CE340 Test 2 Solution Date: 31 Oct. 2017

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Write your name on each sheet of paper that you hand in.

Read all questions very carefully. If the problem statement is not clear, you should ask the proctors present to clarify if possible the question.

Pay particular attention to consistent and appropriate units. Final answers with no or incorrect units will be penalized.

Please attempt to do all work in a neat and organized manner; marks may be deducted otherwise. Draw and label all diagrams neatly if they are to be considered in the grading. Any force or moment balance should be accompanied by a free-body diagram, and the coordinate system used should be clearly indicated. Circle all main results including the final result, which you wish to be considered in the grading. If you write on the back of a sheet, or on additional paper, please note on your first sheet that you have done work on additional sheets and where the additional work is to be found.

For a multi-part problem, in which the answer to a subsequent part depends on the the answer to a preceding part, and you are not able to find a complete solution to the earlier part, but are able to solve the subsequent part, it is recommended that you assume a reasonable value for the answer to the preceding part, and continue solving the subsequent part.

ALL RELEVANT WORK, INCLUDING CLEARLY LABELLED DIAGRAMS, SHOULD BE SHOWN. MARKS MAY BE DEDUCTED IF THE GRADER IS NOT ABLE TO UNDERSTAND HOW AN ANSWER WAS OBTAINED.

Unless otherwise specified, the fluid is water at room temperature (20°C or 70°F) and at standard atmospheric pressure.

There are four (4) problems (one problem of which is multiple choice with three subquestions) in total. It is highly recommended that all problems be attempted, so budget your time accordingly.

Formulae that you may or may not find useful

$$E_v = -\forall \left(\frac{\partial p}{\partial \forall}\right)\bigg|_{T_0}, \qquad h = \frac{2\sigma\cos\theta}{\gamma r} \qquad p = \rho R T \qquad y_{cp} - \overline{y} = \frac{I_{xc}/A}{\overline{y}}, \qquad \frac{dy}{dx} = \frac{v}{u}$$

$$-g\frac{\partial}{\partial s}\left(\frac{p}{\gamma} + z\right) = a_s, \qquad \mathbf{a} = \left(\frac{\partial V_s}{\partial t} + V_s\frac{\partial V_s}{\partial s}\right)\mathbf{e}_s + \left(\frac{\partial V_n}{\partial t} + \frac{V_s^2}{\mathcal{R}}\right)\mathbf{e}_n, \qquad \mathbf{a} = \frac{\partial \mathbf{u}}{\partial t} + u\frac{\partial \mathbf{u}}{\partial x} + v\frac{\partial \mathbf{u}}{\partial y} + w\frac{\partial \mathbf{u}}{\partial z}$$

for orifices and Venturi meters, $$Q = C_d\left(\frac{\pi d^2}{4}\right)\sqrt{\frac{2g\Delta\mathcal{P}_h}{1-(d/D)^4}}$$

for rectangular sharp-crested weirs, $$Q = C_{d,rw}\left[\frac{2}{3}L_w\sqrt{2gH^3}\right], \qquad C_{d,rw} = 0.6 + 0.08(H/P)$$

for triangular sharp-crested weirs, $$Q = C_{d,trw}\left[\frac{8}{15}\tan\left(\frac{\theta}{2}\right)\sqrt{2gH^5}\right]$$

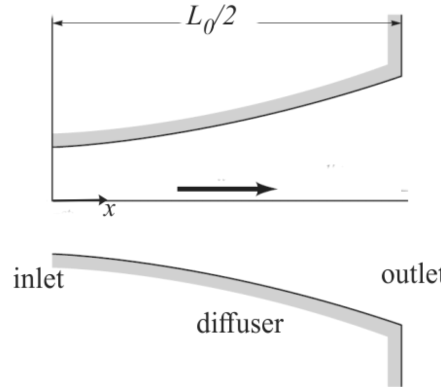
$$\sum \mathbf{F} = \int_{\text{cs}} \rho\mathbf{u}(\mathbf{u}\bullet d\mathbf{A}) = \sum(\beta\dot{m}\mathbf{V})_{\text{out}} - \sum(\beta\dot{m}\mathbf{V})_{\text{in}}$$

$$\text{Volume of an entire sphere} = \frac{4\pi r^3}{3} = \frac{\pi D^3}{6}, \qquad g = 32.2\,\text{ft/s}^2 = 9.8\,\text{m/s}^2$$

1. (18 points) The following are multiple-choice questions with possibly multiple correct answers. **Circle clearly** all responses that you consider correct; if there is any ambiguity in the responses, the responses graded will be decided by the grader. Note that incorrect answers will be penalized but the minimum score for each question is zero.

