MA 174 EXAM I 02/18/2014

Name: ____________ PUID: ____________ Section: ____________

SHOW ALL YOUR WORK. NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED.

Points awarded

1. (8 points) ____________
2. (8 points) ____________
3. (12 points) ____________
4. (8 points) ____________
5. (8 points) ____________
6. (8 points) ____________
7. (8 points) ____________
8. (8 points) ____________
9. (10 points) ____________
10. (10 points) ____________
11. (12 points) ____________
12. (points) ____________

96	68	59	49	43	34
86	64	57	48	40	
75	64	51	45	40	
72	60		45		

Total Points: ____________

1. (8 points) The surface defined by $z^2 = 4x^2 + 9y^2$ is a

A. hyperbolic paraboloid
B. elliptical cone
C. elliptical paraboloid
D. ellipsoid
E. hyperboloid

2. (8 points) Which of the following statements are true for nonzero vectors $\mathbf{u}$ and $\mathbf{v}$?

(i) if $\mathbf{u} \cdot \mathbf{v} = 0$, then $\mathbf{u}$ and $\mathbf{v}$ are orthogonal ✓
(ii) if $\mathbf{u} \times \mathbf{v} = \mathbf{0}$, then $\mathbf{u}$ and $\mathbf{v}$ are orthogonal ✗
(iii) $\mathbf{u} \cdot \mathrm{Proj}_{\mathbf{u}}\mathbf{v} = 0$ ✗
(iv) $\mathbf{u} \times \mathrm{Proj}_{\mathbf{u}}\mathbf{v} = \mathbf{0}$ ✓

A. (i) and (iii) only
B. (i) and (iv) only
C. (ii) and (iii) only
D. (ii) and (iv) only
E. all are true

3. (a)(8 points) Find the plane determined by the lines $x = t,\ y = -t+2,\ z = t+1$ and $x = 2s+2,\ y = s+3,\ z = 5s+6$.

$$\vec{v}_1 = (1, -1, 1) \qquad \vec{v}_1 \times \vec{v}_2 = (-6, -3, 3) = -3\,(2, 1, -1)$$

$$\vec{v}_2 = (2, 1, 5)$$

2

$\underline{t=0}$ $\quad P = (0, 2, 1)$

3

the plane equation $\qquad \vec{n} = (2, 1, -1)$

3

$$0 = (2, 1, -1)\cdot(x, y-2, z-1) = 2x + y - z - 1$$

$$\boxed{\cancel{= 2x + y - z + 1}}$$

(b)(4 points) Find the distance from point $S(3,3,2)$ to the plane in (a).

$$d = \left| \frac{\vec{PS}\cdot\vec{n}}{|\vec{n}|} \right| = \frac{|(3,1,1)\cdot(2,1,-1)|}{\sqrt{4+1+1}}$$

$$= \frac{6}{\sqrt{6}} = \sqrt{6}$$

4. (8 points) A particle starts at the origin with initial velocity $\mathbf{i} - \mathbf{j} + \frac{2}{3}\mathbf{k}$. Its acceleration is $2\,\mathbf{i} + 4\,\mathbf{j} + 2t\,\mathbf{k}$. Find its position at $t = 1$.

$$\vec{r}(0) = (0,0,0), \quad \vec{v}(0) = (1,-1,\tfrac{2}{3})$$

$$\vec{a}(t) = (2,4,2t)$$

$$\vec{v}(t) = (2t,\ 4t,\ t^2) + \vec{v}(0)$$

$$\vec{r}(t) = (t^2, 2t^2, \tfrac{1}{3}t^3) + t\,\vec{v}(0) + \vec{r}(0)$$

$$\vec{r}(1) = (1, 2, \tfrac{1}{3}) + (1,-1,\tfrac{2}{3}) = (2,1,1)$$

5. (8 points) Find the parametric equation for the line through $(0, -7, 0)$ perpendicular to the plane $x + 2y + 2z = 13$.

$\vec{n} = (1,2,2)$ is perpendicular to the plane

$\Longrightarrow$ $\vec{n}$ is parallel to the line

$$\vec{r}(t) = (0,-7,0) + t(1,2,2)$$

$$\begin{cases} x = t \\ y = -7 + 2t \\ z = 2t \end{cases}$$

6. (8 points) Find the parametric equation for the line that is tangent to the curve $\mathbf{r}(t) = (2\sin t)\mathbf{i} + (2\cos t)\mathbf{j} + 5t\mathbf{k}$ at $t_0 = 4\pi$.

$$\vec{r}(t) = (2\sin t,\ 2\cos t,\ 5t), \quad \boxed{\cancel{r(t_0)}}$$

$$\vec{r}(4\pi) = (0,\ 2,\ 20\pi)$$

$$\vec{r}\,'(t) = (2\cos t,\ -2\sin t,\ 5), \quad \vec{r}\,'(4\pi) = (2,\ 0,\ 5)$$

$$\vec{l}(t) = (0,\ 2,\ 20\pi) + t(2, 0, 5)$$

$$= (2t,\ 2,\ 5t + 20\pi)$$

7. (8 points) Let $\mathbf{T}(t)$ be the unit tangent vector, i.e., $\|\mathbf{T}\| = 1$. Prove that $\frac{d\mathbf{T}}{dt}$ is orthogonal to $\mathbf{T}$.

