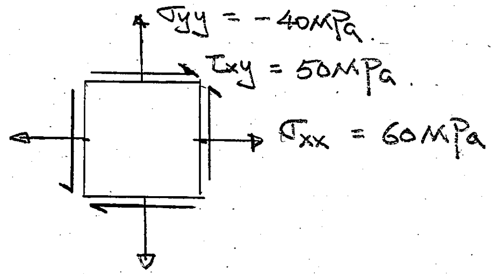
P1

Given :

Material properties : $E = 200\,GPa$ , $\nu = 0.3$ , isotropic material

stresses :




$\sigma_{xx} = 60\ MPa$ , $\sigma_{yy} = -40MPa$ , $\tau_{xy} = 50\ MPa$.

Required : <1>. Calculate strain components ( $\varepsilon_{xx}$, $\varepsilon_{yy}$, and $\gamma_{xy}$ )

<2>. Calculate principal stress ( $\sigma_1$ and $\sigma_2$ ) in MPa using eigenvalue approach.

<3>. Calculate principal direction in degrees using Mohr's circle.

Solutions :

<1>.

The stress vector is: $\sigma = \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} = \begin{Bmatrix} 60 \\ -40 \\ 50 \end{Bmatrix}$

The shear modulus is :

$$G = \frac{E}{2(1+\nu)} = \frac{200{,}000}{2(1+0.3)} = 76923.1\ MPa.$$

The compliance matrix is :

$$[a] = \begin{bmatrix} \frac{1}{E} & -\frac{\nu}{E} & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & 0 \\ 0 & 0 & \frac{1}{G} \end{bmatrix} = \begin{bmatrix} \frac{1}{200{,}000} & -\frac{0.3}{200{,}000} & 0 \\ -\frac{0.3}{200{,}000} & \frac{1}{200{,}000} & 0 \\ 0 & 0 & \frac{1}{76{,}923.1} \end{bmatrix}$$

Therefore, we solved for $\varepsilon_{ij}$ :

$$\boxed{\{\varepsilon\} = [a]\{\sigma\} = \begin{Bmatrix} 3.6\times10^{-4} \\ -2.9\times10^{-4} \\ 6.5\times10^{-4} \end{Bmatrix}}$$

(2). To calculate the principal stress, we need to find the eigenvalues of the stress tensor.

$$\sigma = \begin{bmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{xy} & \sigma_{yy} \end{bmatrix} = \begin{bmatrix} 60 & 50 \\ 50 & -40 \end{bmatrix}.$$

To find eigenvalues, we need to let $\det(\sigma - \lambda I) = 0$

$$\sigma - \lambda I = \begin{bmatrix} 60-\lambda & 50 \\ 50 & -40-\lambda \end{bmatrix}$$

$$\det(\sigma - \lambda I) = 0$$

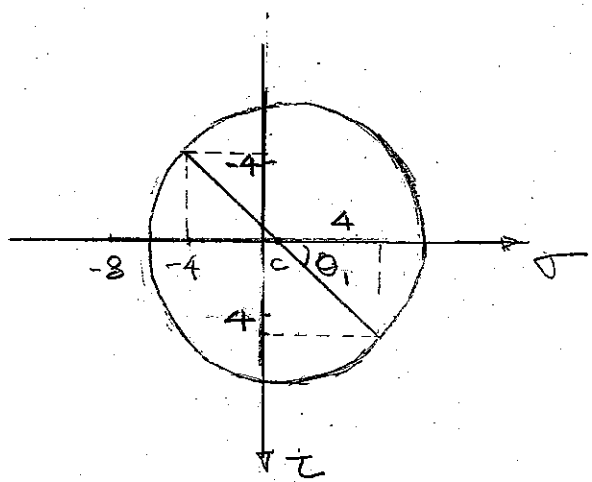
$$\Rightarrow \lambda^2 - 20\lambda - 4900 = 0.$$

$$\Rightarrow \lambda_1 = 80.71 \quad , \lambda_2 = -60.71.$$

Therefore, the principal stresses are:

$$\boxed{\sigma_1 = 80.71\ MPa, \quad \sigma_2 = -60.71\ MPa}$$

(3).