(a) Consider a flow through the diffuser shown due to a valve being opened, starting at time $t = 0$. The flow is defined by a velocity, $\mathbf{u} = u\mathbf{i}$, where $\mathbf{i}$ is the unit vector in the streamwise (x-) direction, and the magnitude u is specified as

$$u = u_0 \left(1 - e^{-t/T}\right) \left(1 - \frac{x}{L_0}\right).$$



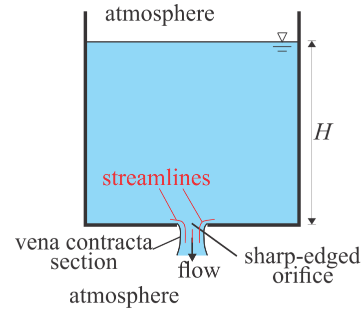
Here u_0 is the ultimate speed at the diffuser inlet (where $x = 0$), T is a time constant, characterizing how fast the speed is changing over time, and L_0 is twice the length of the diffuser (so that the speed at the diffuser outlet is one half of its value at the inlet at any time instant, t).

i. As defined, the flow is unsteady and uniform.
ii. For $t \to \infty$, the flow becomes steady and non-uniform.
iii. The local acceleration is zero for all time $t > 0$.
iv. The convective acceleration is $u_0[1 - (x/L_0)]$.
v. The total acceleration is $u_0(e^{-t/T}/T)[1 - (x/L_0)]$.
vi. For $t \to \infty$, the total acceleration must be positive.

Solution: As defined, the velocity varies spatially in the streamwise (x-)direction, and so is non-uniform, and also varies in time (t) and so is unsteady. Thus, response i) is incorrect. As $t \to \infty$, the exponential term goes to zero, and the flow therefore becomes steady (no dependence on t), but remains non-uniform as the flow still varies in x. Thus, response ii) is correct. Because the flow is unsteady, the local acceleration is not zero for all t. The convective acceleration can be found from $u(\partial u/\partial x)$, but this must necessarily be proportional to u_0^2, so response iv) is incorrect. The total acceleration is the sum of both the local acceleration and the convective acceleration, but as noted in the previous sentence, the convective acceleration would have a term involving u_0^2, and so response v) is incorrect. For $t \to \infty$, the flow is steady, and the total acceleration is entirely the convective acceleration, but it should be clear that as the diffuser expands the velocity decreases as x increases, implying that the convective acceleration must be negative. Thus, response vi) is also incorrect.

In summary, only response ii) is correct.

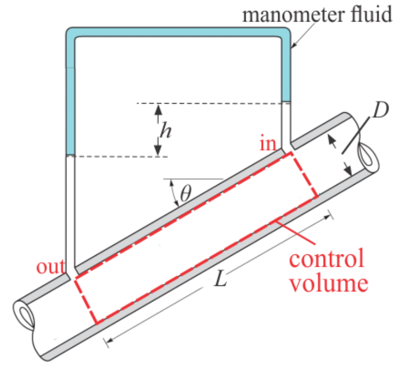
(b) A steady flow of an ideal fluid discharges through a sharp-edged circular orifice at the bottom of a large tank into the atmosphere as shown.



i. The vena contracta section is located at the orifice section.
ii. Streamlines are straight and parallel at the orifice section.
iii. Hydrostatic conditions prevail at the vena contracta section.
iv. The contraction coefficient is the ratio of the diameter of the vena contracta section to the orifice diameter.
v. The contraction coefficient is the ratio of the area of the vena contracta section to the area of the orifice section.
vi. The pressure at the vena contracta section is constant and is equal to γH, where H is the depth of fluid in the tank and γ is the fluid specific weight.