$$1 = \|\vec{T}\|^2 = \vec{T}(t) \cdot \vec{T}(t)$$

$$0 = \frac{d}{dt}\left(\vec{T}(t) \cdot \vec{T}(t)\right)$$

$$= \vec{T}\,' \cdot \vec{T} + \vec{T} \cdot \vec{T}\,'$$

$$= 2\vec{T} \cdot \vec{T}\,'$$

$$\Rightarrow \vec{T} \cdot \vec{T}\,' = 0 \Longrightarrow \vec{T} \perp \vec{T}\,'$$

8. (8 points) Find the arc length parameter along the curve from the point where $t = 0$ by evaluating the integral $s(t) = \int_0^t |\mathbf{v}(\tau)|\, d\tau$. Then find the length of the indicated portion of the curve:

$$\mathbf{r}(t) = (\cos t + t \sin t)\mathbf{i} + (\sin t - t \cos t)\mathbf{j}, \quad \pi/2 \le t \le \pi.$$

$$s(t) = \int_0^t \left| \vec{r}\,'(t) \right| dt$$

$$= \int_0^t \left| \left(-\sin t + \sin t + t\cos t,\ \cos t - \cos t + t \sin t \right) \right| dt$$

$$= \int_0^t \sqrt{(t\cos t)^2 + (t \sin t)^2}\ dt$$

$$= \int_0^t t\, dt = \frac{1}{2}t^2 \Big|_0^t = \frac{1}{2}t^2$$

$$L = s(\pi) - s\left(\frac{\pi}{2}\right)$$

$$= \frac{1}{2}\left(\pi^2 - \frac{\pi^2}{4}\right)$$

$$= \frac{3}{8}\pi^2$$

9. (10 points) Let C be the intersection of $x^2 + y^2 = 4$ and $z = 5$, find the curvature and torsion of C at $(2, 0, 5)$.

$$\vec{r}(t) = (2\cos t, 2\sin t, 5), \quad |\vec{r}\,'(t)| = 2$$

$$\vec{r}\,'(t) = (-2\sin t, 2\cos t, 0),$$

$$\vec{T}(t) = \frac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|} = (-\sin t, \cos t, 0)$$

$$\frac{d\vec{T}}{ds} = \frac{1}{|\vec{r}\,'(t)|}\frac{d\vec{T}}{dt} = \frac{1}{2}(-\cos t, -\sin t, 0) = -\frac{1}{2}(\cos t, \sin t, 0)$$

$$K = \left|\frac{d\vec{T}}{ds}\right| = \frac{1}{2}\left|(\cos t, \sin t, 0)\right| = \frac{1}{2}$$

$$\tau = \begin{vmatrix} -2\sin t & 2\cos t & 0 \\ -2\cos t & -2\sin t & 0 \\ 2\sin t & -2\cos t & 0 \end{vmatrix} \Big/ \left|\vec{r}\,'(t) \times \vec{r}\,''(t)\right| = 0$$

or $$\vec{N} = \cancel{\boxed{K}}\ \frac{1}{K}\frac{d\vec{T}}{ds} = \frac{d\vec{T}}{dt} \Big/ \left|\frac{d\vec{T}}{dt}\right| = \frac{(-\cos t, -\sin t, 0)}{1} = -(\cos t, \sin t, 0)$$

$$\vec{B} = \vec{T} \times \vec{N} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ -\sin t & \cos t & 0 \\ -\cos t & -\sin t & 0 \end{vmatrix} = (0, 0, 1)$$

$$\tau = -\frac{d\vec{B}}{ds} \cdot \vec{N} = -\vec{0} \cdot \vec{N} = 0$$

10. (10 points) For $f(x, y) = 1/\sqrt{16 - x^2 - y^2}$, find the domain, the range, the level curve passing through $(2\sqrt{2}, \sqrt{2})$, and the boundary of the domain; determine if the domain is open, close, or neither; and decide if the domain is bounded or unbounded.

- domain $= \{(x,y) \mid 16 - x^2 - y^2 > 0\} = \{(x,y) \mid x^2 + y^2 < 4^2\}$
- range $= [\frac{1}{4}, +\infty)$ $\quad f(x,y) = \frac{1}{\sqrt{16-x^2-y^2}} \geq \frac{1}{\sqrt{16}} = \frac{1}{4}$
- $f(2\sqrt{2}, \sqrt{2}) = \frac{1}{\sqrt{16-8-2}} = \frac{1}{\sqrt{6}} = \frac{1}{\sqrt{16-x^2-y^2}} \Rightarrow 16 - x^2 - y^2 = 6$

  $\Rightarrow x^2 + y^2 = 10$ level curve
- domain is open
- domain is bounded
- boundary $= \{(x,y) \mid x^2 + y^2 = 4^2\}$

11. (12 points) Compute the following limits:

$$\lim_{(x,y)\to(0,0)} \frac{e^y \sin x}{x}, \qquad \lim_{\substack{(x,y)\to(2,2) \\ x+y\neq 4}} \frac{x+y-4}{\sqrt{x+y}-2}.$$

$$= \lim_{y\to 0} e^y \cdot \lim_{x\to 0} \frac{\sin x}{x}$$

$$= 1 \cdot 1$$

$$= 1$$

$$= \lim \frac{(x+y)-4}{\sqrt{x+y}-2} \cdot \frac{\sqrt{x+y}+2}{\sqrt{x+y}+2}$$

$$= \lim_{(x,y)\to(2,2)} \left(\sqrt{x+y} + 2\right)$$

$$= \sqrt{4} + 2$$

$$= 4$$