AAE 439 Test #1

Fall, 1999

Open Book & Notes

Attempt All Problems

1. In class we have had considerable discussion on the ΔV estimates for launch vehicles. In my last handout I showed that $\Delta V = V_c - V_{surf}$ (with V_c evaluated at earth's surface) works o.k. for orbits very near the surface of the earth (neglecting gravity and drag losses, of course). For Kennedy Space Center, this technique gives a ΔV of 7.5 km/s.

   i) How does this estimate compare with a Hohmann transfer from Kennedy Space Center to a 150 km orbit inclined at 28.5°?

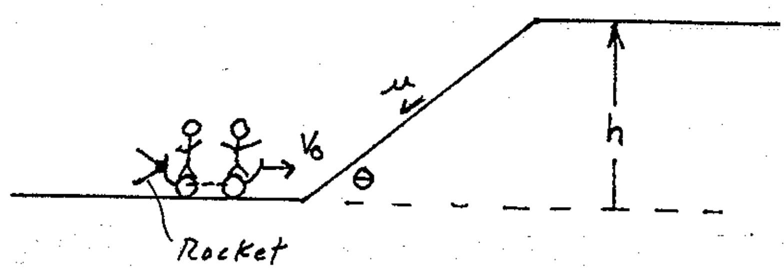
   ii) Which approach do you prefer and why?
   Do you have a better suggestion?

2. Consider a missile to be used for an air-launch application at altitudes from 20,000 - 40,000 ft. Packaging constraints dictate that the nozzle internal diameter be less than (or equal to) 7.0 inches. A minimum total impulse of 12,000 lb-sec is required and the solid propellant utilized generates a characteristic velocity of 4900 f/s. Assuming a constant thrust burn of 10 sec is desired determine the maximum allowable nozzle expansion ratio and minimum allowable chamber pressure for this propulsion system. You may assume $\gamma = 1.2$ and that the critical pressure for flow separation is 1/3 of the local atmospheric pressure.

3. At Rocket World, a new amusement park for rocket scientists, we have abandoned the conventional chain drive used to hoist a roller coaster to the top of a hill. In this case, we fire a rocket which accelerates the vehicle to the required speed to crest the hill just as we encounter the incline (see sketch below). Presume that we know the empty mass of the vehicle, Me, which includes everything except the propellant load. The engine Isp and hill geometry (h, θ) as shown below are also known. In addition, we must account for friction on the rails with a known friction coefficient, μ.

   i) Using a force balance on the roller coaster, derive an expression for the velocity V_o in terms of h, θ, and μ.

   ii) Derive an expression for the propellant mass (M_p) required to accommmplish this mission in terms of Me, h, θ, and μ.



4. In class we noted that for a given Isp and propulsion system mass fraction, λ, propellant mass increased exponentially with increasing ΔV. If the payload mass is fixed, derive an expression for the maximum ΔV attainable for a given rocket with fixed Isp and λ values. You may neglect gravity and drag losses for your analysis.

AAE 439 Test #1 Solns
Fa 1999

1. Hohmann xfer $\quad r_1 = r_e = 6378\ \text{Km} \quad r_2 = r_e + 150\ \text{km}$

$$= 6528\ \text{km}$$

$$V_{c_1} = \sqrt{\mu / r_1} = \left(3.98 \times 10^5 / 6378\right)^{1/2} = 7.9\ \text{Km/s}$$

$$\Delta V_1 = V_{c_1}\left[\sqrt{\frac{2r_2}{r_1 + r_2}} - 1\right] = 7.9\left[\sqrt{\frac{2\ 6528}{6528 + 6378}} - 1\right] = 0.046\ \text{km/s}$$

$$\Delta V_2 = V_{c_1}\left[\sqrt{\frac{r_1}{r_2}} - \sqrt{\frac{2r_1}{r_2(1 + r_2/r_1)}}\right] = 7.9\left[0.9884442 - 0.98268\right]$$