Solution: The vena contracta section is located just downstream of the orifice section (see figure), so that response i) is incorrect. Streamlines are straight and parallel at the vena contracta section (see figure) where the flow cross-sectional area is the narrowest, but the streamlines at the orifice section are highly curved and non-parallel, so response ii) is incorrect. Because streamlines are straight and parallel at the vena contracta section, hydrostatic condition prevails in the direction normal to the streamlines, and so response iii) is correct. The contraction coefficient, C_c, is introduced in order to account for the difference in the area, A_{vc}, of the vena contracta section and the area, A_{or}, of the orifice section, so that the contraction coefficient, $C_c = A_{vc}/A_{or}$, and response iv) is incorrect while response v) is correct. The pressure at the vena contracta section is constant, but is zero (due to the surrounding atmospheric pressure and the hydrostatic condition in the horizontal), so response vi) is incorrect.

In summary, responses iii) and v) are correct.



(c) A fluid of specific gravity, s, flows steadily and uniformly in a constant-diameter pipe inclined at an angle of θ to the horizontal as shown. A differential manometer is installed with taps separated by a distance L. The manometer fluid has a specific gravity of $s_m < s$ and the manometer deflection as shown is h.

i. For an ideal fluid, the manometer deflection would be zero and the fluid in the pipe would be in hydrostatic equilibrium.

ii. For a real fluid, i.e., including frictional losses, given the direction of the manometer deflection as shown, the flow is upwards.

iii. For a real fluid, including frictional losses, given the direction of the manometer deflection, the flow is downward.

iv. For a real fluid, the head loss, h_L, due to pipe friction incurred between the manometer taps is the same as the manometer deflection, i.e., $h_L = h$.

v. For a real fluid, the head loss, h_L, due to pipe friction incurred between the manometer taps, is given by $h_L = [(s_m/s) - 1]h$.

vi. For a real fluid, the head loss, h_L, due to pipe friction incurred between the manometer taps, is given by $h_L = hs_m/s$.

Solution: The energy equation is applied to the control volume (see figure) with inlet and outlet control surfaces located where the legs of the manometer are located (so where information is available):

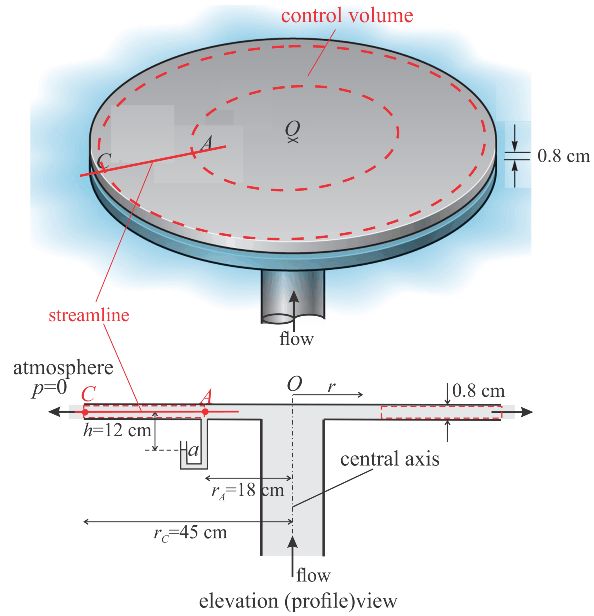
$$\left(\frac{p}{\gamma} + z + \frac{\alpha V^2}{2g}\right)_{in} = \left(\frac{p}{\gamma} + z + \frac{\alpha V^2}{2g}\right)_{out} + h_L \Longrightarrow \left(\frac{p}{\gamma} + z\right)_{in} - \left(\frac{p}{\gamma} + z\right)_{out} = \Delta\mathcal{P}_{h,in\to out} = h_L$$

where it has been assumed that the flow is uniform so that $\alpha V^2/2g$ are equal at both inlet and outlet control surfaces. Thus, the head loss, h_L, is equal to the change in piezometric head, $\Delta\mathcal{P}_{h,in\to out}$, which is related to the manometer deflection, h, by $\Delta\mathcal{P}_{h,in\to out} = [1 - (s_m/s_f)]h = h_L$ (this can be obtained from a manometer analysis). Thus, responses iv), v), and vi) are all incorrect.

