AAE439 Test #1 Solns
Fa 2011

1. ~~B~~ 20pts $\epsilon = 40 \quad \gamma = 1.4 \quad C_{fv} = 1.721$

Target $I_{sp} = \frac{F}{\dot{m}_p} = \frac{10,000}{100} = \sout{60.7}$ ~~100~~ 67 sec.

Required $c^* = \frac{g I_{sp}}{C_{fv}} = \sout{1248}$ ~~1860~~ 1248 f/s

$$c^* = \sqrt{\frac{R_u T_c}{\gamma m}} \left(\frac{2}{\gamma+1}\right)^{-\frac{\gamma+1}{2(\gamma-1)}}$$

$$= \left[\frac{1545 \frac{f}{0R}\ 32.2 \frac{f}{s^2}\ T_c}{28 \quad 1.4}\right]^{1/2} \left(\frac{2}{2.4}\right)^{-\frac{1.2}{0.4}}$$

~~1860~~ ~~1248~~ 1248 $= 1.728 \left[1269 \; T_c\right]^{1/2}$ $\quad T_c = \sout{910}$ 410 R ~~(450°F)~~ (−50°F)

40 pts 2.

$$2CO_2 + N_2 \rightarrow a\,CO_2 + 0.5\,CO + b\,O_2 + N_2$$

Mass Balances

Carbon $2 = a + 0.5 \Rightarrow a = 1.5$

Oxygen $4 = 2a + 0.5 + 2b \qquad b = \frac{4-3.5}{2} = 0.25$ 8 10

Mole Fractions

$n_t = a + 0.5 + b + 1 = 3.25$

$x_{CO_2} = 1.5/3.25 \qquad x_{CO} = 0.5/3.25 \qquad x_{O_2} = 0.25/3.25 \qquad x_{N_2} = \frac{1}{3.25}$

Dissoc Rxn

$$CO_2 \rightleftharpoons CO + \tfrac{1}{2} O_2$$

$$K_p = \frac{x_{CO}\sqrt{x_{O_2}}}{x_{CO_2}}\sqrt{P} = \frac{0.5\sqrt{0.25}}{\sqrt{3.25}\;1.5}\sqrt{10} = 0.292$$ 10

From Table 3.2

$K_p = K_{p_9}/K_{p_{10}} = 0.292$

@ ~~3500~~ 2750 K $\quad K_p = \frac{~~1.184~~\ 0.45}{~~2.811~~\ 3.2} = ~~0.15~~\ 0.14$ interpolating 5

@ ~~3750~~ 3000 K $\quad K_p = \frac{~~9.15~~\ 2.4}{~~3.120~~\ 2.7} = ~~2.92~~\ 0.34$ $\boxed{T = ~~3512~~\ K = 3000}$

Mass Fractions

$m_t = 1.5(44) + 0.5(28) + 0.25\ 32 + 28 = 116$

$y_{CO_2} = m_{CO_2}/m_t = 0.57 \qquad y_{CO} = \frac{14}{116} = 0.12$ 5

$y_{O_2} = 8/116 = 0.07 \qquad y_{N_2} = 28/116 = 0.24$

(ii) Since $T = $ const $K_p = $ const $= \frac{\sqrt{x_{O_2}}\,x_{CO}}{x_{CO_2}}\sqrt{P}$

since P decreases dissoc. increases ~~so~~ CO, O_2 increase and CO_2 decreases. N_2 unaffected.

(i) $2CO_2 + N_2 \rightarrow CO_2 + CO + \frac{1}{2} O_2 + N_2$

$n_t = 3.5$

10

K_p = Same as before $= 0.292 = \frac{1 \sqrt{0.5}}{\sqrt{3.5}\ 1} \sqrt{P}$

$P = 0.6$ Atm

3. Plug noz is altitude compensating, implying that plume expansion adjusts to local P_a. Best estimate (upper bound) is to assume optimal expansion at all altitudes i.e. $C_F = C_{F_{opt}}$

30 4. i $P_a)_{eff} = P_a + \rho g h = 0.1\times10^6 + 1020\ 9.8\ (5) = 150\ KPa$

5 $\frac{P_e}{P_c} = \frac{P_a)_{eff}}{P_c} = 0.03 \xrightarrow{C_f\ Table}_{\gamma=1.2} \epsilon \cong 5 \quad C_{F_{opt}} = 1.48$

5 $I_{sp} = \frac{C_f C^*}{g} = 241\ sec$

5 $\dot{m} = \frac{F}{g I_{sp}} = \frac{4000}{9.8\ 241} = 1.69\ Kg/s$

(ii) $A_t = \frac{\dot{m} C^*}{P_c} = 5.4\times10^{-4}\ m^2 = 5.4\ Cm^2$

8 $D_t = \sqrt{\frac{4A_t}{\pi}} = 2.62\ cm \qquad D_e = \sqrt{\epsilon}\ D_t = 5.86\ cm$

(iii) To increase De must decrease P_c - unfortunately ϵ also decreases in process

7 Guess $\epsilon = 3 \qquad P_c = P_e / 0.0656 = 2.29\ MPa \qquad C_{F_{opt}} = 1.36$

Then

$I_{sp} = 222\ sec \qquad \dot{m} = 1.84\ Kg/s \qquad A_t = 12.8\ cm^2$

$D_t = 4.0\ cm \qquad D_e = 7\ cm \qquad$ Close - need lower ϵ, P_c

& continue ...