$$= 0.046\ \text{Km/s}$$

$$\Delta V = V_{c_1} + \Delta V_1 + \Delta V_2 = 7.99\ \text{Km/s}$$

↑ Need this to start the Hohmann xfer

2. $F = C_f P_c A_t = 1,200\ lb_f$

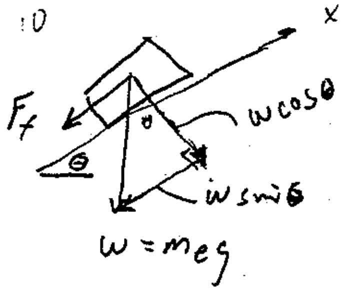
$P_a = 6.75$

$P_e = 1/3\, P_a$ = known $= 2.25$ psi $\qquad A_t = \frac{A_e}{\epsilon} = \frac{38.5}{\epsilon}$

Guess $P_c \rightarrow \frac{P_e}{P_c} \xrightarrow{C_f \text{ Tables}} C_{f_v},\ \epsilon \rightarrow C_f = C_{f_v} - \epsilon\, P_a/P_c \rightarrow A_t = \frac{38.5}{\epsilon} \rightarrow F$

P_c	P_e/P_c	ϵ	C_{f_v}	C_f	$A_t \sim in^2$	F	
1000	2.25×10^{-3}	38	1.88	1.624	1.01	1639	
600	0.00375	25	1.84	1.559	1.54	1440	
300	0.0075	15	1.79	1.452	2.57	1118	close enuf.

3.



$$F_f = \mu m_e g \cos\theta$$

i) $\Sigma F_x = m_e \frac{dv}{dt} = -m_e g\,(\sin\theta + \mu\cos\theta)$

integ. — i.c. at $t=0$ $V=V_0$

$$V = -gt\,(\sin\theta + \mu\cos\theta) + V_0 \qquad (1)$$

when $V=0$ want Coaster to Crest the hill

$$\therefore t_A = \text{Time to Ascend Hill} = \frac{V_0}{g(\sin\theta + \mu\cos\theta)} \qquad (2)$$

now integ. (1) again $v = dx/dt$

$$x = \cancel{x_0}^{\,0} + V_0 t - \frac{gt^2}{2}(\sin\theta + \mu\cos\theta) \qquad (3)$$

Plug (2) in (3)

$$x = \frac{h}{\sin\theta} = \frac{V_0^2}{2g(\sin\theta + \mu\cos\theta)} \Rightarrow \boxed{V_0^2 = 2gh\,(1 + \mu\cot\theta)} \quad 15$$

ii) From Rocket Eq.

$$\Delta V = V_0 = g I_{sp} \ln \frac{m_e + m_p}{m_e}$$

or

$$e^{V_0/gI_{sp}} - 1 = \frac{m_p}{m_e}$$

Using Result from Above for V_0, we get

$$10 \quad \boxed{m_p = m_e\left[e^{\frac{[2h}{I_{sp}}(1+\mu\cot\theta)/g]^{1/2}} - 1\right]}$$

4. $$\mathcal{R} = \frac{m_{PL} + m_p/\lambda}{m_{PL} + m_p\left(\frac{1}{\lambda} - 1\right)} \qquad m_p \mathcal{R}\left(\frac{1}{\lambda} - 1\right) + m_{PL}\mathcal{R} = m_{PL} + m_p/\lambda$$

$$m_p\left[\frac{(\mathcal{R}-1)}{\lambda} - \mathcal{R}\right] = m_{PL}(1 - \mathcal{R})$$

$$m_p = m_{PL}\left[\frac{\mathcal{R}-1}{\mathcal{R} - \frac{(\mathcal{R}-1)}{\lambda}}\right]$$

$m_p \to \infty$ when $\mathcal{R} - \frac{(\mathcal{R}-1)}{\lambda} = 0$

or $\lambda \mathcal{R} = \mathcal{R} - 1 \qquad \mathcal{R} = \frac{1}{1-\lambda} = e^{\Delta V / g I_{sp}}$

$$\boxed{\Delta V\big|_{max} = -g\, I_{sp} \ln(1-\lambda)}$$