PUID: ______________________ Name: ______________________

Industrial Engineering 546 Spring 2019
Economic Decisions in Engineering

Exam 1 – Solutions

Directions:

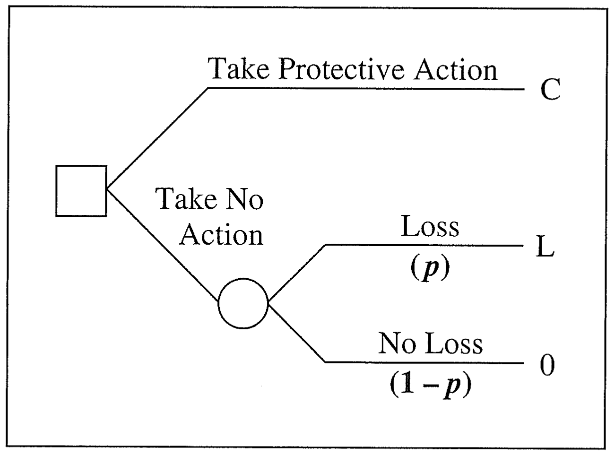
- You have 75 minutes to complete this exam. If you open the test before stated or do not turn it in on time you will automatically lose 20 points.
- You are allowed a single $8\frac{1}{2} \times 11$ inch sheet, with notes front and back. Otherwise, this exam is closed book and notes.
- Your are allowed to use a calculator (graphing or simple). No other electronic devices are permitted.
- Your answers must be legible. Circle, underline, or leave sufficient white-space to distinguish your answers from intermediate work.
- Show all your work.
- Write your name and PUID on each sheet.
- Do not write along the edge of the paper or on the back-side. The final pages are blank and you may use them for scratch work or overflow.

Grade:

1. [35] ______________________
2. [30] ______________________
3. [15] ______________________
4. [5] ______________________
5. [15] ______________________

Total: ______________________

Problem 1. [35 points] A farmer hears of potential bad weather next week. He can pay $\$C$ to take protective action to save his crop. If he does not take protective action, with probability p his crop will be severely damaged and he will incur a loss of $\$L$. The following decision tree models his decision.



	low	base	high
p	10%	20%	50%
C	10k	20k	40k
L	50k	100k	200k

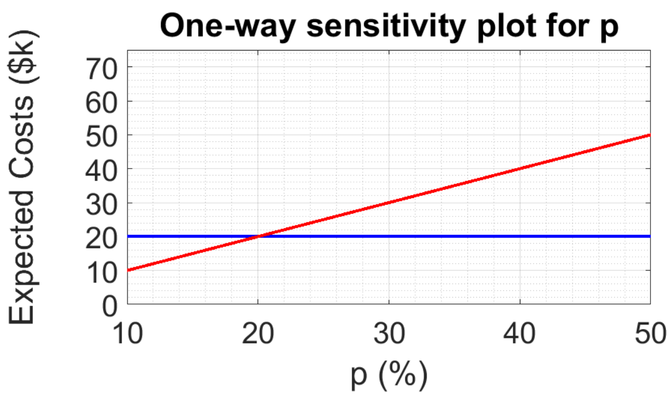
Conduct sensitivity analysis. First determine the expected costs of each alternative using the base-case values:

$$\text{E}\big[\text{ cost of taking protective action }\big] = C = \$20k$$

$$\text{E}\big[\text{ cost of taking no action }\big] = p * L + (1 - p) * 0 = (0.2)(\$100k) = \$20k$$

	low	base	high
p	10%	20%	50%
C	10k	20k	40k
L	50k	100k	200k

Draw the one-way sensitivity plots for p and L. Label the curves.



