Solution

November 8, 2006 Name ______________________

ECE608 Computational Models and Methods
Fall 2006
Exam #3

Solve the following six problems.

Circle Yes/No answers. Provide explanations following the word "Explanation." Use the graphs provided when needed.

The number of points for each problem is shown in the table below.

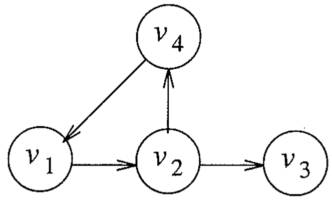
Problem		Points
1	(a)	/ 5
	(b)	/ 5
	(c)	/ 5
	(d)	/ 5
2	(a)	/ 5
	(b)	/ 5
3	(a)	/ 10
	(b)	/ 10
4	(a)	/ 10
	(b)	/ 10
	(c)	/ 10
5		/ 10
6		/ 10
total		/ 100

Problem 1

Let $G = (V,E)$ be a directed graph with n vertices. The transitive closure of G is the graph $G^* = (V,E^*)$ where $E^* = \{(v_i,v_j)$:*there is a path from* v_i *to* v_j *in* $G\}$.

Let C^m be an $n \times n$ matrix. Let the entry in row i column j of C^m be c_{ij}^m. Our goal is to compute C^m such that $c_{ij}^m = 1$ if there is a path of length $\leq m$ from v_i to v_j in G; otherwise $c_{ij}^m = 0$.

Consider the following graph in all the parts of this problem.



(a) Fill the matrix C^1.

C^1	1	2	3	4
1	1	1	0	0
2	0	1	1	1
3	0	0	1	0
4	1	0	0	1

(b) Write a general equation for computing c_{12}^2 as a function of the entries of C^1, and compute the value of c_{12}^2.

Answer: $c_{12}^2 = c_{11}^1 \,\&\, c_{12}^1$ or $c_{12}^1 \,\&\, c_{22}^1$ or $c_{13}^1 \,\&\, c_{32}^1$ or $c_{14}^1 \,\&\, c_{42}^1 = 1$

(c) Write a general equation for computing c_{13}^2 as a function of the entries of C^1, and compute the value of c_{13}^2.

Answer: $c_{13}^2 = c_{11}^1 \,\&\, c_{13}^1$ or $c_{12}^1 \,\&\, c_{23}^1$ or $c_{13}^1 \,\&\, c_{33}^1$ or $c_{14}^1 \,\&\, c_{43}^1 = 1$

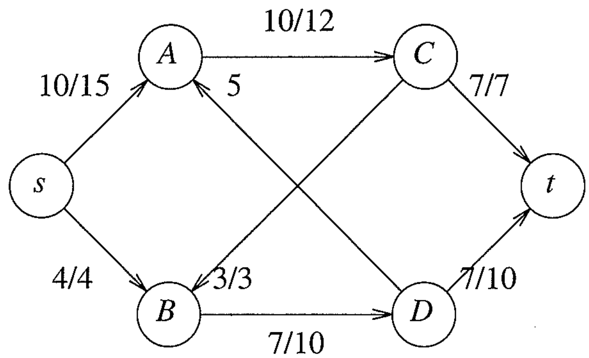
(d) What is the largest value of m that needs to be considered in order to compute the transitive closure of G?

Answer: $m = n-1 = 3$

Explanation: The longest simple path has n-1 edges

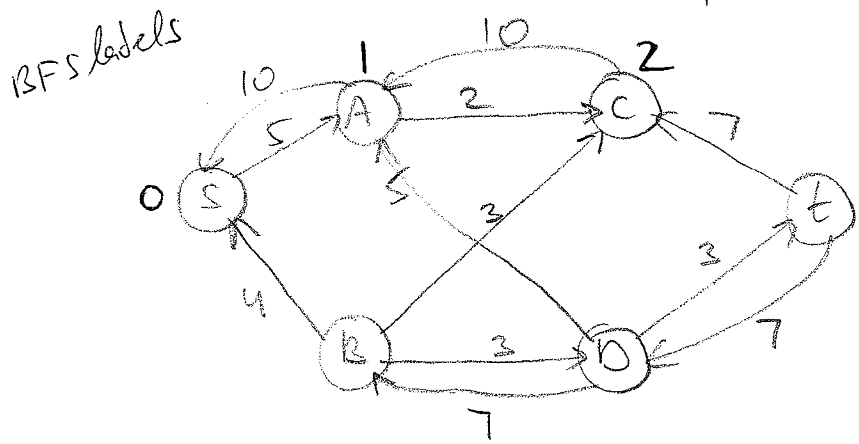
Problem 2

Argue in two different ways that the flow in the following network is maximum. Your arguments should be concise, accurate and specific. For example, if you say that there is an augmenting path, specify the augmenting path.



