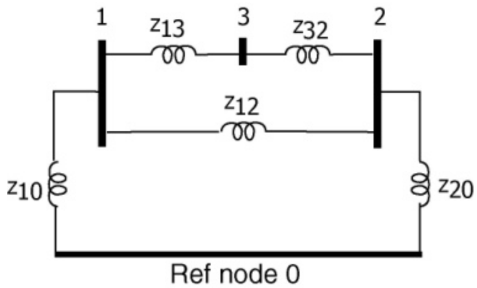
ECE43200 **Test 3** **Dec 18, 2105**

Q.1 (33 pts.)



The Z_{bus} for the above 3-bus network with bus 0 as reference, in per unit, is given to be

$$\begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ Z_{bus} = & \begin{bmatrix} 0.8702j & 0.7298j & 0.8j \\ 0.7298j & 0.8702j & 0.8j \\ 0.8j & 0.8j & 1j \end{bmatrix} \end{array}$$

Assuming that the prefault values of all buses are 1 per unit, that is $\tilde{V}_1 = \tilde{V}_2 = \tilde{V}_3 = 1\angle 0^o$, and ignoring all load currents, determine

a) (9 pts) the short-circuit current, in per unit, that would flow out of bus 3 through a fault of zero fault impedance connected between bus 3 and the reference bus

b) (15 pts) estimate the voltages of buses 1, 2, and 3 during the short-circuit fault on bus

c) (9 pts) If the impedance of the branch z_{12} directly connecting buses 1 and 2 is $j0.5$ per unit and you are asked to remove z_{12} from the Zbus given. Determine the augmented matrix in this process, that is the intermediate matrix before you perform a Kron's reduction to get the update Zbus. Do NOT perform the Kron's reduction.

Assuming prefault bus voltages are all unity $V_{1pf} := 1$ $V_{2pf} := 1$ $V_{3pf} := 1$

(a) Short-circuit current at bus 3 modelled by superimposing a equal and opposite prefault voltage

$\Delta V_3 := -V_{3pf}$ $Z_{th3} := Zbus_{3,3} = j$ $I_{3sc} := \dfrac{V_{3pf}}{Z_{th3}} = -j$

(b) With a direct short-circuit on bus 3, the new bus voltages can be estimated using

$$\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} := \begin{pmatrix} V_{1pf} + \Delta V_1 \\ V_{2pf} + \Delta V_2 \\ V_{3pf} + \Delta V_3 \end{pmatrix} = \blacksquare \cdot \begin{pmatrix} Zbus_{1,1} & Zbus_{1,2} & Zbus_{1,3} \\ Zbus_{2,1} & Zbus_{2,2} & Zbus_{2,3} \\ Zbus_{3,1} & Zbus_{3,2} & Zbus_{3,3} \end{pmatrix} \cdot \begin{pmatrix} I_{1pf} \\ I_{2pf} \\ I_{3pf} + \Delta I_3 \end{pmatrix}$$

For a direct short at bus 3, $\Delta V_3 = -V_{3pf}$ and $\Delta I_3 = -I_{3sc}$.
The prefault currents produce the prefault voltages, leaving

$$\begin{pmatrix} \Delta V_1 \\ \Delta V_2 \\ -V_{3pf} \end{pmatrix} = \blacksquare \cdot \begin{pmatrix} Zbus_{1,1} & Zbus_{1,2} & Zbus_{1,3} \\ Zbus_{2,1} & Zbus_{2,2} & Zbus_{2,3} \\ Zbus_{3,1} & Zbus_{3,2} & Zbus_{3,3} \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ -I_{3sc} \end{pmatrix} \qquad \begin{pmatrix} \Delta V_1 \\ \Delta V_2 \\ -V_{3pf} \end{pmatrix} := V_{3pf} \cdot \begin{pmatrix} \dfrac{-Zbus_{1,3}}{Zbus_{3,3}} \\ \dfrac{-Zbus_{2,3}}{Zbus_{3,3}} \\ -1 \end{pmatrix}$$

Using Zbus matrix values that were computed earlier, the bus voltages during the fault is given by

$$\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} := \begin{pmatrix} V_{1pf} - \dfrac{Zbus_{1,3}}{Zbus_{3,3}} \\ V_{2pf} - \dfrac{Zbus_{2,3}}{Zbus_{3,3}} \\ 0 \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.2 \\ 0 \end{pmatrix}$$