E[cost act]

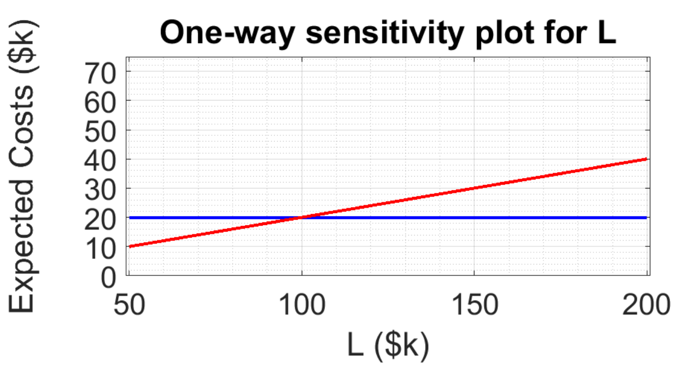
$= C = \$20k$

E[cost no act]

$= p * L + (1 - p) * 0$

$= [0.1 \;\; 0.2 \;\; 0.5](\$100k)$

$= [\$10k \;\; \$20k \;\; \$50k]$



E[cost act]

$= C = \$20k$

E[cost no act]

$= p * L + (1 - p) * 0$

$= (0.2)[\$50k \;\; \$100k \;\; \$200k]$

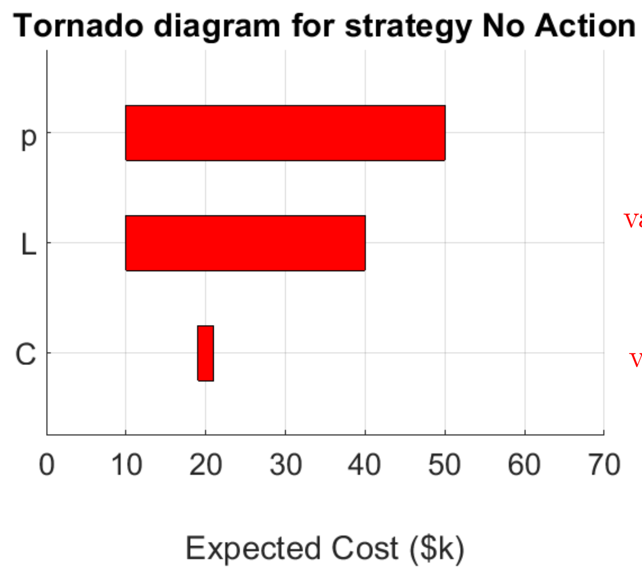
$= [\$10k \;\; \$20k \;\; \$40k]$

What is the advantage of a one-way sensitivity plot compared to a tornado diagram?

One-way sensitivity plots show the objective values of all the strategies together.

	low	base	high
p	10%	20%	50%
C	10k	20k	40k
L	50k	100k	200k

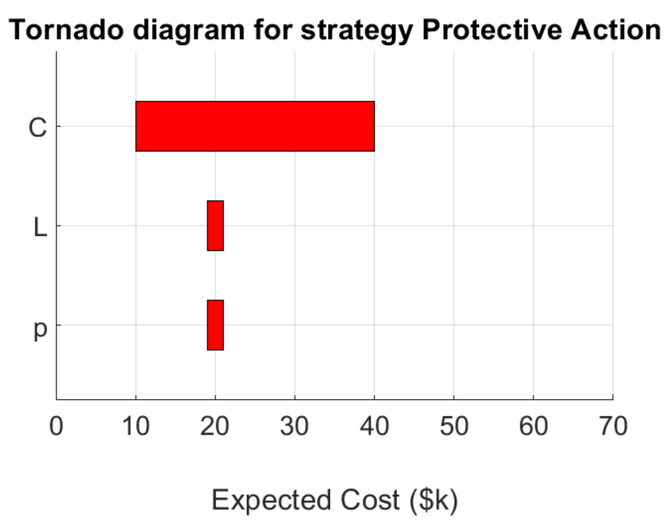
Draw the tornado diagrams for strategies 'no action' and 'protective action.' Label the axes.



$E[\text{ cost no act }] = p * L$

$\text{vary } L = (0.2)[\$50k \;\; \$100k \;\; \$200k]$
$= [\$10k \;\; \$20k \;\; \$40k]$

$\text{vary } p = [0.1 \;\; 0.2 \;\; 0.5](\$100k)$
$= [\$10k \;\; \$20k \;\; \$50k]$



