

1. (20 pts)

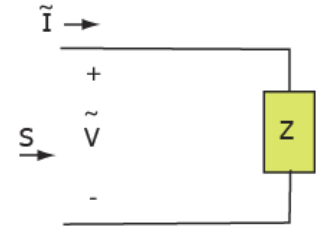
Figure (a) shows the rms phasor voltage V , rms phasor current I , and the complex power $S = P + jQ$ at the terminals of a network component.

Given the following measurements at the terminals:

$$\underline{V} := 220 \angle 0^\circ \text{ V} \quad P := 5000 \text{ W} \quad \text{input power factor is 0.8 leading}$$

Find (a) current drawn I

(b) the equivalent impedance Z looking into the terminals of the component .



(a) Original network component

1.

$$\underline{V} := 220 \times e^{j \times \frac{0}{180} \times \pi} = 220 \quad P := 5000 \quad \text{pf} := 0.8 \quad \text{leading}$$

$$\phi := \arccos(\text{pf}) \quad \underline{S} := \frac{P}{\text{pf}} \times e^{j \times (-\phi)} = 5 \times 10^3 - 3.75j \times 10^3$$

$$\text{Current drawn} \quad \underline{I} := \frac{\underline{S}}{\underline{V}} = 22.727 + 17.045j \quad |I| = 28.409 \quad \arg(I) \times \frac{180}{\pi} = 36.87$$

$$\text{Input impedance and power factor} \quad \underline{Z} := \frac{\underline{V}}{\underline{I}} = 6.195 - 4.646j \quad |Z| = 7.744 \quad \arg(Z) \times \frac{180}{\pi} = -36.87$$

$$\text{pf} := \frac{\text{Re}(S)}{|S|} = 0.8 \quad \text{leading since } Q < 0$$

Since P absorbed is positive, component is a load drawing real power from ac supply

Q absorbed is negative, hence the component is capacitive

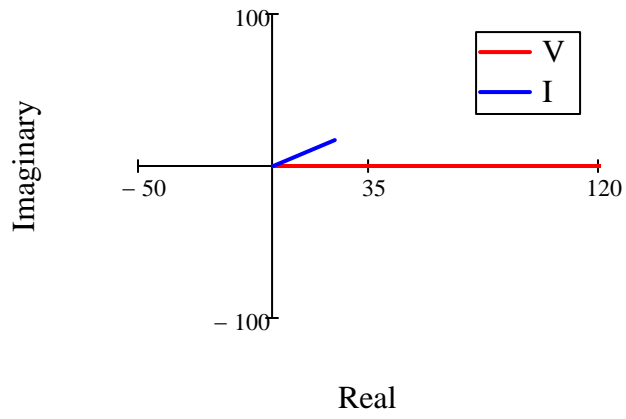
Specify a range variable from 0 to 1 for plotting the two phasors

$$i := 0..1$$

Define the vectors for plotting

$$\text{VR} := \begin{pmatrix} 0 \\ \text{Re}(V) \end{pmatrix} \quad \text{VI} := \begin{pmatrix} 0 \\ \text{Im}(V) \end{pmatrix} \quad \text{IR} := \begin{pmatrix} 0 \\ \text{Re}(I) \end{pmatrix} \quad \text{II} := \begin{pmatrix} 0 \\ \text{Im}(I) \end{pmatrix}$$

Voltage and Current Phasors



2. (40 pts)

A single-phase, 2-winding transformer rated 25 kVA, 7620V/120V, 60 Hz has the following equivalent circuit parameters:

Combined equivalent series resistance, $r_1 + r'_2$, on the 7620 V side is 3.06 Ω

Combined equivalent leakage reactance, $x_{l1} + x'_{l2}$, on the 7620 V side is 6.5 Ω

Core losses at rated voltage is $P_{\text{core}} = 400$ W

Ignoring the effects of magnetizing reactance which is not given, determine

- a)(18 pts) the rated current on the 7620 V side, then estimate the efficiency of the transformer delivering rated volts and current to a 0.9 power factor load connected to the 120 V side.
- b)(14 pts) the per unit values of the total winding resistance and total leakage reactance using the rated values of voltages and volt-ampere of the transformer as the base;
- c)(8 pts) What would be the 3-phase ratings, that is rated line-to-line voltage and total 3-phase VA, when 3 units of the above single-phase transformers are connected in Y-Y ?



$$V_{1\text{rated}} := 7620 \quad V_{2\text{rated}} := 120 \quad S_{\text{rated}} := 25 \times 10^3 \quad f_{\text{rated}} := 60 \quad \omega := 2 \times \pi \times f_{\text{rated}}$$

$$I_{1\text{rated}} := \frac{S_{\text{rated}}}{V_{1\text{rated}}} = 3.281 \quad I_{2\text{rated}} := \frac{S_{\text{rated}}}{V_{2\text{rated}}} = 208.333$$

$$r_{1\text{series}} := 3.06 \quad x_{1\text{series}} := 6.5 \quad P_{\text{core}} := 400$$