(a) **Argument 1:**

The residual network has no path from s to t.



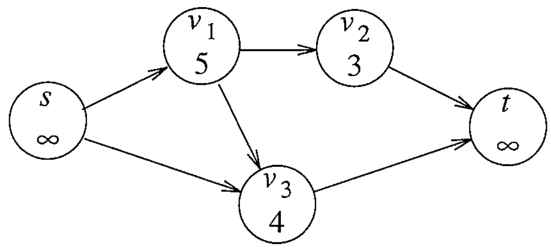
(b) **Argument 2:**

The size of the flow ($f(s,V) = 10+4=14$) is equal to the capacity of a cut $S=\{s,A,C\}$, $T=\{B,D,t\}$

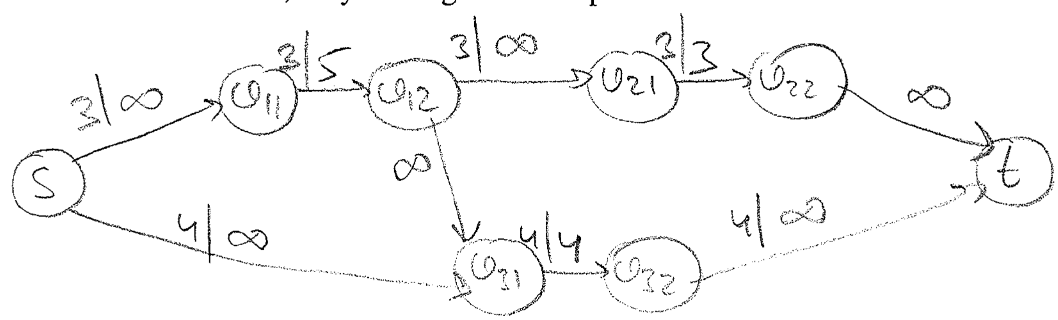
$c(S,T) = 4+3+7=14$.

Problem 3

In a flow network, the edges have infinite capacities. However, the flow through a vertex v is limited by the capacity of the vertex. The capacity of a vertex v is denoted by $c(v)$. The source s and the sink t have infinite capacities. The following is an example of such a flow network.



(a) For the flow network FN above, show a standard flow network FN' such that a maximum flow in FN' translates into a maximum flow in FN.
In a standard flow network, only the edges have capacities.



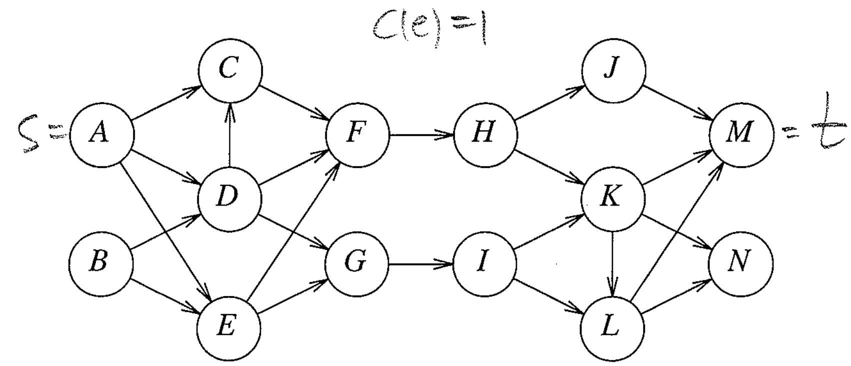
(b) Solve the maximum flow problem in FN' that you found in (a). Show the maximum flow on the network. What is the size of the maximum flow?