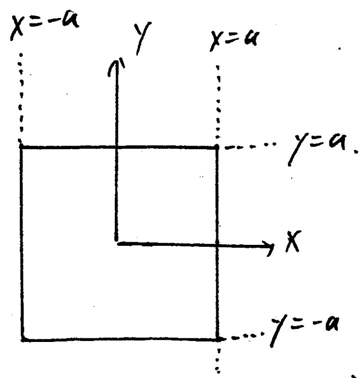

The center of the circle: $r = \frac{60+(-40)}{2} = 10.$

Then, $\tan\theta_1 = \frac{50}{(60-10)} = 1 \Rightarrow \theta_1 = 45^\circ$

$$\boxed{\begin{array}{l} \text{Therefore. } \theta_{1p} = \theta_1/2 = 22.5^\circ \\ \text{Thus, we solved } \theta_p = \theta_{1p} + 90^\circ = 112.5^\circ \end{array}}$$

P2

Given :




$$\sigma_{xx} = -\frac{P(x^2-y^2)}{a^2} \qquad \sigma_{yy} = \frac{P(x^2-y^2)}{a^2} \qquad \tau_{xy} = \frac{2Pxy}{a^2}.$$

Required:

(1). Traction $\{t\}$ on the positive $y$ face (i.e, $y=a$)

(2). Traction $\{t\}$ on the negative $y$ face (i.e., $y=-a$).

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Solution:

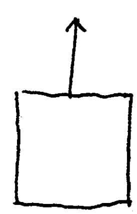
$$\{t\} = [\sigma]\{n\}, \quad \begin{Bmatrix} t_x \\ t_y \end{Bmatrix} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} \\ \tau_{yx} & \sigma_{yy} \end{bmatrix} \begin{Bmatrix} n_x \\ n_y \end{Bmatrix} \text{ where } \tau_{xy} = \tau_{yx}$$

(1). On surface $y=a$:

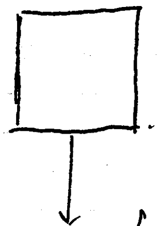
$$\begin{Bmatrix} t_x \\ t_y \end{Bmatrix} = \underbrace{\begin{bmatrix} -\frac{P(x^2-y^2)}{a^2} & \frac{2Pxy}{a^2} \\ \frac{2Pxy}{a^2} & \frac{P(x^2-y^2)}{a^2} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{} = \begin{Bmatrix} \frac{2Pxy}{a^2} \\ \frac{P(x^2-y^2)}{a^2} \end{Bmatrix} \overset{y=a}{=} \begin{Bmatrix} \frac{2Px}{a} \\ \frac{P(x^2-a^2)}{a^2} \end{Bmatrix}$$

from

$$n = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix}.$$



(2). On Surface $y=-a$.



$$n=\begin{Bmatrix}0\\-1\end{Bmatrix}.$$

$$\begin{Bmatrix}t_x\\t_y\end{Bmatrix}=\begin{bmatrix}-\frac{p(x^2-y^2)}{a^2} & \frac{2pxy}{a^2}\\ \frac{2pxy}{a^2} & \frac{p(x^2-y^2)}{a^2}\end{bmatrix}\begin{Bmatrix}0\\-1\end{Bmatrix}=\begin{Bmatrix}-\frac{2pxy}{a^2}\\-\frac{p(x^2-y^2)}{a^2}\end{Bmatrix}\overset{y=-a}{=}\begin{Bmatrix}\frac{2px}{a}\\-\frac{p(x^2-a^2)}{a^2}\end{Bmatrix}.$$

## Problem 3

Given) $[\sigma] = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix} = \begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (MPa)

sol'n)

(a) $\{t\} = [\sigma]\{n\}$, where $\{n\} = \begin{bmatrix} \sqrt{2}/2 \\ \sqrt{2}/2 \\ 0 \end{bmatrix}$

$$= \begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \sqrt{2}/2 \\ \sqrt{2}/2 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 2.8284 \\ 2.1213 \\ 0 \end{Bmatrix} \text{ (MPa)}$$

(b) $\theta = 38^\circ$

$$[\beta] = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$[\sigma'] = [\beta][\sigma][\beta^T]$$

$$= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} 3.5913 & -0.2432 \\ -0.2432 & 1.4087 \end{bmatrix} = \begin{bmatrix} \sigma_{xx}' & \tau_{xy}' \\ \tau_{yx}' & \sigma_{yy}' \end{bmatrix} \text{ (MPa)}$$

or we can simply use coordinate transformation formula

$$\begin{Bmatrix} \sigma_{xx}' \\ \sigma_{yy}' \\ \tau_{xy}' \end{Bmatrix} = \begin{bmatrix} \cos^2\theta & \sin^2\theta & \sin 2\theta \\ \sin^2\theta & \cos^2\theta & -\sin 2\theta \\ -\frac{1}{2}\sin 2\theta & \frac{1}{2}\sin 2\theta & \cos 2\theta \end{bmatrix} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix}$$

$$= \begin{Bmatrix} 3.5913 \\ 1.4087 \\ -0.2432 \end{Bmatrix} \text{ (MPa)}$$

$\Rightarrow$ $\sigma_{xx}' = 3.5913$ (MPa) $\sigma_{yy}' = 1.4087$ (MPa) $\tau_{xy}' = -0.2432$ (MPa)