$E[\text{ cost prot act }] = C$

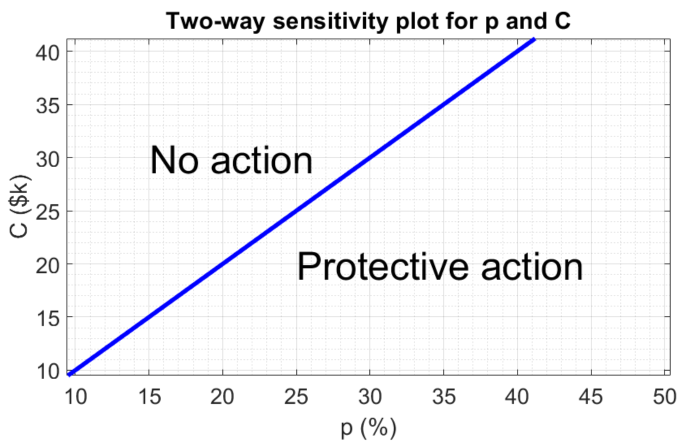
$= [\$10k \;\; \$20k \;\; \$40k]$

What is the advantage of a tornado diagram to a one-way sensitivity plot?

Tornado diagrams show how a strategy's expected value varies due to ranges of all of the parameters.

	low	base	high
p	10%	20%	50%
C	10k	20k	40k
L	50k	100k	200k

Draw the two-way sensitivity plot for p and C. For each region, denote the preferred strategy. (Recall the values are *costs*, so smaller is better.)



The boundary is the line where

$$\text{E[cost no act]} = \text{E[cost prot. act]}$$
$$pL = C$$
$$p(\$100k) = C$$

for instance, $C = \$30k$ means $p = 30\%$. To determine preferred strategy, at $p = 20\%$ and $C = \$40k$, E[cost no act] $= \$20k$ and E[cost prot. act] $= \$40k$, so no action would be preferred.

What is the preferred alternative on the boundary itself?

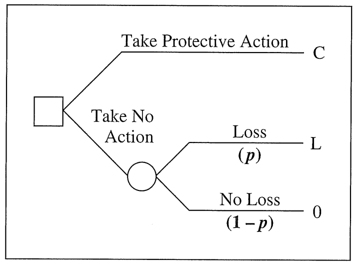
Neither. For any point along the boundary, we are indifferent to the two strategies.

Based on the results of the sensitivity analysis, what should the farmer do next in modeling this decision, if anything?

The decisions are sensitive to each of the parameters. It would be worthwhile revising the ranges of the parameters.

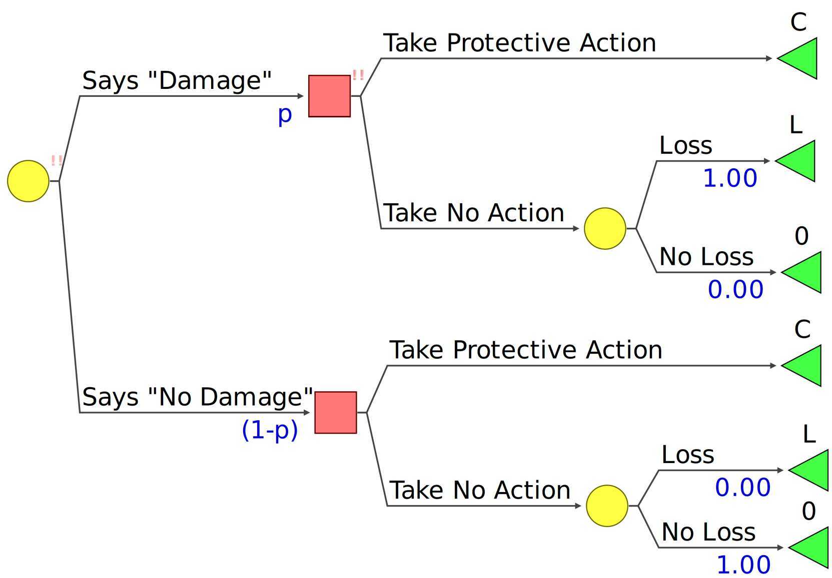
Problem 2. [30 points]

We'll stick with helping out our farmer friend. He is considering consulting a clairvoyant[1].



	low	base	high
p	10%	20%	50%
C	10k	20k	40k
L	50k	100k	200k

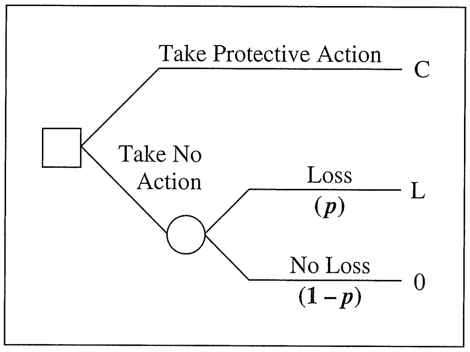
Suppose he has access to perfect information about whether there would be damage. Draw the augmented decision tree.