(c) To remove the link z12 between buses 1 and 1, add a link of equal but opposite impedance value

$$z_{12a} := -z_{12} = -0.5j$$

$$vadd4 := Zbus^{\langle 1 \rangle} - Zbus^{\langle 2 \rangle} \qquad z_{bb} := Zbus_{1,1} + Zbus_{2,2} - 2 \cdot Zbus_{1,2} + z_{12a} = -0.2193j$$

$$Zbusaug := stack\left(augment(Zbus, vadd4), augment\left(vadd4^T, z_{bb}\right)\right)$$

Answer asked for

$$Zbusaug = \begin{pmatrix} 0.8702j & 0.7298j & 0.8j & 0.1404j \\ 0.7298j & 0.8702j & 0.8j & -0.1404j \\ 0.8j & 0.8j & j & 0 \\ 0.1404j & -0.1404j & 0 & -0.2193j \end{pmatrix}$$

double-checking

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eliminate(Z, nrow, ncol, pv) := | for  i ∈ 1 .. nrow
                                |   for  j ∈ 1 .. ncol
                                |     Zreduce[i,j] ← Z[i,j] − (Z[i,pv]·Z[pv,j]) / Z[pv,pv]
                                | Zreduce
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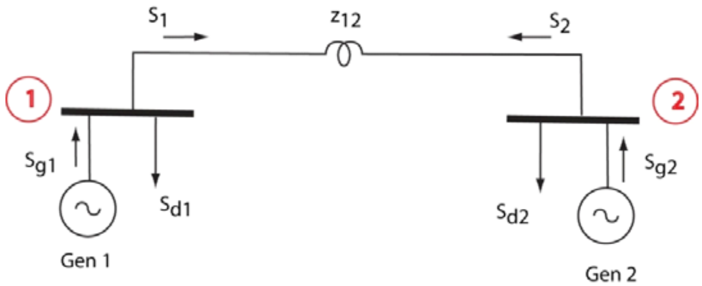
$\text{Zbus} := \text{eliminate}(\text{Zbusaug}, 3, 3, 4)$

$$\text{Zbus} = \begin{pmatrix} 0.96j & 0.64j & 0.8j \\ 0.64j & 0.96j & 0.8j \\ 0.8j & 0.8j & j \end{pmatrix}$$

Q.2 (34 pts.)



2-bus system

The 2-bus system with line impedance z_{12}=j0.2 per unit is to be operated with the following desired operating condition:

The bus type and their scheduled/desired operating condition are given in the table below:

Bus No. i	**Bus type**	**Bus voltages in pu**		**Generation into bus** $P_{gi} + jQ_{gi}$ in pu		**Loading on bus** $P_{di} + jQ_{di}$ in pu	
		Magnitude $\lvert V_i \rvert$	**Angle** δ_i	P_{gi}	Q_{gi}	P_{di}	Q_{di}
1	Slack bus	1.02	0	?	?	0.2	0.1
2	Gen bus	1.0	?	0.6	?	1.0	0.4

a) (25 pts) Starting with an initial guess of $\tilde{V}_1 = 1.02\angle 0^o$ and $\tilde{V}_2 = 1\angle 0^o$, perform one complete iteration of updating $\tilde{V}_2$ and Q_{g2}.

b) (9 pts) Also compute the S_{g1} of the slack generator at the end of this first iteration.

Desired operation conditions: $V_1 := 1.02$ $\quad S_{d1} := 0.2 + j\cdot 0.1$

$V_{2sch} := 1 \quad P_{g2} := 0.6 \quad S_{d2} := 1.0 + j\cdot 0.4$

$z_{12} := 0 + j\cdot 0.2 \qquad y_{12} := \dfrac{1}{z_{12}} = -5j$

By inspection, the Y-bus is $Y_{1,1} := y_{12} \quad Y_{1,2} := -y_{12} \quad Y_{2,1} := Y_{1,2} \quad Y_{2,2} := y_{12}$

$$Y = \begin{pmatrix} -5j & 5j \\ 5j & -5j \end{pmatrix}$$

Equations to compute net power injected

Gen bus 2, given P_{g2} and S_{d2} $\quad S_{d2} := P_{d2} + jQ_{d2}$ $\quad P_2 := P_{g2} - P_{d2}$