For an ideal fluid, the head loss is zero, and so h must be zero, so response i) is correct as a zero deflection implies that the piezometric head at the inlet and outlet are equal, which in turn must mean that the inlet and outlet must be in hydrostatic equilibrium. For a real fluid, with frictional losses, the higher piezometric head is at the upper leg (this can be found by a manometer analysis, or more quickly, by the fact that the level is higher in the upper than the lower leg), so response ii) is incorrect and response iii) is correct.

In summary, only responses i) and iii) are correct.

2. (24 points) A fountain is produced by water that flows up a central tube, and eventually, radially outward between two circular (radius, 45 cm) end plates sandwiched together with a gap through which the water flows. The vertical height of the gap is 0.8 cm. At a point A at a radial distance of 18 cm, a piezometer is installed below the lower end plate, and the piezometer level is observed to be 12 cm below the middle of the gap. The water is discharged into the atmosphere. Assuming that water is an ideal fluid and that the flow velocity is constant over any relevant control surface, determine the radial velocity at the exit (at C).



Solution: The Bernoulli equation is applied on the streamline shown between the point A and a point at the exit C (these points are chosen because information is available at A and at C, and information is wanted at C):

$$\left(\frac{p}{\gamma} + z + \frac{V_s^2}{2g}\right)_A = \left(\frac{p}{\gamma} + z + \frac{V_s^2}{2g}\right)_C,$$

which can be simplified because the elevations are the same and $p_C = 0$ as the discharge is into the atmosphere, so that the velocity at C can be solved for as

$$\frac{V_{sC}^2}{2g} = \frac{p_A}{\gamma} + \frac{V_{sA}^2}{2g}.$$

We have two other unknowns (in addition to the asked-for V_{sC}), and so we need two other equations. The first is mass conservation over the control volume shown (a 'donut' region with one cylinderical surface through point A, and the other cylindrical surface through point C – thus, the control volume is chosen with surfaces where information is either known or where information is wanted):

$$\dot{m}_{out} - \dot{m}_{in} = 0 \Longrightarrow Q_{out} = Q_{in} \Longrightarrow (2\pi r_C h)V_{out} = (2\pi r_A h)V_{in}.$$

Assuming that the velocity is the same across any cross-section, then $V_{sA} = V_{in}$ and $V_{sC} = V_{out}$, so that $V_{sA} = (r_C/r_A)V_{sC}$. The second equation concerns p_A, which can be related to the piezometer information, using hydrostatics (the level 0 designates the level in the piezometer open to the atmosphere):

$$p_A - p_0 = -\gamma(z_A - z_0) \rightarrow \frac{p_A}{\gamma} = -(z_A - z_0) = -h = -0.12\,\text{m}.$$

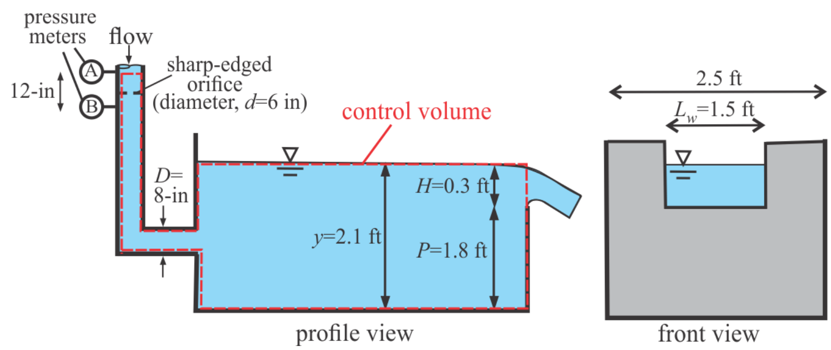
Note the negative sign, as p_A must be lower than p_C (it has a higher velocity), and so must be less than atmospheric.

