end{pmatrix} \right\}$, $\beta = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$.

Then $[1_{\mathbb{R}^2}]_\beta = [1_{\mathbb{R}^2}]_\alpha = I_2$, so we have

$$[1_{\mathbb{R}^2}]_\beta = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} [1_{\mathbb{R}^2}]_\alpha \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}^{-1},$$

without $\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}$ being equal to $[1_{\mathbb{R}^2}]_\alpha^\beta$ which is $\begin{pmatrix}1 & 5\\ -1 & 3\end{pmatrix}$.

(13) True.

If $\exists$ A s.t. $[T]_\alpha \cdot A = I_n$, it means $[T]_\alpha$ is invertible. (A "one-sided" inverse of $[T]_\alpha$ will be a "two-sided" inverse $[T]_\alpha$).

But $[T]_\alpha$ invertible iff T invertible (th. 2.18).

So T is invertible.

(14) False.

The j^{th} column of the change of coordinates matrix from basis α to basis β is $[\vec{v_j}]_\beta$, not $[\vec{w_j}]_\beta$ (which by the way is e_j).

(15) False.

It's the other way around: $[\vec{v}]_\beta = Q[\vec{v}]_\alpha$.

(16) False.

It's the other way around: $[T]_\beta = Q[T]_\alpha Q^{-1}$.

17. As A is 4×4 matrix, $p_A(\lambda)$ has degree 4. As $p_A(\lambda)$ has also the root 7, the max order of multiplicity possible for the root 2 is 3, is $m_2 \leq 3$. But on the other hand

$$3 = \dim(E_2) \leq m_2,$$

so $m_2 = 3$, so $\dim(E_2) = m_2$.
$\dim(E_7) = m_7$ is satisfied also (this is always satisfied for the eigenvalues of multiplicity one), so A is diagonalizable.

True.

18. True (th. 5.1)

19. True.

<u>Pf. 1</u>: A diagonalizable $\overset{def}{\Longleftrightarrow}$ L_A diagonalizable $\Longleftrightarrow$ $\overset{\text{ex. 12b/5.2}}{\Longleftrightarrow}$ L_A^{-1} diagonalizable $\Longleftrightarrow$ $L_{A^{-1}}$ diagonalizable

$\overset{def}{\Longleftrightarrow}$ A^{-1} diagonalizable

<u>Pf. 2</u>: A diagonalizable $\Longleftrightarrow$ $A = \overset{\text{invertible}}{Q} \underset{\text{diagonal}}{D} Q^{-1}$ $\Longleftrightarrow$

$\Longleftrightarrow$ $A^{-1} = (QDQ^{-1})^{-1}$ $\Longleftrightarrow$ $A^{-1} = \overset{\text{invertible}}{Q} \underset{\text{diagonal}}{D^{-1}} Q^{-1}$ $\Longleftrightarrow$

$\Leftrightarrow A^{-1}$ diagonalizable

(We used that if $D = \begin{pmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{pmatrix}$ is diagonal, its

inverse D^{-1} is also diagonal and $D^{-1} = \begin{pmatrix} \frac{1}{\lambda_1} & & 0 \\ & \ddots & \\ 0 & & \frac{1}{\lambda_n} \end{pmatrix}$,

which is easy to prove).

(20) True.

As $(A|b)$ has m rows and $n+1$ columns, we'll

have:

$m = \text{rank}(A) \leq \text{rank}(A|b) \leq$ both m and $n+1$,

so rank $(A|b)$ is squeezed to be equal to m.

Thus both rank(A) and rank $(A|b)$ are equal

to m, so the system is consistent.

(21) False.

A upper-triangular, eigenvalues: 4 (multiplicity 2), 5 (multiplicity 1)

For $\lambda_1 = 4$:

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$$\begin{pmatrix} 4-4 & 1 & 0 \\ 0 & 4-4 & 0 \\ 0 & 0 & 5-4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \iff x_2 = x_3 = 0 \longrightarrow$$

$\longrightarrow$ eigenspace $E_4 = \left\{ a\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, a \in \mathbb{R} \right\}$ (dimension 1).

So $\dim(E_4) < m_4 = 2$, thus A will fail to be diagonalizable.

22 True.

$[T]_\alpha = \left([T(1)]_\alpha \ [T(x)]_\alpha \ [T(x^2)]_\alpha \right) = \left([1+x+x^2]_\alpha \ [x+x^2]_\alpha \ [x+x^2]_\alpha \right)$

canonical : $\{1, x, x^2\}$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \Rightarrow p_{[T]_\alpha}(\lambda) = \begin{vmatrix} 1-\lambda & 0 & 0 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{vmatrix} =$$

$= (1-\lambda)(-\lambda)(2-\lambda)$ $\left\langle \begin{matrix} 1 \\ 0 \\ 2 \end{matrix} \right.$ real and distinct

eigenvalues $\Rightarrow$ the condition $\dim(E_{\lambda_i}) = m_{\lambda_i}$ each i is automatically verified. Hence $[T]_\alpha$ and thus T are diagonalizable.

(23) False.

Counterexample: $A = \begin{pmatrix} -1 & 2 \\ -1 & 2 \end{pmatrix}$ has $p_A(\lambda) = \lambda^2 - \lambda <^{0}_{1}$

so it is diagonalizable. But even if the order of A is 2 (A is a 2×2 matrix), the rank$(A) = 1$.

(24) True.

(Corollary 2c of th. 3.6 / section 3.2)

Both dim (Col space A) and dim (Row space A) represent the rank(A).

(25) False.

Counterexp.: Let $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$.

Then $AB = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, so rank$(AB) = 1$, but

$BA = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, so rank$(BA) = 0$. Thus rank$(AB) \neq$ rank(BA).

(26) True.

By multiplying D with the invertible matrices Q

and Q^{-1} we do not change the rank(D). (th. 3.4c / section 3.2). Thus rank(A) = rank(D).

27 True.

Any elementary matrix E is invertible (th. 3.2 / section 3.1) so if we call its inverse B we have of course $E \cdot B = I_n$.