Calculate $\quad Q_2 := \mathrm{Im}\left[V_2 \cdot \overline{\left(Y_{21} \cdot V_1\right)}\right]$ $\quad Q_{g2} := Q_2 + Q_{d2}$

$S_2 := P_2 + jQ_2$

Slack bus 1 $\quad I_1 := Y_{1,1} \cdot V_1 + Y_{1,2} \cdot V_2$ $\quad S_1 := V_1 \cdot \overline{I_1}$

$S_{g1} := S_1 + S_{d1}$

Equations for updating voltage of Gen bus 2

$$V_2 := \frac{1}{Y_{2,2}}\left[\frac{\overline{S_2}}{\overline{V_2}} - \left(Y_{2,1} \cdot V_1\right)\right] \qquad \arg\left(V_2\right) \cdot \frac{180}{\pi} = \blacksquare$$

Update voltage of bus 2 to scheduled value using $\quad V_2 := V_{2sch} \cdot e^{j \cdot \arg\left(V_2\right)}$

(a) Perform one Gaussian iteration

Initial and scheduled values

$V_{1sch} := 1.02 \qquad V_1 := V_{1sch} \cdot e^{j \cdot 0} \qquad S_{d1} = 0.2 + 0.1j$

$V_{2sch} := 1.0 \qquad V_2 := V_{2sch} \cdot e^{j \cdot 0} \qquad P_{g2} = 0.6 \qquad S_{d2} = 1 + 0.4j$

First store values for voltage convergence check later $\quad V_{2old} := V_2$

$P_2 := P_{g2} - \mathrm{Re}\left(S_{d2}\right) = -0.4$

$Q_2 := \mathrm{Im}\left[V_2 \cdot \overline{\left(Y_{2,1} \cdot V_1 + Y_{2,2} \cdot V_2\right)}\right] = -0.1$

$S_2 := P_2 + j \cdot Q_2 = -0.4 - 0.1j$

$Q_{g2} := Q_2 + \mathrm{Im}\left(S_{d2}\right) = 0.3$

Iterating for voltage of Gen bus 2

$$V_2 := \frac{1}{Y_{2,2}}\left[\frac{\overline{S_2}}{\overline{V_2}} - \left(Y_{2,1} \cdot V_1\right)\right] = 1 - 0.08j \qquad \arg\left(V_2\right) \cdot \frac{180}{\pi} = -4.5739$$

Adjust voltage magnitude back to scheduled value

$$V_2 := V_{2sch} \cdot e^{j \cdot \arg(V_2)} = 0.9968 - 0.0797j$$

Compute Q_2

$$Q_2 := \operatorname{Im}\left[V_2 \cdot \overline{\left(Y_{2,1} \cdot V_1 + Y_{2,2} \cdot V_2\right)}\right] = -0.0838$$

$$Q_{g2} := Q_2 + \operatorname{Im}(S_{d2}) = 0.3162$$

Check change in V's for convergence

$$\left|V_2 - V_{2old}\right| = 0.0798$$

Calculate slack bus injection and generation

$$I_1 := Y_{1,1} \cdot V_1 + Y_{1,2} \cdot V_2 = 0.3987 - 0.1159j$$

$$S_1 := V_1 \cdot \overline{I_1} = 0.4067 + 0.1182j$$

$$S_{g1} := S_1 + S_{d1} = 0.6067 + 0.2182j$$

Q.3 (33 pts.)

The fuel cost in dollars per hour and allowable operating range of two generators in the same power station are given as

$$F(P_1) = 0.5P_1^2 + 12P_1 + 500 \quad \$/\text{hour} \qquad 10 \le P_1 \le 100 \text{ MW}$$
$$F(P_2) = 0.4P_2^2 + 6P_2 + 980 \quad \$/\text{hour} \qquad 10 \le P_2 \le 120 \text{ MW}$$

where the output powers of the two generators, P_1 and P_2 , are in MW.

If the total demand to be met by the power stations, that is $P_t = P_1 + P_2$, is 180 MW,

a) (18 pts) Determine the incremental fuel cost of each generator and the total operating fuel cost in $ per hour.

b) (15 pts) Determine the incremental fuel cost, $ per MWh, for the next MW of total demand beyond 180 MW?