Both of these results can be substituted into the original equation involving V_{sC}:

$$\frac{V_{sC}^2}{2g} = -h + \frac{](r_C/r_A)V_{sC}]^2}{2g} \Longrightarrow V_{sC} = \sqrt{\frac{2gh}{[(r_C/r_A)^2 - 1]}} = \sqrt{\frac{2(9.8\,\text{m/s}^2)(0.12\,\text{m})}{(45\,\text{cm}/18\,\text{cm})^2 - 1}} = 0.67\,\text{m/s}.$$

3. (28 points) An 8-in pipe supplies water into a channel of rectangular cross-section with a horizontal bottom, and the water eventually flows over a rectangular weir. The weir does not span the entire channel (see figure), but rather has a length 1.5 ft whereas the width of the channel is 2.5 ft. The weir crest is located at an elevation that is 1.8 ft above the channel bottom. Under design conditions, the depth in the channel is 2.1 ft. A sharp-edged orifice (diameter 6-in) is installed in the supply pipe. For the design condition, what is the expected pressure difference ($p_A - p_B$) measured across the orifice meter if the elevation difference between the pressure meters is 12-in? Indicate clearly which meter (A or B) records the higher pressure.

Solution: We evaluate the flow over the weir, which by continuity must be the same as the the flow through the orifice, and so the related asked-for pressure difference can be obtained. The flow over the weir, Q_{weir}, can be found as



$$Q_{weir} = \underbrace{\left[0.6 + 0.08\frac{H}{P}\right]}_{C_{d,rw}} \sqrt{2gH^3} = \left[0.6 + 0.08\left(\frac{0.3\,\text{ft}}{1.8\,\text{ft}}\right)\right]\sqrt{2(32.2\,\text{ft/s}^2)(0.3\,\text{ft})^3} = 0.81\,\text{ft}^3/\text{s},$$

where the head over the weir, $H = 0.3$ ft, not 2.1 ft. Due to mass conservation over a control volume from the section at the pressure meter A to the weir, it is found that $Q_{weir} = Q_{orif} = Q$, so that we apply the equation for a standard orifice,

$$Q = 0.81\,\text{ft}^3/\text{s} = C_{d,o}\left(\frac{\pi d^2}{4}\right)\sqrt{\frac{2g\Delta\mathcal{P}_h}{1-(d/D)^4}}, = 0.6\left[\frac{\pi(0.5\,\text{ft})^2}{4}\right]\sqrt{\frac{2(32.2\,\text{ft/s}^2)\Delta\mathcal{P}_h}{1-(0.5\,\text{ft}/0.67\,\text{ft})^4}}$$

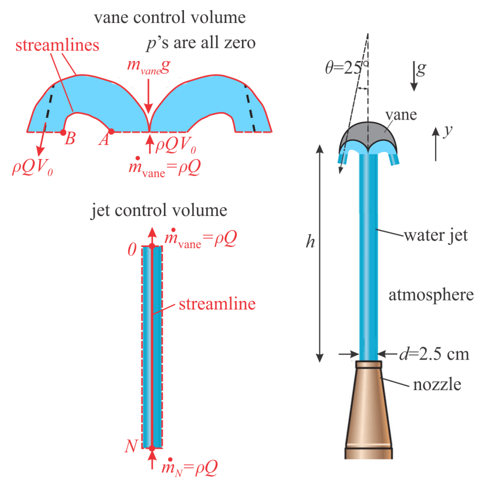
$$\Longrightarrow \Delta\mathcal{P}_h = 0.50\,\text{ft} = \left(\frac{p_a}{\gamma}+z\right)_A - \left(\frac{p_a}{\gamma}+z\right)_B = \left(\frac{p_A - p_B}{\gamma}\right) + z_A - z_B.$$

Here we have chosen $C_{d,o} = 0.6$ as the discharge coefficient appropriate for a sharp-edged orifice (values between 0.5 and 0.7 were accepted as appropriate). This allows the solution for $p_A - p_B$ as

$$p_A - p_B = \gamma(\Delta\mathcal{P}_h - (z_A - z_B)) = 62.4\,\text{lb/ft}^3(0.5\,\text{ft} - 1\,\text{ft}) = -31.2\,\text{lb/ft}^2,$$

with the pressure at B being higher, as our result for the pressure difference is negative.

