Industrial Engineering 546 Spring 2017
Economic Decisions in Engineering

Exam 1

Student Information (print neatly)

Name: _____

Purdue email: _____

Directions:

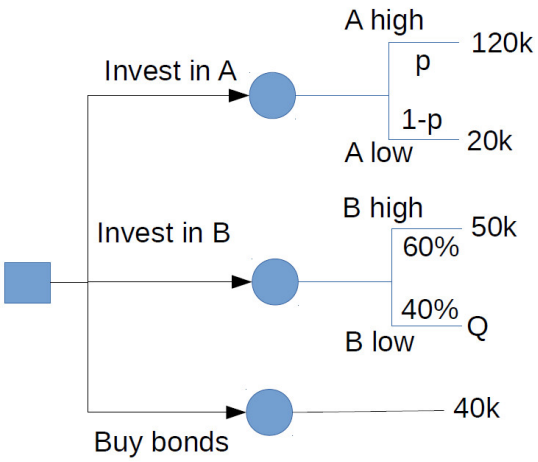
- You have 60 minutes to complete this exam. If you open the test before stated or do not turn it in on time you will lose 20 points.
- This exam is closed book and notes. You will receive a zero for this exam for using books, notes, electronic devices (cell phone, ipad, calculator, laptop, etc.). You will also be reported and appropriate disciplinary action will be taken.
- Your answers must be legible. Circle, underline, or leave sufficient white-space to distinguish your answers from intermediate work.
- Show all your work.

Grade:

1. [30] _____
2. [20] _____
3. [50] _____

Total: _____

Problem 1. [30 points] Consider the following decision tree.



There are two parameters, p and Q, which are not known precisely.

	low	base	high
p	10%	30%	50%
Q	25k	30k	40k

Conduct sensitivity analysis. First, determine the expected earnings as a function of the variables p and Q:

$$\text{E}[\text{ earnings of } A\ ] = 120p + 20(1-p) \qquad = 100p + 20$$

$$\text{E}[\text{ earnings of } B\ ] = 50 * 0.6 + Q * 0.4 \qquad = 30 + 0.4Q$$

$$\text{E}[\text{ earnings of } C\ ] = 40$$

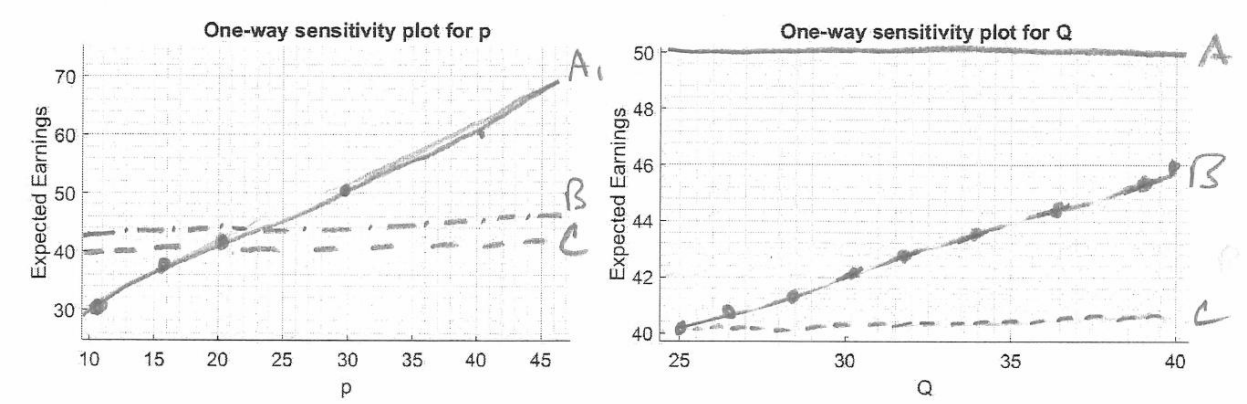
Second, determine the expected earnings using the base-case values for p and Q:

