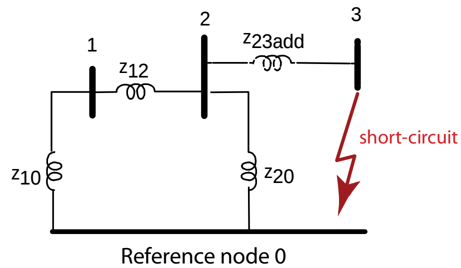
**ECE432** **Test 3** **Dec 17, 2014**

**Q.1 (32 pts.)**




The $Z_{bus}$ with bus 0 as reference for the 2-bus network without the branch $z_{23add}$ is

$$
\begin{array}{cc}
 & \begin{array}{cc} 1 & 2 \end{array} \\
Z_{bus} = j & \begin{bmatrix} 0.5833 & 0.5 \\ 0.5 & 0.6 \end{bmatrix}
\end{array} \quad \text{pu}
$$

a) (10 pts) Expansion to the 2-bus network is carried out with the addition of the $z_{23add} = j0.2$ pu resulting in a 3-bus network. Determine the $Z_{bus}$ for the newly created 3-bus network.

b) (22 pts) As part of a short-circuit study performed on the newly created 3-bus network, determine

   i. the short-circuit capacity at bus 3 for a zero-impedance fault between bus 3 and the reference bus 0.

   ii. the approximate bus voltages at all 3 buses during the short-circuit at bus 3, assuming their pre-fault values were 1 per unit.

$ORIGIN := 1$

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Part a)

$$Zbus = \begin{pmatrix} 0.5833j & 0.5j \\ 0.5j & 0.6j \end{pmatrix} \qquad j := \sqrt{-1}$$

Add new branch z23add between buses 2 and 3 $\qquad z_{23add} := j \cdot 0.2$

$$vadd := Zbus^{\langle 2 \rangle} = \begin{pmatrix} 0.5j \\ 0.6j \end{pmatrix}$$

$$Zbb := Zbus_{2,2} + z_{23add} = 0.8j$$

$$Zbus := \text{stack}\left(\text{augment}(Zbus, vadd), \text{augment}\left(vadd^T, z_{bb}\right)\right) = \begin{pmatrix} 0.5833j & 0.5j & 0.5j \\ 0.5j & 0.6j & 0.6j \\ 0.5j & 0.6j & 0.8j \end{pmatrix}$$

Part b)

i ) Thevenin's impedance between bus 3 and ground is $Z_{3,3}$

$$Z_{th3} := Zbus_{3,3} = 0.8j$$

Short circuit capacity at bus 1, which assumes rated or 1 pu voltage is

$$SCR := \left|\frac{1}{Z_{th3}}\right| = 1.25 \qquad \text{pu}$$

ii ) Bus voltages during a short-circuit fault at bus 3, assuming their prefault values are 1 per unit

Prefault values $V_{pf} := \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

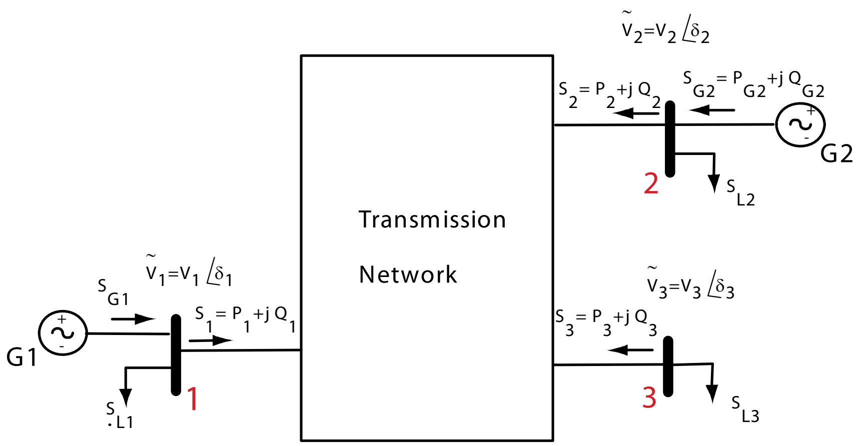
Change in bus voltages due to short-circuit at bus 3

$$\Delta V := Zbus \cdot \begin{pmatrix} 0 \\ 0 \\ \dfrac{-V_{pf_3}}{Z_{th3}} \end{pmatrix} = \begin{pmatrix} -0.625 \\ -0.75 \\ -1 \end{pmatrix}$$

