AAE 439 Test #1 Soln
Fall, 2018

30 pts 1) @@ $\epsilon_1 = \left(\frac{16}{4}\right)^2 = 16$ $\epsilon_2 = \left(\frac{24}{4}\right)^2 = 36$

10 i) @ ϵ_1 $C_{f,v} = 1.797$ $C_f = C_{fv} - \epsilon_1 P_a/P_c$

$C_{fse} = 1.797 - 16\ 14.7/1200 = 1.6$

$F_{se} = C_{fse} P_c A_t = 1.6\ 1200\ 4\pi = 24,140$ lbf

$I_{sp_{se}} = C_{fse} C^*/g = 1.6\ 5000/32.2 = 248$ sec

10 ii) $C_{fv} = C_{fv}$ @ $\epsilon = 36 = 1.875$

$F_v = C_{fv} P_c A_t = 28,300$ lbf

$I_{sp_v} = C_{fv} C^*/g = 291$ sec

10 iii) Deploy skirt when $F_1 = F_2 \Rightarrow C_{f_1} P_c A_t = C_{f_2} P_c A_t$

$C_{f_1} = C_{f_{1v}} - \epsilon_1 P_a/P_c = C_{f_{2v}} - \epsilon_2 P_a/P_c$

$1.797 - 16 P_a/1200 = 1.875 - 36 P_a/1200$

$20 P_a = 1200\ (1.875 - 1.797)$ $P_a = 4.68$ psi

From Alt. Table @ $P_a/P_{sl} = 0.318$ $h \cong 29$ kft

AOE 439 Test #1 Solns
Fall 2019

2. 2 pts

$$V_c = \sqrt{\mu/r} = \left[3.986\times10^5/(6378+150)\right]^{1/2} = 7.8\ km/s$$

$$V_{surf} = 2\pi r_e \cos\theta/(1\ day) = 2\pi(6378)\cos 28/(24(3600))$$
$$= 0.411\ km/s$$

10 c) $\Delta V = V_c - V_{surf} - V_{spin\,launch} = 7.8 - 0.4 - 2.2 = 5.2\ km/s$

$$m_R = e^{\Delta V/gI_{sp}} = e^{5200/9.8(300)} = 5.86$$

8 ii) $$m_p = m_{pl}\,\frac{m_R - 1}{(m_R - (m_R-1))\lambda} = 100\,\frac{4.86}{5.86 - 4.86/0.9}$$
$$= 1056\ kg$$

$$m_i = m_p/\lambda - m_p = 118\ kg$$

7 iii) $m_p \to \infty$ when $m_R - (m_R - 1)/\lambda = 0$

or $\lambda = \frac{m_R - 1}{m_R} = \frac{4.86}{5.86} = 0.83$

$$m_0 = m_{pl} + m_p + m_i = 1274\ kg$$

30 pts 3. $m_p = \int_0^{t_b} \frac{g P_c A_t}{c^*}$ $\quad c^* = \frac{g A_t}{m_p} \int_0^{t_b} P_c(t)\, dt$

$$\int_0^{t_b} P_c\, dt = 600t + \frac{633}{2}t^2 - \frac{1.22}{3}t^3 \Big|_0^{60}$$

$$= 62{,}100 \ \frac{lb\text{-}s}{in^2}$$

Thus $c^* = \frac{32.2 \cdot \pi \cdot 1.5^2}{2900} \cdot 62{,}100 = 4873$ ft/s

i) $\eta_{c^*} = c^*/c^*_{th} = 4873/5000 = 97.5\%$

10 pts i) $m_p = \frac{32.2 \cdot \pi \cdot 1.5^2 \cdot 62{,}000}{5000} = 2830$ lbf

10 pts (c) $I = \int_0^{60} F(t)\, dt = m_p I_{sp}$

$$\int_0^{60} F\, dt = 6600t + 649\frac{t^2}{2} - 13.4\frac{t^3}{3} \Big|_0^{60}$$

$$= 684{,}000 \ lb_f\text{-sec}$$

Thus

$$I_{sp} = 241.7 \text{ sec}$$

10 pts iii) $F_v = F + P_a A_e \Rightarrow I_v = \int_0^{60} [F(t) + P_a A_e]\, dt$

$$= I + P_a A_e t_b$$

$$= 777{,}500 \ lb_f\text{-s}$$

Thus

$$I_{sp_v} = I_v / m_p = 274.7$$

2.) $V_c = \sqrt{\mu/r} = [3.986\times10^5/(6378+150)]^{1/2}$

$= 7.8$ km/s

$V_{surf} = (2\pi r_e \cos\theta)/1\,day = 2\pi(6378)\cos 28/[24(3600)]$

$= 0.41$ km/s

$= V_{spin}$

10 i) $\Delta V = V_c - V_{surf} = 7.4$ km/s $- 2.2 = 5.2$ km/s

$MR = e^{\Delta V/g_0 I_{sp}} = e^{7400/(9.8\,350)} = 8.45$

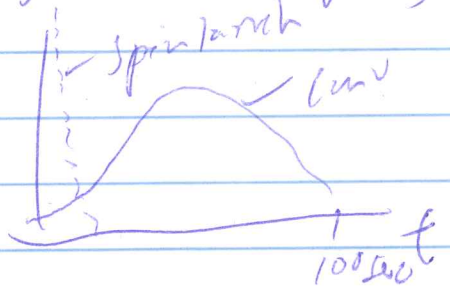
$M_p = M_{fa}\,\frac{MR}{MR-(MR-1)(\lambda)} = 100\,\frac{8.45}{8.45-7.45}$

$M_p = 5100$ kg $M_i = 5647$ kg

10 ii) $G_{low} = M_{pl} + M_p + M_i = 5770$ kg

Q. 5 iii) g-t losses much smaller than conventional system

10 iv) Drag loss is complex, but likely less



Huge accel placed on payload - leads to overdesign