AAE 439 Final Exam
Fall, 2000

Open Book & Notes
Attempt All Problems

1. A liquid rocket engine/propulsion system is to be employed to accelerate a space probe weighing 1000 lb to a ΔV of 10,000 f/s. The engine has an expansion ratio of 50 and the propellants produce a C^* of 6000 f/s at $\gamma = 1.2$. Assuming the propellant mass fraction of the propulsion system is 0.8 determine

   i) The propellant and inert masses associated with the propulsion system.

   ii) The nozzle manufacturer proposes to put a skirt on the end of the nozzle to increase the expansion ratio to 150. This change will add 100 lb of inert weight to the existing design. The manufacturer argues that this change will increase the ΔV capabilities of the propulsion system - a desirable feature for this application. Should you implement this change? i.e. is the nozzle manufacturer correct?

2. A ground launched interceptor utilizes a cylindrical fuel tank 30 inches in diameter and 100 inches long. The tank has a ullage pressure of 30 psia and a maximum internal pressure of 120 psig. The vehicle has a maximum acceleration of 20 g's and the fuel tank supports 1000 lb in upper stage propulsion, interstage and fairings. The tank is to be built from aluminum 6061 alloy which has the following properties:

Ultimate tensile strength	= 42 Ksi
Yield strength (tension)	= 36 Ksi
Modulus	= 10×10^6 Psi
Density	= 0.098 lb/in^3

   i) Determine the minimum acceptable wall thickness for this tank.

   ii) Assuming constant wall thickness throughout, estimate the weight of the tank skin.

3. Consider the solid rocket ballistic test motor shown in the sketch below. The small motor uses a cylindrical grain with a center perforation in order to achieve a nearly neutral burn.

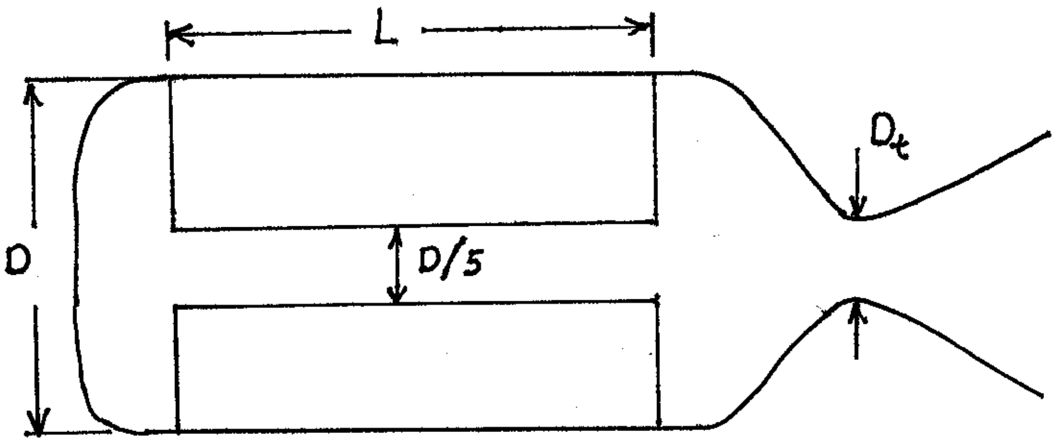
   i) Determine the L/D ratio which will provide identical chamber pressures at ignition and just prior to burnout of the propellant grain.

   ii) Assume we want a total of 0.5 Kg of propellant in the test motor and we desire a chamber pressure of 3 MPa for the test. Determine the dimensions L, D, and D_t to produce these results. The propellant characteristics are:

   Burn Rate = 1 cm/s , Density = 1700 Kg/m^3

   $C^* = 1500$ m/s, $\gamma = 1.2$,

   Hint: if you can't get an answer in Part ii), you may assume L/D = 2 to work this part of the problem.



4. In our discussion of hydrazine performance, we noted that the decomposition reaction could be written in terms of the fraction of NH_3 (ammonia) dissociation, x:

$$3\,N_2H_4 \rightarrow 4\,(1-x)NH_3 + (1+2x)N_2 + 6\,x\,H_2$$

For optimal performance (Isp), we desire $x \cong 0.2$. Suppose we ran a test of the thruster at 500 psi and determined that $x \cong 0.3$ for this condition. Can you predict (i.e. calculate) the chamber pressure required to reduce x to the desired value? You may assume that a negligible variation in flame temperature occurs over this narrow range of x values.