4. (30 points) A nozzle of diameter 2.5 cm discharges water at a rate of 0.006 m^3/s vertically into atmosphere as a circular jet, which impinges on an axially symmetric vane that deflects the jet in an axially symmetric manner. The flow exiting the vane makes an angle of 25° with the vertical as shown. If the vane has a mass of 12 kg, determine the height (h) above the nozzle at which the vane is suspended by the jet. It may be assumed that the size of the vane is negligible (compared to h) and also that frictional effects are negligible.



Solution: A control volume about the jet, with control surfaces at the nozzle and at the inflow to the vane as shown, is chosen for a mass balance, so that $\dot{m}_{\text{vane}} - \dot{m}_N = 0$ or $\dot{m}_{\text{vane}} = \dot{m}_N = \rho Q$, where $\dot{m}_N = \rho Q = 10^3\,\text{kg/m}^3(0.006\,\text{m}^3/\text{s}) =$ 6 kg/s is the mass flow rate from the nozzle, and $\dot{m}_{\text{vane}}$ is the mass flow rate into the vane region (vane control volume). A Bernoulli equation is also applied along the centerline of the jet from the nozzle (point N) and to a point 0 at the inflow section of the vane:

$$\left(\frac{p}{\gamma} + z + \frac{V_s^2}{2g}\right)_N = \left(\frac{p}{\gamma} + z + \frac{V_s^2}{2g}\right)_0 \Longrightarrow \frac{V_{s0}^2}{2g} = \frac{V_{sN}^2}{2g} - (z_0 - z_N) = \frac{V_{sN}^2}{2g} - h,$$

where h is the height of the jet. While we can find

$$V_{sN} = Q/(\pi d^2/4) = 0.006\,\text{m}^3/\text{s}/[\pi(0.025\,\text{m})^2/4 = 12.22\,\text{m/s},$$

we still need to find V_{s0} in order to solve for h.

A momentum balance is applied to a control volume around the vane alone as shown, not including the jet. Due to the symmetry about the jet axis, only the vertical balance (in y), i.e., in the y-direction (see defined coordinate system), is relevant, and is expressed as

$$\uparrow\!+ \sum F_y = -W_{\text{vane}} = -m_{\text{vane}}g = \dot{m}_{out}V_{out,y} - \dot{m}_{in}V_{in,y} = \dot{m}(-V_0\cos\theta - V_0) = -\dot{m}V_0(1+\cos\theta),$$

where by mass conservation, $\dot{m}_{out} = \dot{m}_{in} = \dot{m}_{\text{vane}} = \rho Q$, while by the application of the Bernoulli equation,e.g., along the streamline shown between points A and B, the magnitude of the outflow velocity, V_{out}, must be equal to the magnitude of the inflow velocity, $V_{in} = V_0 = V_{s0}$. Note the effect of the angle θ and the sign of the outflow momentum, consistent with the chosen coordinate system (see figure). This allows the solution for

$$V_{s0} = \frac{m_{\text{vane}}g}{\rho Q(1+\cos\theta)} = \frac{12\,\text{kg}(9.8\,\text{m/s}^2)}{6\,\text{kg/s}(1+\cos(25°)} = 10.28\,\text{m/s}.$$

A solution for h can then be found as

$$h = \frac{V_{sN}^2 - V_{s0}^2}{2g} = \frac{(12.22\,\text{m/s})^2 - (10.28\,\text{m/s})^2}{2(9.8\,\text{m/s}^2)} = 2.22\,\text{m/s}.$$

A common error was to assume that the inflow (and outflow) velocity, V_0, into the vane region was the same as the nozzle velocity, V_N. While elevation differences within the vane region can be neglected, elevation differences from the nozzle to the vane cannot be neglected, and the velocities therefore are different. A subtle error is made by replacing the correct $\dot{m}V_0$ with $\rho V_0^2(\pi d^2/4)$, in effect assuming that $Q = V_0(\pi d^2/4)$ but $Q = V_N(\pi d^2/4)$ and $V_0 < V_N$, so using $\rho V_0^2(\pi d^2/4)$ violates mass conservation. The area must increase as the velocity decreases with increasing elevation (y).