Given conditions, with P's in MW

$F_1(P_1) := 0.5P_1^{\ 2} + 12 \cdot P_1 + 500$ $/h $\qquad P_{1min} := 10 \qquad P_{1max} := 100$

$F_2(P_2) := 0.4P_2^{\ 2} + 6 \cdot P_2 + 980$ $/h $\qquad P_{2min} := 10 \qquad P_{2max} := 120$

Minimum and maximum total demand from station $\quad P_{tmin} := 30 \qquad P_{tmax} := 180$

a)

Obtain the incremental fuel cost equations by differentiating F_1 and F_2 with respect to P_1 and P_2, respectively

$\lambda_1(P_1) := 1 \cdot P_1 + 12$ $/MWh

$\lambda_2(P_2) := 0.8 \cdot P_2 + 6$ $/MWh

At a total load $\quad P_t := 180 \quad$ Using an initial estimates $\quad P_1 := \dfrac{P_t}{2} = 90 \qquad P_2 := P_t - P_1 = 90$

$\lambda := \lambda_1(P_1) = 102$

Given $\quad 1 \cdot P_1 - \lambda = -12$

$0.8 \cdot P_2 - \lambda = -6$

$P_1 + P_2 = P_t$

$$\text{Soln} := \text{Find}(P_1, P_2, \lambda) = \begin{pmatrix} 76.6667 \\ 103.3333 \\ 88.6667 \end{pmatrix}$$

$P_1 := \text{Soln}_1 = 76.6667 \qquad P_2 := \text{Soln}_2 = 103.3333$

$\lambda := \text{Soln}_3 = 88.6667$ $/MWh

Above solution is feasible because P_1 and P_2 do not exceed their limits.

$$\lambda := 1 \cdot P_1 + 12 = 88.6667$$

$$F_t := F_1(P_1) + F_2(P_2) = 1.023 \times 10^4$$ \$/hr

b) For the next MW of additional demand $P_t := 181$

Given $1 \cdot P_1 - \lambda = -12$

$$0.8 \cdot P_2 - \lambda = -6$$

$$P_1 + P_2 = P_t$$

$$Soln := \mathrm{Find}(P_1, P_2, \lambda) = \begin{pmatrix} 77.1111 \\ 103.8889 \\ 89.1111 \end{pmatrix} \qquad P_1 := Soln_1 = 77.1111 \qquad P_2 := Soln_2 = 103.8889$$

$$\lambda_1 := Soln_3 = 89.1111$$ \$/MWh

Total fuel cost to generate 1 more MW $F_{t1} := F_1(P_1) + F_2(P_2) = 1.0319 \times 10^4$ \$

Thus, the differential cost to generate the extra MW $F_{t1} - F_t = 88.8889$ \$

Note that the difference in total fuel cost is equal to the average value of the incremental costs $\dfrac{\lambda + \lambda_1}{2} = 88.8889$ \$/MWh

Using alternate solution method for parts a) and b)

$$a_1 := 1 \qquad b_1 := 12 \qquad a_2 := 0.8 \qquad b_2 := 6$$

$$a_t := \left(\frac{1}{a_1} + \frac{1}{a_2}\right)^{-1} = 0.4444 \qquad b_t := a_t \cdot \left(\frac{b_1}{a_1} + \frac{b_2}{a_2}\right) = 8.6667$$

For a total demand $P_t := 180$ $\lambda := a_t \cdot P_t + b_t = 88.6667$ \$/MWh

Check limits $P_1 := \dfrac{\lambda - b_1}{a_1} = 76.6667$ $P_2 := \dfrac{\lambda - b_2}{a_2} = 103.3333$

Total fuel cost $F_t := (F_1(P_1) + F_2(P_2)) = 1.023 \times 10^4$

For a total demand $P_t := 181$ $\lambda_1 := a_t \cdot P_t + b_t = 89.1111$ \$/MWh

Check limits $P_1 := \dfrac{\lambda_1 - b_1}{a_1} = 77.1111$ $P_2 := \dfrac{\lambda_1 - b_2}{a_2} = 103.8889$

Total fuel cost $\quad F_{t1} := \left(F_1\left(P_1\right) + F_2\left(P_2\right)\right) = 1.0319 \times 10^4$

Differential cost to generate the extra MW $\quad F_{t1} - F_t = 88.8889$

$$\frac{\lambda + \lambda_1}{2} = 88.8889$$

Note that the difference in total fuel cost is equal to the average value of the incremental costs