Bus voltages during a short-circuit fault at bus 3

$$V_{fault} := V_{pf} + \Delta V = \begin{pmatrix} 0.375 \\ 0.25 \\ 0 \end{pmatrix} \qquad \text{per unit}$$

**Q.2 (32 pts.)**




**3-bus system**

The Y-bus of the 3-bus transmission network shown above is

$$Y_{bus} = \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} -j5 & 0 & j5 \\ 0 & -j4 & j4 \\ j5 & j4 & -j8.8 \end{bmatrix} \end{array} \quad \text{pu}$$

a) (6 pts) What is the role of the slack bus in the formulation of a load flow problem?

b) (6 pts) Give the expression for the reactive power, $Q_2$, flowing into the network at bus 2, in terms the bus voltages and the Y-bus elements.

c)(20 pts) In a load flow study of the 3-bus system shown above, bus 1 is chosen to be the slack bus. Starting with initial guesses of $\tilde{V}_2 = 1.02\angle 0^o$, and $\tilde{V}_3 = 1\angle 0^o$, perform one Gaussian iteration that includes updating $\tilde{V}_2$, $\tilde{V}_3$ and $Q_{G2}$ and checking on the changes in the voltages at buses 2 and 3.

| Bus Type | Scheduled Bus Voltage (pu) | | Generation into Bus (pu) | | Loading on Bus (pu) | |
|---|---|---|---|---|---|---|
| | \|V\| | δ | $P_g$ | $Q_g$ | $P_L$ | $Q_L$ |
| 1 slack | 1.03 | 0 | | | 0.12 | 0.08 |
| 2 generator | 1.02 | | 0.6 | | 0.1 | 0.05 |
| 3 load | | | 0 | 0 | 1.0 | 0.4 |

a ) Role of the slack bus is to absorb uncertainty of the network losses

b ) $Q_i := \mathrm{Im}\left(V_i \cdot \overline{I_i}\right) = \blacksquare \cdot \mathrm{Im}\left[V_i \cdot \overline{\left(Y_{i1} \cdot V_1 + Y_{i2} \cdot V_2 + Y_{i3} \cdot V_3\right)}\right]$

$Y_{11} := -j \cdot 5$ $\quad Y_{12} := 0$ $\quad Y_{13} := j \cdot 5$ $\quad j := \sqrt{-1}$

$Y_{21} := 0$ $\quad Y_{22} := -j \cdot 4$ $\quad Y_{23} := j \cdot 4$

$Y_{31} := j \cdot 5$ $\quad Y_{32} := Y_{23}$ $\quad Y_{33} := -j \cdot 8.8$

c )

Given condition

$V_1 := 1.03$ $\quad \delta_1 := 0$ $\quad P_{L1} := 0.12$ $\quad Q_{L1} := 0.08$

$P_{L2} := 0.1$ $\quad Q_{L2} := 0.05$ $\quad P_{G2} := 0.6$ $\quad V_{2sch} := 1.02$

$P_{L3} := 1.0$ $\quad Q_{L3} := 0.4$ $\quad P_{G3} := 0$ $\quad Q_{G3} := 0$

$P_2 := P_{G2} - P_{L2} = 0.5$

$P_3 := P_{G3} - P_{L3} = -1$ $\quad Q_3 := Q_{G3} - Q_{L3} = -0.4$

**First iteration** $\quad V_2 := V_{2sch}$ $\quad V_3 := 1$ $\quad V_{2old} := V_{2sch}$ $\quad V_{3old} := V_3$

$$Q_2 := \mathrm{Im}\left[V_2 \cdot \overline{\left(Y_{21} \cdot V_1 + Y_{22} \cdot V_2 + Y_{23} \cdot V_3\right)}\right] = 0.0816$$

$$Q_{G2} := Q_2 + Q_{L2} = 0.1316$$

$$V_2 := \frac{1}{Y_{22}}\left[\frac{P_2 - j \cdot Q_2}{\overline{V_2}} - \left(Y_{21} \cdot V_1 + Y_{23} \cdot V_3\right)\right] = 1.02 + 0.1225j \qquad \left|V_2\right| = 1.02$$

Enforce scheduled $V_2$ magnitude $\quad V2_{ang} := \dfrac{V_2}{\left|V_2\right|}$ $\quad \arg\left(V_2\right) \cdot \dfrac{180}{\pi} = 6.851$