439 Practice Final Solns

(1) C_F tables @ $\gamma = 1.2$ $\quad C_{F_v} = 1.902$ @ $\epsilon = 50$

$= 1.977$ @ $\epsilon = 150$

12 i) $I_{sp} = C_{F_v} C^*/g = 1.902 \; 6000/32.2 = 354.4$ sec. 4

$$MR = e^{\Delta V/g I_{sp}} = e^{\frac{10000}{32.2 \; 354.4}} = 2.402$$

$$M_p = M_{PL}\left[\frac{MR - 1}{\frac{-MR+1}{\delta} + MR}\right] = \cancel{3277.5} \; 2158 \text{ lb} \qquad 4$$

$$M_{IN} = M_p\left(\frac{1}{\delta} - 1\right) = \cancel{655.5} \; 539.6 \text{ lb} \qquad 4$$

13 ii) Bigger nozzle $\quad I_{sp_{new}} = 1.977 \; 6000/32.2 = 368.4$ sec

New weights $\quad M_0 = M_{PL} + M_p + M_{IN} + 100 = \cancel{4233.0} \; 3797.6$

$M_f = M_0 - M_p = \cancel{1099.4} \; 1639.6$

$MR_{new} = 2.316$

10

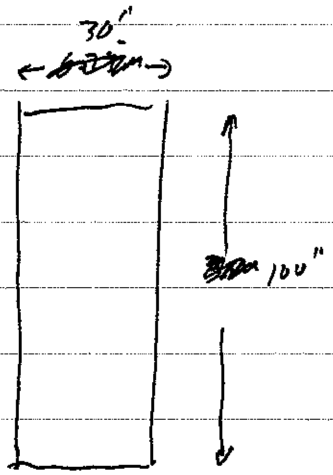
new $\Delta V = g I_{sp_{new}} \ln MR_{new} = 32.2 \; 368.4 \; \ln 2.316 = 9963$ f/s

Don't Put on Skirt! 3

TOPS FORM

② Alum 6061 alloy $F_{tu} = 42$ ksi $F_{ty} = 36$ ksi $\rho = 0.098$ lb/in^3 $E = 10 \times 10^6$ psi

25 pts



$P_{t\,max} = 120$ psi $\qquad M_{PL} = 1000$ lb

$P_a = 70$ psi

~~$M_{PL} = 100$ kg~~

$n_{max} = 20$ g's

i) $\sigma_w = \min\left(\frac{36}{1.1}, \frac{42}{1.25}\right) = \min(32.7, 33.6) = 32.7$ ksi

7 $\quad t = \frac{P_{mx} R}{\sigma_w} = \frac{120 \;15}{32{,}700} = 0.055''$

Buckling Check

$\sigma_{cr} = -10 \times 10^6 \left[9 \left(\frac{0.055}{15}\right)^{1.6} + 0.16 \left(\frac{0.055}{100}\right)^{1.3} \right]$

$= -11{,}500$ psi

10

$\sigma = \frac{\pi R^2 (P_u - P_a) - F_{ax}}{2\pi R t} = \frac{\pi\, 15^2 (60-15) - 1000(20)}{30\,\pi\; 0.055} = \frac{-9397}{5.18} = -1810$ psi

$\sigma > \sigma_{cr}$ - Tank won't Buckle

ii) $S = 30\pi\; 100 + 2\pi\; 15^2 = 10{,}840$ in^2

8

$W = \rho\, S\, t = 0.098 \;\; 10{,}840 \;\; 0.055 = 58$ lb

TOPS FORM
7523

25 pts 2. $NH_3 \rightleftharpoons \frac{1}{2} N_2 + \frac{3}{2} H_2$ Dissoc. Rxn

$$K_p = \frac{\sqrt{X_{N_2}}\ X_{H_2}^{3/2}}{X_{NH_3}} \underbrace{P^1}_{Atm}$$

$X_1 = 0.2$ $\quad 3 N_2H_4 \rightarrow 3.2\ NH_3 + 1.4\ N_2 + 1.2\ H_2 \quad n_{tot} = 5.8$

$X_{NH_3} = 0.552 \quad X_{N_2} = 0.241 \quad X_{H_2} = 0.207$

8 $$K_{p_{0.2}} = \frac{\sqrt{0.241}\ 0.207^{1.5}}{0.552} \left(\frac{P_c}{P_a}\right) = 0.0838 \left(P_c/P_a\right)$$

$X_2 = 0.3$ $\quad 3 N_2H_4 \rightarrow 2.8\ NH_3 + 1.6\ N_2 + 1.8\ H_2 \quad n_{tot} = 6.2$