Part a) rated current on the 7620 V side is $I_{1\text{rated}} = 3.281$

Winding copper losses with rated current flowing $P_{\text{cu}} := I_{1\text{rated}}^2 \times r_{1\text{series}} = 32.938$

Output power at rated voltage and current to load of 0.9 power factor lagging $\text{pf} := 0.9$

$$P_{\text{out}} := V_{1\text{rated}} \times I_{1\text{rated}} \times \text{pf} = 2.25 \times 10^4$$

Efficiency at the given operating condition $P_{\text{losses}} := P_{\text{cu}} + P_{\text{core}} = 432.938$

$$\eta := \left(1 - \frac{P_{\text{losses}}}{P_{\text{out}} + P_{\text{losses}}} \right) \times 100 \quad \eta = 98.112$$

Part b) $S_{\text{base}} := S_{\text{rated}}$ Select base voltages for the 2 regions according to transformer ratio

$$V_{1\text{base}} := V_{1\text{rated}} \quad Z_{1\text{base}} := \frac{V_{1\text{base}}^2}{S_{\text{base}}} = 2.323 \times 10^3 \quad I_{1\text{base}} := \frac{S_{\text{base}}}{V_{1\text{base}}} = 3.281$$

total series impedance in per unit

$$z_{\text{seriespu}} := \frac{r_{1\text{series}} + j \times x_{1\text{series}}}{Z_{1\text{base}}} = 1.318 \times 10^{-3} + 2.799j \times 10^{-3}$$

Part c) rated line voltages of Y-Y connected transformer are $V_{1\text{rated}} \times \sqrt{3} = 1.32 \times 10^4$

$$V_{2\text{rated}} \times \sqrt{3} = 207.846$$

rated 3-phase power is $S_{3\text{phrated}} := 3 \times S_{\text{rated}} = 7.5 \times 10^4$



3 (40 pts)

A 3-phase, round-rotor, synchronous generator, connected directly to a large power network, has synchronous reactance X_s of 1.2 per unit on its own base and negligible stator resistance. The magnitude of the generator's terminal bus voltage may be assumed to be constant at 1 per unit on the generator base.

Given that the above generator is initially delivering 0.8 real power into the power network with the magnitude of its internal emf, $|E_f|$, adjusted to 1.8 per unit.

(22 pts) Calculate the power angle δ and the power factor ϕ , and sketch the phasor diagram for this initial operating condition, clearly labelling all the components of the phasor diagram.

(8 pts) At the generator terminal, is the power network appearing as a inductive or capacitive load?

(10 pts) Assuming that you have control of $|E_f|$ via the generator's excitation field and the generator's output via its input mechanical power, indicate how you would change the generator's output power factor to unity power factor while holding its active output power constant at rated value.



Given initial operating condition: $X_s := 1.2$ $V_t := 1 \times e^{j \times 0}$ $E_{f\text{mag}} := 1.8$ $P := 0.8$

$$\sin_delta := \frac{P}{|V_t \times E_{f\text{mag}}| X_s} \quad \delta := \text{asin}(\sin_delta) \quad \delta_{\text{deg}} := \frac{180}{\pi} \times \delta = 32.231$$

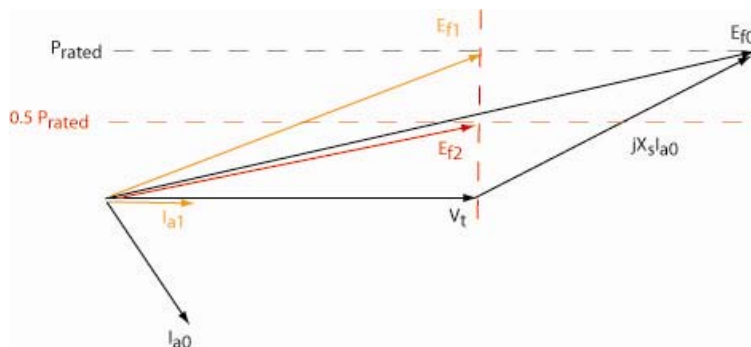
$$E_f := E_{f\text{mag}} \times e^{j \times \delta} \quad \text{Output phase current: } I_a := \frac{E_f - V_t}{j \times X_s} = 0.8 - 0.436j \quad \text{current lags terminal voltage}$$

$$|I_a| = 0.911 \quad \phi := \text{atan}\left(\frac{\text{Im}(I_a)}{\text{Re}(I_a)}\right) \quad \cos(\phi) = 0.878$$

Alternatively $\text{Complex power output } S := V_t \times \bar{I}_a = 0.8 + 0.436j$

$Q := \text{Im}(S) = 0.436$ since $Q > 0$, lagging power factor

Output power factor $\text{pf} := \frac{P}{|S|} = 0.878$ lagging because Q is negative



raise lagging power factor to unity: reduce field excitation from E_{f0} to E_{f1}
 lower output power, keeping pf at unity: E_{f1} to E_{f2} and adjust governor setting to lower P_{mech} by approximately half

At the given operating condition, the network is absorbing inductive reactive power produced by the generator.

To change power factor from lagging to unity, holding output power constant, we lower the field excitation of the generator changing from E_{f0} to E_{f1} while leaving governor control untouched.