What is the Value of Perfect Information? Using base case values for variables p, C, and L. (Recall the expected values are *costs*, so smaller is better. You may also convert the costs to gains by writing "-C" and "-L" in the tree above.)

From Problem 1, we know the expected value without information is $\$20k$. Plugging in base case values in the above tree and solving, we get expected value to be $\$4k$. So $VoPI = \$16k$.

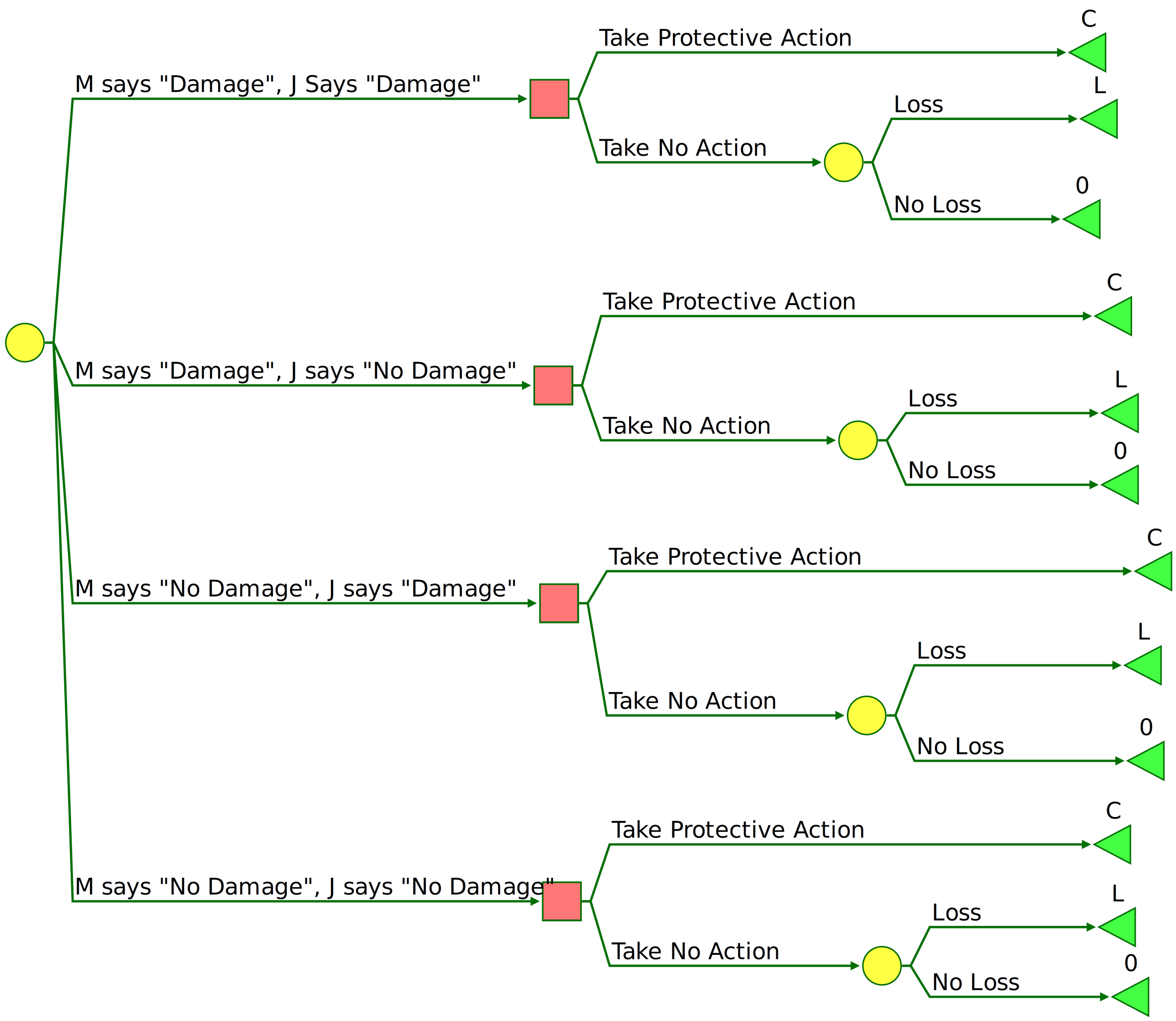
[1] Since our farmer does not have money for consulting firms, he'd look to Miss Cleo, John Edward, or a fortune cookie from that Chinese restaurant down the street.