$V_2 := V_{2sch} \cdot V2_{ang} = 1.0127 + 0.1217j$ or $V_2 := V_{2sch} \cdot e^{j \cdot \arg\left(V_2\right)} = 1.0127 + 0.1217j$ $\quad \left|V_2\right| = 1.02$

$$V_3 := \frac{1}{Y_{33}}\left[\frac{P_3 - j \cdot Q_3}{\overline{V_3}} - \left(Y_{31} \cdot V_1 + Y_{32} \cdot V_2\right)\right] = 1.0001 - 0.0583j \qquad \left|V_3\right| = 1.0018$$

$$Q_2 := \mathrm{Im}\left[V_2 \cdot \overline{\left(Y_{21} \cdot V_1 + Y_{22} \cdot V_2 + Y_{23} \cdot V_3\right)}\right] = 0.1387$$

$$Q_{G2} := Q_2 + Q_{L2} = 0.1887$$

Check change in V's against specified tolerance

$$\Delta V_2 := \left|V_{2old} - V_2\right| = 0.1219$$

$$\Delta V_3 := \left|V_{3old} - V_3\right| = 0.0583$$

Since $\Delta V_2 > \varepsilon_V$ , continue iteration

Store values $V_{2old} := V_2$ $V_{3old} := V_3$

more iterations

**Q.3 (36 pts.)**

The fuel cost in \$/h of two thermal plants are

$$F(P_1) = 0.008P_1^2 + 6.8P_1 + 300, \qquad 50 \le P_1 \le 300 \text{ MW}$$

$$F(P_2) = 0.0085P_2^2 + 6.2P_2 + 280, \qquad 50 \le P_2 \le 200 \text{ MW}.$$

a) (6 pts) Determine the equation for the incremental fuel costs of each generator.

b) (8 pts) Ignoring network losses, determine the optimal dispatch of the two generators to meet a total demand of 400 MW.

c) (6 pts) For the optimal dispatch obtained in b) what would be the total operating fuel cost for the two generators ?

d) (16 pts) If network losses, given by $P_{loss} = 0.00022P_1^2 + 0.00023P_2^2$ $MW$, are to be included in your economic dispatch, perform one iteration to refine the lossless dispatch solution obtained in part b) using the bisection method to update the incremental cost $\lambda$ .

**Setting up the parameter values** $\quad ORIGIN := 1$

$$a := 2\begin{pmatrix} 0.008 \\ 0.0085 \end{pmatrix} \quad b := \begin{pmatrix} 6.8 \\ 6.2 \end{pmatrix} \quad c := \begin{pmatrix} 300 \\ 280 \end{pmatrix} \quad Pmax := \begin{pmatrix} 300 \\ 200 \end{pmatrix} \quad Pmin := \begin{pmatrix} 50 \\ 50 \end{pmatrix}$$

$$B := \begin{pmatrix} 0.00022 & 0 \\ 0 & 0.00023 \end{pmatrix} \quad B_0 := \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad B_{00} := \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad P_D := 400$$

$$F_1(P_1) := 0.008 \cdot (P_1)^2 + 6.8 \cdot P_1 + 300 \qquad F_2(P_2) := 0.0085 \cdot (P_2)^2 + 6.2 \cdot P_2 + 280$$

**a ) Incremental fuel costs**:

$$\frac{d}{dP_1} F_1 = 0.016P_1 + 6.8 \qquad \frac{d}{dP_2} F_2 = 0.017P_2 + 6.2$$

**b ) Optimal dispatch ignoring network loss**

$$a_t := \left( \sum_{i=1}^{2} \frac{1}{a_i} \right)^{-1} \quad a_t = 0.0082 \qquad b_t := a_t \cdot \sum_{i=1}^{2} \frac{b_i}{a_i} \quad b_t = 6.5091$$

$$\lambda := a_t \cdot P_D + b_t = 9.8061$$

Using vectorize operator

$$P := \frac{\lambda}{\vec{a}} - \overrightarrow{\frac{b}{a}} \qquad P = \begin{pmatrix} 187.8788 \\ 212.1212 \end{pmatrix} \qquad \sum P = 400$$

Since $P_2 > P_{2max}$, enforce its limit

$$P_2 := Pmax_2 = 200 \qquad P_1 := P_D - P_2 = 200$$

**c ) Fuel costs for above dispatch**

$$F_1(P_1) = 1980 \qquad F_2(P_2) = 1860$$

$$F_{total} := F_1(P_1) + F_2(P_2) = 3840$$

**d ) Dispatch including network losses using λ iteration - bisection method**

Set inital values: $\lambda_{old} := 0$ $\qquad P_{told} := 0$ $\qquad P_{loss} := 0$ $\qquad P_D := 400$