$$\text{E}[\text{ earnings of } A\ ] = 100 * 0.3 + 20 \qquad = 50k$$

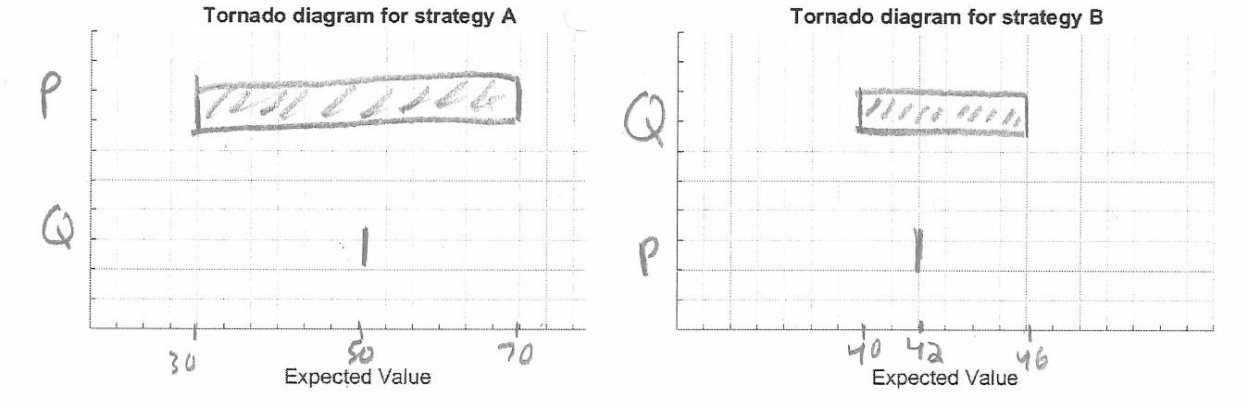
$$\text{E}[\text{ earnings of } B\ ] = 30 + 0.4 * 30 = 30 + 12 \qquad = 42k$$

$$\text{E}[\text{ earnings of } C\ ] = 40k$$

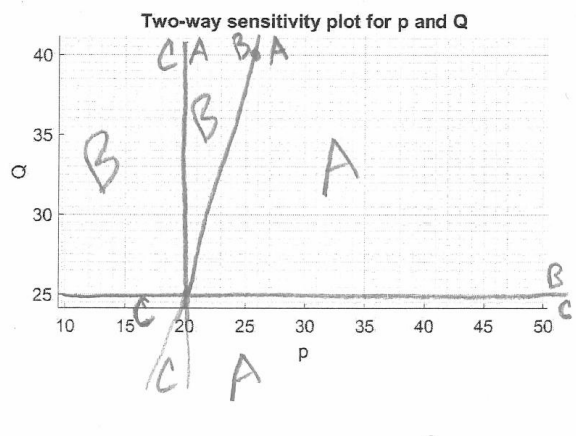
Draw the one-way sensitivity plots for p and Q. Label the curves.



Draw the tornado diagrams for strategies A and B. Label the axes.



Draw the two-way sensitivity plot for p and Q. For each boundary denote which strategy is preferred and for each region denote the preferred strategy.



$E[A] = 100p + 20 = 40 = E[C]$

$\Rightarrow 100p = 20$

$p = 0.2$

$E[B] = 30 + 0.4Q = 40 = E[C]$

$\Rightarrow 0.4Q = 10$

$Q = 25$

$E[A] = 100p + 20 = 30 + 0.4Q = E[B]$

$100p = 10 + 0.4Q$

$Q = 20 \Rightarrow p = .18 \quad Q = 40 \Rightarrow p = .26$

$Q = 25 \Rightarrow p = .20$

Rubric

To the extent errors do not over-simplify the work, each stage should be graded for correctness based on previous stage. For instance, if there's an error in the expectation formula, and the boundary line is correctly drawn for that (wrong) formula, then the only penalty should be for the initial formula.

- Expectations [4 points total]
  - -1 per formula error (with p and Q); they do not need to simplify
  - -0.5 per base-case error (max -1 total)
- One way plots [8 total]
  - -1 per plot label error
  - -1 per line drawn wrong
- Tornado plots [8 total]
  - -1 per plot x-axis label error
  - -1 per plot rectangle label error
  - -1 per rectangle wrong
- Two way plot [10]
  - -2 per boundary line formula error (1 point for setting up, 1 point for deriving)
  - -0.5 per boundary line drawing error
  - -0.5 per boundary line labeling error
  - -1 region labeling error (only need the top three regions B, B, and A)

Problem 2. [20 points] The following questions involve calculating how money changes value over time. You do not need to calculate the final answer. You do need to simplify your answer so the quantity of interest is on the left hand side and all numbers are on the right. For instance, if asked to find a return rate r, your final answer should look like "$r = \ldots$" Recall the formula $FV = PV(1+r)^n$.