$X_{NH_3} = 0.452 \quad X_{N_2} = 0.258 \quad X_{H_2} = 0.29$

8 $$K_{p_{0.3}} = \frac{\sqrt{0.258}\ .29^{1.5}}{0.452} \left(\frac{P_{c\,old}}{P_a}\right) = 0.175 \frac{P_{c\,old}}{P_a}$$

now, since $K_p = K_p(T)$ alone and $T \equiv$ const we can set $K_{p_{0.2}} = K_{p_{0.3}}$

$$0.0838\ P_c = 0.175\ P_{c_{old}} = 0.175\ (500)$$

9 $$\boxed{\therefore\ P_c = 1044\ psi}$$

(3) i) want $A_{b_i} = A_{b_f}$ for neutrality

25 pts

$$A_{b_i} = \pi(D/5)L + 2\pi\left(D^2/4 - (D/5)^2\frac{1}{4}\right)$$

$\cancel{A_{b_f} =}$

$$A_{b_f} = \pi D(L - 4D/5)$$

so

10 $$\pi\left(\frac{L}{D}\right)\frac{D^2}{5} + \frac{\pi}{2}D^2\left[1 - 1/25\right] = \pi D^2\left(\frac{L}{D} - \frac{4}{5}\right)$$

$$\therefore L/D = \frac{5}{4}\left[\frac{4}{5} + \frac{12}{25}\right] = 1 + \frac{3}{5} = \frac{8}{5}$$

(ii) $$\bar{V}_p = \frac{\pi}{4}\left(D^2 - (D/5)^2\right)L = \frac{\pi}{2}D^3\frac{24}{25}L/D = \pi\frac{\overset{48}{\cancel{96}}}{125}D^3$$

15

Know we want $\rho_p \bar{V}_p = M_p = 0.5$ Kg

$$\therefore 1700\,\pi\,\frac{\overset{48}{\cancel{96}}}{125}D^3 = 0.5 \qquad \therefore D = \sqrt[3]{1.219\times10^{-4}} = 0.0496 \text{ m}$$

$\boxed{D = \cancel{4.96}\ 6.2 \text{ cm}}$ $\cancel{4.6}$ 5.8 $(L/D = 2)$

$$L = \frac{8}{5}D = 0.079 \text{ m} \quad (\cancel{7.9}\ 10 \text{ cm}) \quad 11.6\ (L/D = 2)$$

$$\dot{m}_{in} = r\rho_p A_{b_i} = 0.01\frac{m}{s}\,1700\frac{Kg}{m^3}\,\pi(\cancel{0.0496}\ 0.062)^2\left(\frac{8}{5} - \frac{4}{5}\right)m^2$$

or final $= \cancel{0.105}$ Kg/s 0.164

$$\dot{m}_{out} = \dot{m}_{in} = \frac{P_c A_t}{c^*} \quad \therefore A_t = \frac{c^*\dot{m}_{in}}{P_c} = \frac{1560\frac{m}{s}\ \cancel{0.105}\ 0.164\ Kg/s}{3\times10^6\ N/m^2}$$

$$\therefore A_t = \cancel{5.25}\ 9.56\times10^{-5}\ m^2 \qquad D_t = \frac{4}{\pi}\sqrt{A_t} = \cancel{9.22}\ 1.1\times10^{-2}\ m$$

$$= \cancel{0.92}\ 1.1 \text{ cm}$$

Rocket Launch Project Grading Form

As I had mentioned in class, your grade for the project will be determined based on the reviews of fellow team members working the project. On this form, please evaluate all other members on your team **except yourself**. Please consider the following items in assessing your grade:

- i) Did he/she attend all the meetings/functions required by the team?
- ii) Did he/she provide inputs in a timely fashion?
- iii) Did he/she provide high-quality information for use by other team members?
- iv) Did he/she provide adequate leadership/organization of team activities? (team leader assessment only)
- v) Did he/she come up with innovative approaches to solving the problems posed?

Grade each member on a 100 point scale. As a general guideline, only Isaac Newton (or an amazing facsimile thereof) should receive 100 points. Students doing an average job might range from 60-70 points; gross omissions or neglect of duties would correspond to scores below 50 points. Above average and exceptional work is left for the remaining range from 70-99 points (remember, 99 is very Newton-like).

Team Member Name________________________________ Score______

Team Member Name________________________________ Score______

Team Member Name________________________________ Score______

Team Member Name________________________________ Score______