$|f| = 7$

Problem 4

Let $G(V,E)$ be a directed graph. An (a,b) *edge separator* is a set of edges T such that every directed path from a to b goes through at least one edge of T. In other words, if all the edges of T are removed from the graph, b remains disconnected from a.

The following is an example of a graph where we will want to find an (A,M) edge separator.

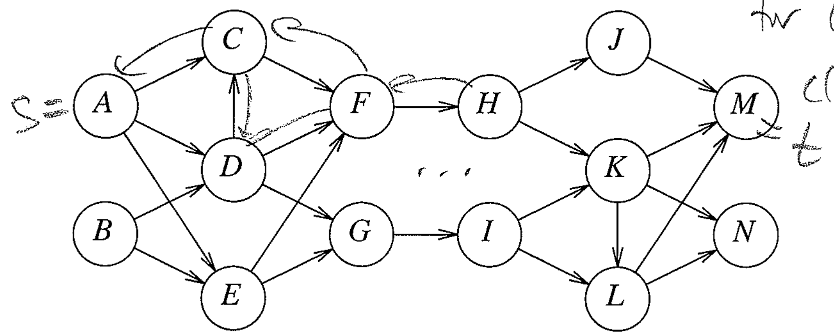


(a) Using the graph above, show a flow network such that the solution to the maximum flow problem in this network will yield a minimum (A,M) edge separator.

(b) How does one find the minimum (A,M) edge separator based on the maximum flow? find a minimum cut (S,T). The edges from S to T are the required set.

(c) The graph $\hat{G}$ is the undirected version of the graph above (in the undirected version of a graph, a directed edge is replaced by an undirected one). Use the following copy of the graph to show a flow network such that the solution to the maximum flow problem in this network will yield a minimum (A,M) edge separator in $\hat{G}$.

for (u,v) add (v,u).



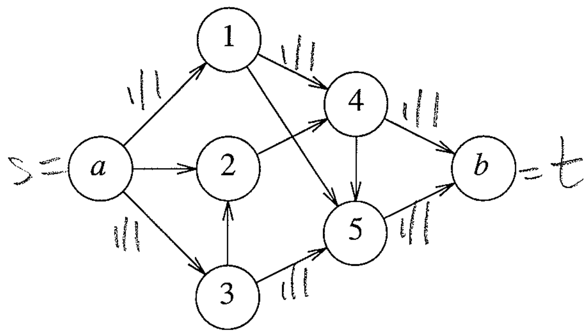
c(e) = 1

Problem 5

Let $G(V,E)$ be a directed graph. For $a,b \in V$, let $p(a,b)$ be the maximum number of edge disjoint paths from a to b (two paths are said to be edge disjoint if they do not have any edge in common).

The following is an example of a graph where we will want to find $p(a,b)$.

c(e)=1



Show a flow network such that the solution to the maximum flow problem in this network will yield $p(a,b)$. What is the value of $p(a,b)$ in this case?

p(a,d)=2

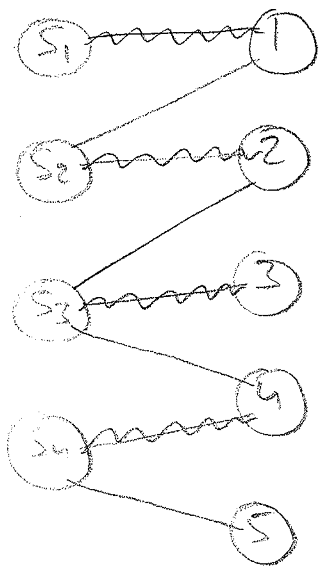
Problem 6

The following sets of elements are given. $S_1 = \{1\}$, $S_2 = \{1,2\}$, $S_3 = \{2,3,4\}$ and $S_4 = \{4,5\}$. It is required to find a set $S = \{s_1, s_2, s_3, s_4\}$ such that $s_i \in S_i$ for $i = 1,2,3,4$ (S should contain four different elements).

Show a bipartite graph such that a maximum matching in the bipartite graph will yield a set S defined as above.

Label the vertices of the graph in a way that will make it clear how they correspond to the sets and their elements.

Show a solution to the bipartite matching problem and the corresponding set S.



$S = \{1, 2, 3, 4\}$