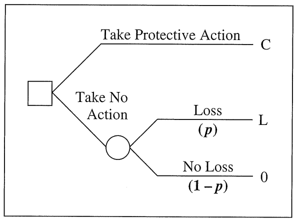


	low	base	high
p	10%	20%	50%
C	10k	20k	40k
L	50k	100k	200k

Now suppose he is considering consulting with two imperfect clairvoyants, M and J.

Draw the augmented decision tree (without probabilities) for this decision. Do not calculate probabilities here.





	low	base	high
p	10%	20%	50%
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Consider the two imperfect clairvoyants, M and J. Their statements are independent conditioned on the event. They have accuracies of 70% and 90% respectively. For example

$$\begin{aligned} P(M \text{ says loss }, \ J \text{ says no loss} \mid \text{loss }) &= P(M \text{ says loss} \mid \text{loss }) \ P(J \text{ says no loss} \mid \text{loss }) \\ &= (0.70)(1 - 0.90) \end{aligned}$$

What is $P(M \text{ says loss }, \ J \text{ says no loss })$? (use $p = 0.2$)

$$\begin{aligned} &P(M \text{ says loss }, J \text{ says no loss }) \\ &\quad = P(M \text{ says loss }, \ J \text{ says no loss }, \ \text{ loss }) \\ &\qquad + P(M \text{ says loss }, \ J \text{ says no loss }, \ \text{ no loss }) \\ &\quad = P(\text{ loss })P(M \text{ says loss }, \ J \text{ says no loss} \mid \text{loss }) \\ &\qquad + P(\text{ no loss })P(M \text{ says loss }, \ J \text{ says no loss} \mid \text{no loss }) \\ &\quad = P(\text{ loss })P(M \text{ says loss} \mid \text{loss })P(J \text{ says no loss} \mid \text{loss }) \\ &\qquad + P(\text{ no loss })P(M \text{ says loss} \mid \text{no loss })P(J \text{ says no loss} \mid \text{no loss }) \\ &\quad = 0.2 * 0.7 * 0.1 + 0.8 * 0.3 * 0.9 = 0.23 \end{aligned}$$

What is $P(\text{ no loss} \mid M \text{ says loss }, \ J \text{ says no loss })$? (use $p = 0.2$)

$$\begin{aligned} &P(\text{ no loss} \mid M \text{ says loss }, \ J \text{ says no loss }) \\ &\quad = \frac{P(\text{ no loss }, \ M \text{ says loss }, \ J \text{ says no loss })}{P(\ M \text{ says loss }, \ J \text{ says no loss })} \\ &\quad = \frac{P(\text{ no loss })P(\ M \text{ says loss }, \ J \text{ says no loss} \mid \ \text{no loss })}{P(\ M \text{ says loss }, \ J \text{ says no loss })} \\ &\quad = \frac{P(\text{ no loss })P(\ M \text{ says loss} \mid \ \text{no loss })P(\ J \text{ says no loss} \mid \ \text{no loss })}{P(\ M \text{ says loss }, \ J \text{ says no loss })} \\ &\quad = \frac{0.8 * 0.3 * 0.9}{0.23} = 0.93 \end{aligned}$$

Problem 3. [15 points]

A. Your friend Al wants to borrow \$100k for his business. He promises to pay you \$125k 5 years from now. What is the net present value of this deal? Assume 3% inflation.

$$NPV = -\$100k + \frac{\$125k}{(1+0.03)^5} = -\$100k + \$107.83 = \$7.83k$$

B. Your friend Al wants to borrow \$100k for his business. He promises to pay you back 5 years from now. How much should he pay you so you can make a profit of \$25k (in 2019 dollars)? Assume 3% inflation.