A. 10 years ago, you invested \$1000 in a savings account with 5% yearly compounded interest. Inflation has been stable at 2%.

1. What is the current dollar amount of your investment?

$$FV = 1000(1+0.05)^{10}$$

2. What is the current purchasing power in 2007 dollars?

$$FV = 1000\frac{(1+0.05)^{10}}{(1+0.02)^{10}}$$

B. Your friend asks to borrow \$10,000. He says he'll pay you back \$5000 after five years and \$7500 after 10 years. Suppose inflation is 2%. What is the net present value of this deal?

$$NPV = -10000 + \frac{5000}{(1+0.02)^5} + \frac{7500}{(1+0.02)^{10}}$$

C. You have \$1000 to invest. Inflation is 2%.

1. You invest in a savings account with 5% interest. How many years n of compounding will it take for the dollar amount to double?

$$2000 = 1000(1 + 0.05)^n$$

$$2 = (1.05)^n$$

$$\begin{aligned} \log(2) &= \log((1.05)^n) \\ &= n \log(1.05) \end{aligned}$$

$$n = \frac{\log(2)}{\log(1.05)}$$

2. What interest rate r should you invest at so the purchasing power of your money will double after 25 years?

$$2000 = 1000 \frac{(1 + r)^{25}}{(1 + 0.02)^{25}}$$

$$2(1 + 0.02)^{25} = (1 + r)^{25}$$

$$\begin{aligned} \log(2(1 + 0.02)^{25}) &= \log((1 + r)^{25}) \\ &= 25 \log(1 + r) \end{aligned}$$

$$1 + r = exp\left(\frac{\log(2(1 + 0.02)^{25})}{25}\right)$$

$$r = exp\left(\frac{\log(2(1 + 0.02)^{25})}{25}\right) - 1$$

Rubric

A. 1. [2 points]

- -1 for using inflation
- -1 for repeated investment

2. [3 points]

- -1.5 for not using inflation
- -1 for repeated investment

B. [5 points]

- -1 for (correctly) setting it up for repeated payments each year
- -1 for missing the "−$10000"
- -2 for missing inflation or using it incorrectly

C. 1. [5 points]

- -2 for using inflation
- -2 for using rule of 72
- -2 for major mistake in derivation
- -1 for minor mistake in derivation [max -2]
- -1 for doing most of the work but not simplifying to "$n = \ldots$"

2. [5 points]

- -0 for (correctly) using roots instead of logarithms
- -2 for missing inflation or using it incorrectly
- -2 for major mistake in derivation
- -1 for minor mistake in derivation [max -2]
- -2 for using rule of 72
- -1 for doing most of the work but not simplifying to "$r = \ldots$"

Problem 3. [50 points] **Show your work, even if you cannot finish the calculations.**

Using the profit matrix below (rows correspond to strategies $\{A_i\}$ and columns correspond to scenarios $\{S_j\}$), as well as the table of scenario probabilities, determine which strategy is preferred for each of the following criteria (if any):

	S_1	S_2	S_3	S_4
Probability (%):	20	40	30	10

	S_1	S_2	S_3	S_4
A_1	10	50	50	30
A_2	40	30	70	40
A_3	20	20	40	60

A. Expected value

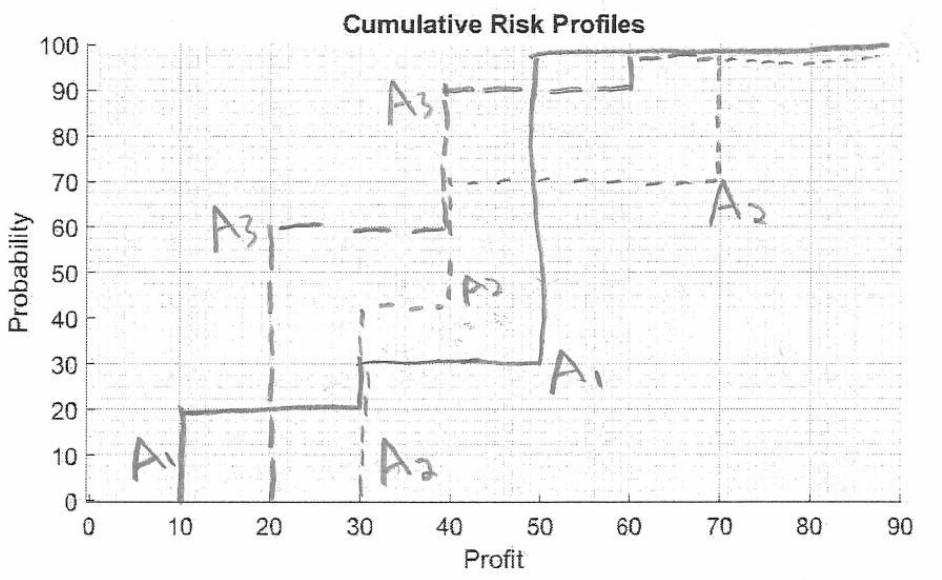
$$\text{E[earnings } A_1] = 0.2 * 10 + 0.4 * 50 + 0.3 * 50 + 0.1 * 30 = 2 + 20 + 15 + 3 \quad = 40$$

