

Q. 1 (32 pts)

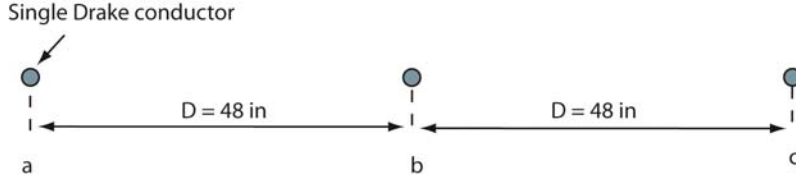


Fig. 1 Conductor arrangement of a 138 kV, 3-phase, compact line

Figure 1 shows the conductor configuration of a 138 kV, 60 Hz, 3-phase, compact line. Each phase conductor has an envelope radius of 0.554 inch and a geometric mean radius of 0.45 inch.

Determine

- (a) (8 pts) the geometric mean distance between phase conductors,
- (b) (8 pts) the per-phase inductance per kilometer of the line,
- (c) (8 pts) the phase-to-neutral capacitance per kilometer of the line, neglecting the effect of ground, and
- (d) (8 pts) the surge impedance, Z_o , of the line.

Given $\mu_o = 4\pi \times 10^{-7} \text{ H/m}$ and $\epsilon_o = 8.854 \times 10^{-12} \text{ F/m}$.

$$\begin{aligned}
 V_{\text{rated}} &:= 138 \times 10^3 & \text{freq} &:= 60 & \mu_o &:= 4 \times \pi \times 10^{-7} & \epsilon_o &:= 8.854 \times 10^{-12} \\
 D_{ab} &:= 48 & D_{bc} &:= 48 & D_{ac} &:= 2 \times D_{ab} = 96 & \omega &:= 2 \times \pi \times \text{freq} = 376.991 \\
 \text{cond_rad} &:= 0.554 & \text{gmr} &:= 0.45
 \end{aligned}$$

(a) $\text{GMD} := \sqrt[3]{D_{ab} \times D_{bc} \times D_{ac}} \quad \text{GMD} = 60.476 \text{ inch}$

(b) GMR for inductance calculation $\text{GMRL} := \text{gmr} = 0.45 \text{ inch}$

per-phase inductance $L_{\text{ph}} := \frac{\mu_o}{2 \times \pi} \times \ln\left(\frac{\text{GMD}}{\text{GMRL}}\right) \times 10^3 \quad L_{\text{ph}} = 9.802 \times 10^{-4} \frac{\text{H}}{\text{km}}$

(c) GMR for capacitance calculation $\text{GMRC} := \text{cond_rad} = 0.554$

per-phase capacitance $C_n := \frac{2 \times \pi \times \epsilon_o}{\ln\left(\frac{\text{GMD}}{\text{GMRC}}\right)} \times 10^3 \quad C_n = 1.185 \times 10^{-8} \frac{\text{F}}{\text{km}}$

(d) Characteristic impedance of line

$$Z_o := \sqrt{\frac{L_{ph}}{C_n}} = 287.544 \quad \text{Ohms}$$

Q. 2 (36 pts)

A 138 kV, 60 Hz, 3-phase transmission line has per phase series impedance of $z = 0.073 + j0.37 \text{ ohms per km}$ and shunt admittance of $y = j4.47 \times 10^{-6} \text{ mhos per km}$.

(16 pts) A 50 km-long section of the above 138 kV line is to be represented by a simple series RL circuit model. Determine the ABCD parameters of the transmission matrix for the short section of 50 km long line.

(20 pts) Find the total line losses of all 3 phases of the 50 km line when it is delivering power to a balanced 3-phase load of 60 MVA, 0.85 power factor lagging with the receiving-end line-to-line voltage is held at 138 kV (line-to-line).

$$(a) \quad z := 0.073 + j \times 0.37 \frac{\Omega}{\text{km}} \quad y := j \times 4.47 \times 10^{-6} \frac{\text{mhos}}{\text{km}} \quad V_{\text{rated}} := 138 \times 10^3$$

$$\text{Length} := 50 \text{ km}$$

$$\text{Series impedance of series RL circuit} \quad Z := z \times \text{Length} = 3.65 + 18.5j$$

$$\text{Shunt admittance of series RL circuit} \quad Y := 0$$

ABCD parameters of series RL circuit

$$A := \frac{Z \times Y}{2} + 1 = 1 \quad B := Z = 3.65 + 18.5j$$

$$C := Y \times \left(\frac{Z \times Y}{4} + 1 \right) = 0 \quad D := \frac{Z \times Y}{2} + 1 = 1$$