You want $NPV = \$25k$.

$$\$25k = NPV = -\$100k + \frac{x}{(1+0.03)^5}$$

$$x = \$125k(1+0.03)^5 = \$144.91k$$

C. Instead of lending your \$100k to Al, you consider investing it in an account with 6% annual interest. How many years will it take for the value (in 2019 dollars) to triple? Assume 3% inflation.

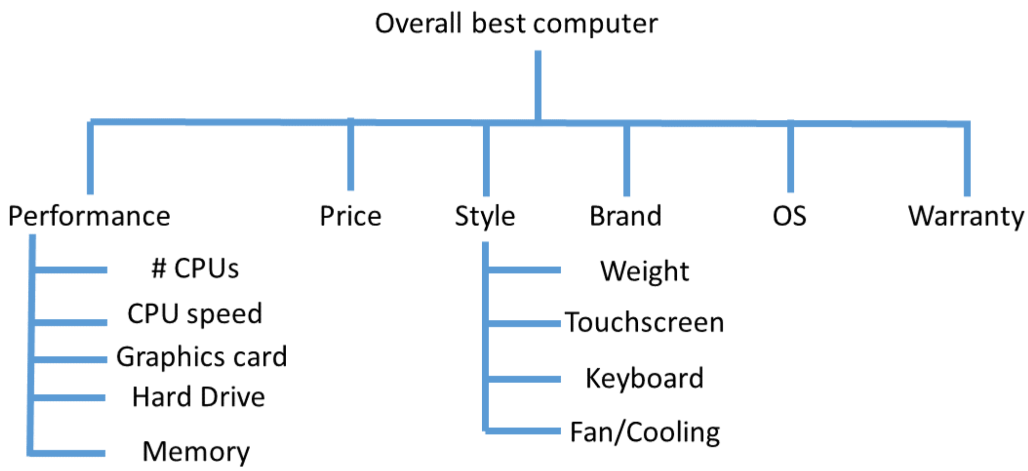
You want $NPV = \$200k$.

$$\$200k = NPV = -\$100k + \frac{\$100k(1+0.06)^n}{(1+0.03)^n}$$

$$3 = \left(\frac{1.06}{1.03}\right)^n$$

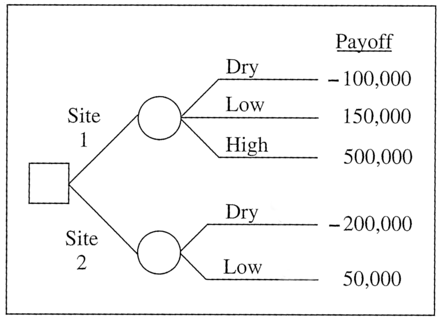
$$\log 3 / \log 1.0291 = n = 38.3$$

Problem 4. [5 points] Draw a fundamental objective hierarchy for buying personal computer. The hierarchy should have at least two layers (below overall objective) and at least ten total objectives (in addition to the overall objective). Do not determine trade-off coefficients (e.g. calculate an overall score).



Problem 5. [15 points] Consider you are a manager at an oil firm deciding where to drill for oil. There are two sites you are considering drilling at, and payoffs are listed in the tree below.

Construct a regret table for this decision and determine what strategy is best according to minimax regret.



There are two strategies, which we will call "Site 1" and "Site 2". There are six scenarios, and we will refer to them as "1D,2L" for site 1 dry and site 2 low, for example.

First, the payoff matrix:

	1D,2D	1D,2L	1L,2D	1L,2L	1H,2D	1H,2L
Site 1	-\$100k	-\$100k	\$150k	\$150k	\$500k	\$500k
Site 2	-\$200k	\$50k	-\$200k	\$50k	-\$200k	\$50k

Now we can calculate the regret matrix,

	1D,2D	1D,2L	1L,2D	1L,2L	1H,2D	1H,2L
Site 1	0	\$150k	0	0	0	0
Site 2	\$100k	0	\$350k	\$100k	\$700k	\$450k

So strategy "Site 1" has a worst case regret of \$150k, while strategy "Site 2" has a worst case regret of \$700k. Using minimax regret, strategy "Site 1" is best.

Explain in your own words what minimax regret is.

Minimax regret is a criteria to minimize the worst case losses *in hindsight*, once the scenario is revealed.