$$\text{E[earnings } A_2] = 0.2 * 40 + 0.4 * 30 + 0.3 * 70 + 0.1 * 40 = 8 + 12 + 21 + 4 \quad = 45$$

$$\text{E[earnings } A_3] = 0.2 * 20 + 0.4 * 20 + 0.3 * 40 + 0.1 * 60 = 4 + 8 + 12 + 6 \quad = 30$$

A_2 is preferred.

B. Risk profile dominance (label the curves)



No strategy is dominant (though A_2 stochastically dominates A_3).

	S_1	S_2	S_3	S_4
A_1	10	50	50	30
A_2	40	30	70	40
A_3	20	20	40	60

C. Laplace

$$\text{Value}(A_1) = \frac{1}{4}(10 + 50 + 50 + 30) = 10 + 25 \qquad = 35$$

$$\text{Value}(A_2) = \frac{1}{4}(40 + 30 + 70 + 40) = 10 + 25 + 10 \qquad = 45$$

$$\text{Value}(A_3) = \frac{1}{4}(20 + 20 + 40 + 60) = 5 + 5 + 10 + 15 \qquad = 35$$

A_2 is preferred.

D. Maximax

$$\text{Value}(A_1) = 50$$

$$\text{Value}(A_2) = 70$$

$$\text{Value}(A_3) = 60$$

A_2 is preferred.

E. Maximin

$$\text{Min}(A_1) = 10$$

$$\text{Min}(A_2) = 30$$

$$\text{Min}(A_3) = 20$$

A_2 is preferred.

	S_1	S_2	S_3	S_4
Probability (%):	20	40	30	10

	S_1	S_2	S_3	S_4
A_1	10	50	50	30
A_2	40	30	70	40
A_3	20	20	40	60

	S_1	S_2	S_3	S_4
A_1	30	0	20	30
A_2	0	20	0	20
A_3	20	30	30	0

F. Minimax regret

$$\text{Min regret}(A_1) = 30$$

$$\text{Min regret}(A_2) = 20$$

$$\text{Min regret}(A_3) = 30$$

A_2 is preferred.

G. Minimum expected regret

$$\text{E[regret } A_1] = 0.2 * 30 + 0.4 * 0 + 0.3 * 20 + 0.1 * 30 = 6 + 0 + 6 + 3 \qquad = 15$$

$$\text{E[regret } A_2] = 0.2 * 0 + 0.4 * 20 + 0.3 * 0 + 0.1 * 20 = 0 + 8 + 0 + 2 \qquad = 10$$

$$\text{E[regret } A_3] = 0.2 * 20 + 0.4 * 30 + 0.3 * 30 + 0.1 * 0 = 4 + 12 + 9 + 0 \qquad = 25$$

A_2 is preferred.

Rubric

To the extent errors do not over-simplify the work, each stage should be graded for correctness based on previous stage. For instance, if there's an error in the regret table, the calculations for minimax regret and expected regret will lose points for correctly using that wrong value.

For each question,

- -1 for not specifying dominant strategy (circling is fine)
- -0.5 per arithmetic mistake, max -2 per problem
- -3 for wrong formula in subproblem

A. [5 points]

B. [10 points]

- -1 if not clearly labelled
- -1 per curve for one minor mistake
- -2 per curve for multiple mistakes
- -2 for wrong conclusion (they don't need to specify A_2's stochastic dominance over A_3)

C. [5 points]

D. [5 points]

E. [5 points]

F. [10 points – 5 points for regret table, 5 points for minimax regret]

- -0.5 per mistake in regret table (max -5)

G. [5 points]