(b)

$$V_{\text{rated}} := 138 \times 10^3 \quad V_R := \frac{V_{\text{rated}}}{\sqrt{3}} = 7.967 \times 10^4$$

Complex power of 60 MVA load at 0.85 pf lagging $\text{pf} := 0.85$

$$S_R := 60 \times 10^6 \times (\text{pf} + j \times \sqrt{1 - \text{pf}^2}) = 5.1 \times 10^7 + 3.161j \times 10^7$$

$$I_R := \frac{\overline{S_R}}{3 \times V_R} = 213.369 - 132.234j \quad |I_R| = 251.022 \quad \arg(I_R) \times \frac{180}{\pi} = -31.788$$

Sending-end phase to neutral voltage

$$V_S := A \times V_R + B \times I_R = 8.29 \times 10^4 + 3.465j \times 10^3$$

$$|V_S| = 8.297 \times 10^4 \quad \arg(V_S) \times \frac{180}{\pi} = 2.393$$

Sending-end phase current

$$I_S := C \times V_R + D \times I_R \quad I_S = 213.369 - 132.234j$$

$$|I_S| = 251.022 \quad \arg(I_S) \times \frac{180}{\pi} = -31.788$$

Sending-end 3-phase complex power

$$S_S := 3 \times V_S \times \overline{I_S} \quad S_S = 5.169 \times 10^7 + 3.51j \times 10^7$$

$$|S_S| = 6.248 \times 10^7 \quad \cos(\arg(S_S)) = 0.827 \quad \text{lagging}$$

Total line losses in all 3 phases

$$P_{\text{loss}} := \text{Re}(S_S) - \text{Re}(S_R) = 6.9 \times 10^5 \quad \text{Watts}$$

Q. 3 (32 pts)

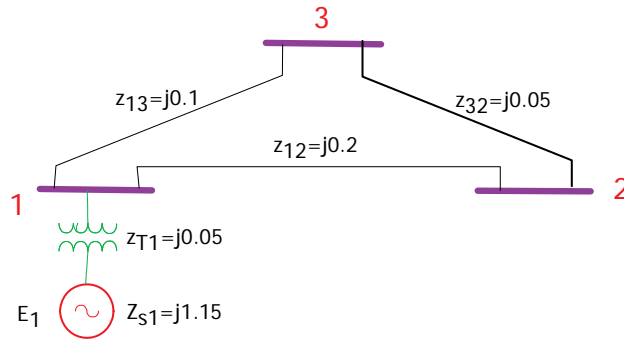


Fig. 3 Single-line diagram of a 3-bus network

Given on the one-line diagram of a 3-bus network are the per unit value of the primitive impedances of the 3 lines, line charging of the lines has been ignored, and also those of the transformer and generator.

- (a) (16 pts) Sketch the equivalent single-phase circuit representation retaining only buses 1, 2 and 3 with the generator and transformer represented by a Norton equivalent. On this diagram give the values of the primitive admittances for each element.
- (b) (16 pts) Determine the Y-bus for the 3-bus network.

ORIGIN := 1

- (a) Convert impedances to admittances and Thevenin to Norton $j := \sqrt{-1}$

$$z_{13} := j \times 0.1 \quad z_{12} := j \times 0.2 \quad z_{32} := j \times 0.05 \quad z_t := j \times 0.05 \quad z_s := j \times 1.15$$

$$y_{13} := \frac{1}{z_{13}} = -10j \quad y_{12} := \frac{1}{z_{12}} = -5j \quad y_{32} := \frac{1}{z_{32}} = -20j \quad y_{10} := \frac{1}{z_t + z_s} = -0.833j$$

$$E_1 := 1.6 \quad I_1 := \frac{E_1}{z_t + z_s} = -1.333j$$

$$Y_{1,1} := y_{10} + y_{12} + y_{13} = -15.833j \quad Y_{1,2} := -y_{12} = 5j \quad Y_{1,3} := -y_{13} = 10j$$

$$Y_{2,1} := Y_{1,2} \quad Y_{2,2} := y_{12} + y_{32} = -25j \quad Y_{2,3} := -y_{32} = 20j$$

$$Y_{3,1} := Y_{1,3} \quad Y_{3,2} := Y_{2,3} \quad Y_{3,3} := y_{13} + y_{32} = -30j$$

$$Y = \begin{pmatrix} -15.833j & 5j & 10j \\ 5j & -25j & 20j \\ 10j & 20j & -30j \end{pmatrix}$$