With any λ iteration method, expressions for updating the P's for different values of λ are needed.
This scheme begin with some guess value of λ and use it in the above expressions to obtain the corresponding P's.
Check whether | ΣP's - $P_D$ | is near zero, within some convegence tolerance ε.
If it is stop, if not raise or lower λ by some small Δλ
Repeat until convergence is reached.

$$P_1(\lambda) := \frac{1 - \frac{b_1}{\lambda} - 2\cdot\left(B_{1,2}\cdot P_2\right)}{\frac{a_1}{\lambda} + 2\cdot B_{1,1}} \qquad P_2(\lambda) := \frac{1 - \frac{b_2}{\lambda} - 2\cdot\left(B_{2,1}\cdot P_1\right)}{\frac{a_2}{\lambda} + 2\cdot B_{2,2}}$$

$$P_{loss}(\lambda) := \begin{pmatrix} P_1(\lambda) \\ P_2(\lambda) \end{pmatrix}^T \cdot B \cdot \begin{pmatrix} P_1(\lambda) \\ P_2(\lambda) \end{pmatrix}$$

$$f(\lambda) := P_1(\lambda) + P_2(\lambda) - P_{loss}(\lambda) - P_D$$

**λ iteration using bisection of λ**

Lambda irteration, losses not included in coordination

**Total cost with network losses included in coordination**

To get hold of a pair of λ values for starting the bisection method:

Use lossless P's to determine approx network losses than substitue back in equivalent dispatch model to get approx λ

$$P = \begin{pmatrix} 200 \\ 200 \end{pmatrix} \qquad B = \begin{pmatrix} 2.2\times 10^{-4} & 0 \\ 0 & 2.3\times 10^{-4} \end{pmatrix} \qquad P_{lossapprox} := P^T \cdot B \cdot P = 18$$

$$\lambda_{approx} := a_t \cdot \left(P_D + P_{lossapprox}\right) + b_t = 9.9544 \qquad f\left(\lambda_{approx}\right) = -83.4672$$

Since f(λ)<0, use a value close ot it as lower bound, that is $\lambda L := 10$ $\quad f(\lambda L) = -79.7431$

For the upper bound, f(λH) must be > 0, try $\lambda H := 12$ $\quad f(\lambda H) = 73.5168$

$$P_{Dapprox} := P_D + P_{lossapprox} = 418$$

$\varepsilon := 1$ $\qquad imax := 1$ $\qquad$ Start with the lossless P's, that is $P := \begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = \begin{pmatrix} 200 \\ 200 \end{pmatrix}$

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IC2(a, b, ε, imax, λH, λL, Pmax, Pmin, P, B, P_D) :=
  n ← length(Pmax)
  yH ← f(λH)
  yL ← f(λL)
  error("Function has same sign at endpoints")  if  yH·yL > 0
  λ ← (λH + λL)/2
  for i ∈ 0 .. imax
    for j ∈ 1 .. n
      P_j ← [λ − b_j − 2·λ·(B_j,1·P_1 + B_j,2·P_2 − B_j,j·P_j)] / (a_j + 2·B_j,j·λ)
      P_j ← Pmax_j  if  P_j > Pmax_j
      P_j ← Pmin_j  if  P_j < Pmin_j
    P_loss ← B_1,1·(P_1)^2 + B_2,2·(P_2)^2 + B_1,2·P_1·P_2
    f ← ΣP − P_D − P_loss
    break  if  |f| < ε
    λL ← λ  if  f < 0
    λH ← λ  if  f > 0
    λ ← (λH + λL)/2
  (λ
   P
   i)
```

**Result after 1 iteration**

$$IC2\left(a, b, \varepsilon, imax, \lambda H, \lambda L, Pmax, Pmin, P, B, P_D\right) = \begin{pmatrix} 11.25 \\ \{2,1\} \\ 1 \end{pmatrix}$$

$$\lambda := IC2\left(a, b, \varepsilon, imax, \lambda H, \lambda L, Pmax, Pmin, P, B, P_D\right)_1 = 11.25$$

$$P_{loss}(\lambda) = 21.8544$$

$$PG := IC2\left(a, b, \varepsilon, imax, \lambda H, \lambda L, Pmax, Pmin, P, B, P_D\right)_2 = \begin{pmatrix} 223.1719 \\ 200 \end{pmatrix}$$

number of iteration taken: $IC2\left(a, b, \varepsilon, imax, \lambda H, \lambda L, Pmax, Pmin, P, B, P_D\right)_3 = 1$

Total generation $PG_1 + PG_2 = 423.1719$

Total cost $F_{total2} := F_1\left(PG_1\right) + F_2\left(PG_2\right) = 4076.0144$ $ per hr

Alternative method