Industrial Engineering 546 — Spring 2017
Economic Decisions in Engineering

Exam 2

Student Information (print neatly)

Name: ________________________________

Purdue email: ___________________________

Directions:

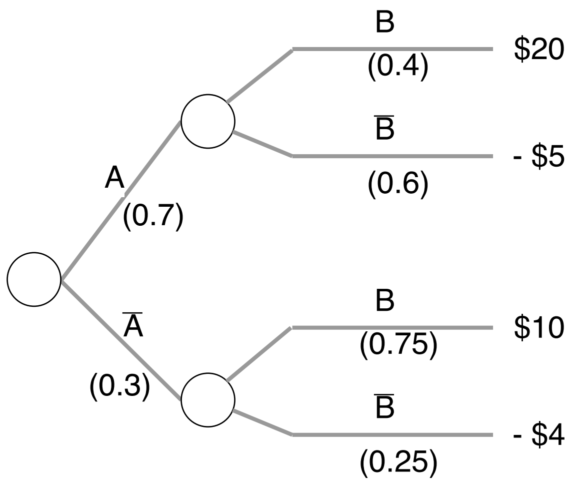
- You have 60 minutes to complete this exam. If you open the test before stated or do not turn it in on time you will lose 20 points.
- This exam is closed book and notes. You will receive a zero for this exam for using books, notes, electronic devices (cell phone, ipad, calculator, laptop, etc.). You will also be reported and appropriate disciplinary action will be taken.
- Your answers must be legible. Circle, underline, or leave sufficient white-space to distinguish your answers from intermediate work. The last page is scratch paper and may be torn out.
- Show all your work.

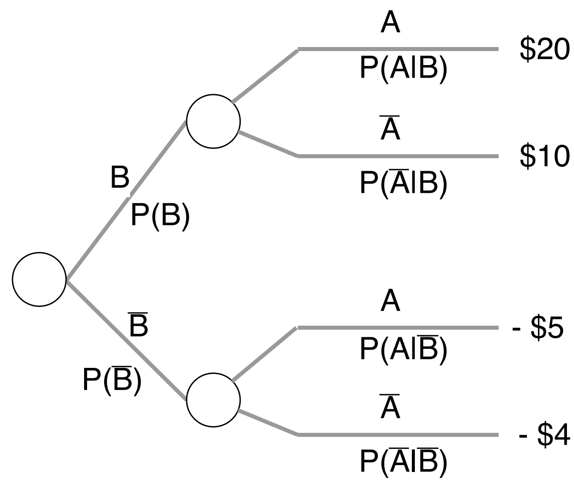
Grade:

1. [25] ___________________________
2. [25] ___________________________
3. [30] ___________________________
4. [20] ___________________________

Total: ___________________________

Problem 1. [25 points] In this problem you will flip a decision tree.





A. Write down the formulas for each of the probabilities in the left tree. (eg "$P(\dots) = 0.92$")

$$P(A) = 0.7 \quad P(\bar{A}) = 0.3$$

$$P(B|A) = 0.4 \quad P(\bar{B}|A) = 0.6 \qquad P(B|\bar{A}) = 0.75 \quad P(\bar{B}|\bar{A}) = 0.25$$

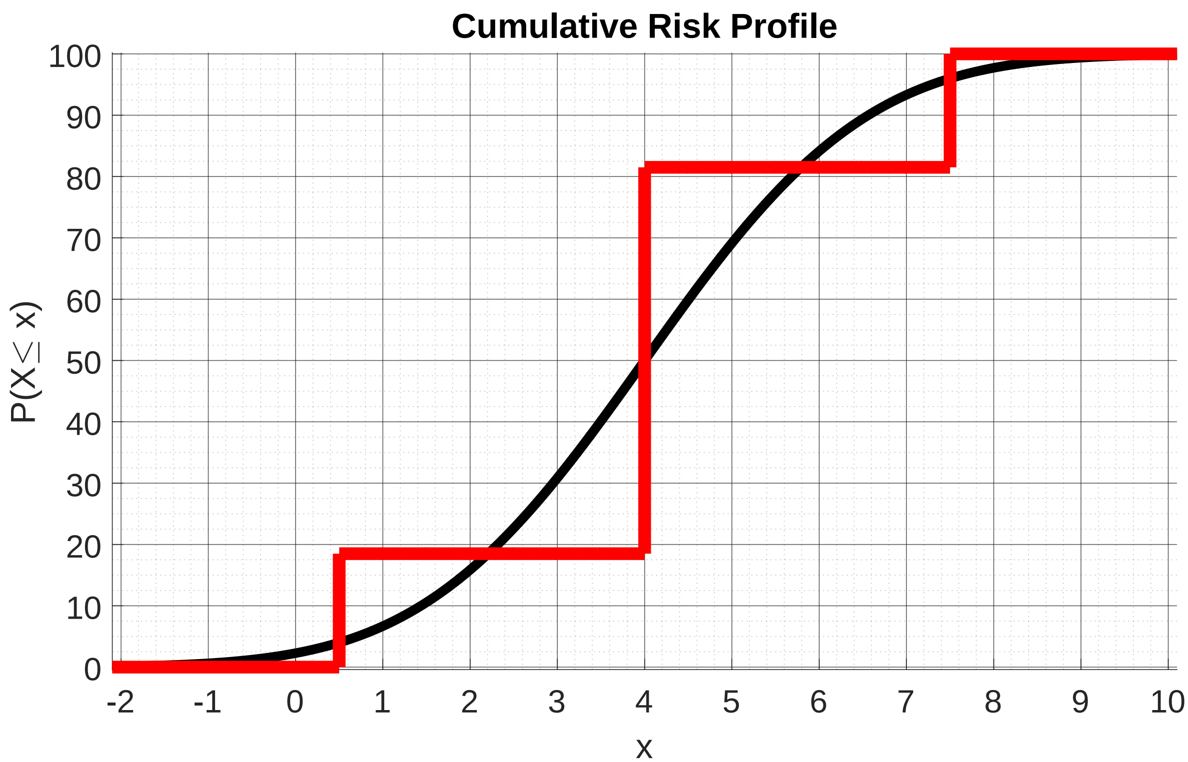
B. Label the outcome branches, final values, and the probability formulas (no #'s) for the tree on the right.

C. Solve for each of the probabilities in the right hand tree using probabilities in the left hand tree. Leave in terms of formulas (eg $P(\dots) = P(\dots)P(\dots) - \dots$). Don't plug-in values.

$$P(B) = P(B, A) + P(B, \bar{A}) = P(A)P(B|A) + P(\bar{A})P(B|\bar{A})$$
$$P(\bar{B}) = 1 - P(B)$$

$$P(A|B) = \frac{P(A, B)}{P(B)} = \frac{P(A)P(B|A)}{P(B)} \qquad P(\bar{A}|B) = 1 - P(A|B)$$
$$P(A|\bar{B}) = \frac{P(A, \bar{B})}{P(\bar{B})} = \frac{P(A)P(\bar{B}|A)}{P(\bar{B})} \qquad P(\bar{A}|\bar{B}) = 1 - P(A|\bar{B})$$

Problem 2. [25 points] You will approximate the following cumulative risk profile using discrete approximations.



For the following, round x-coordinates to the nearest half (eg, $\{\ldots, 4,\ 4.5,\ 5,\ 5.5,\ 6, \ldots\}$).

A. Find the Pearson-Tukey approximation Q_1. Write down the distribution. Also plot the corresponding cumulative risk profile of Q_1 on the graph above.

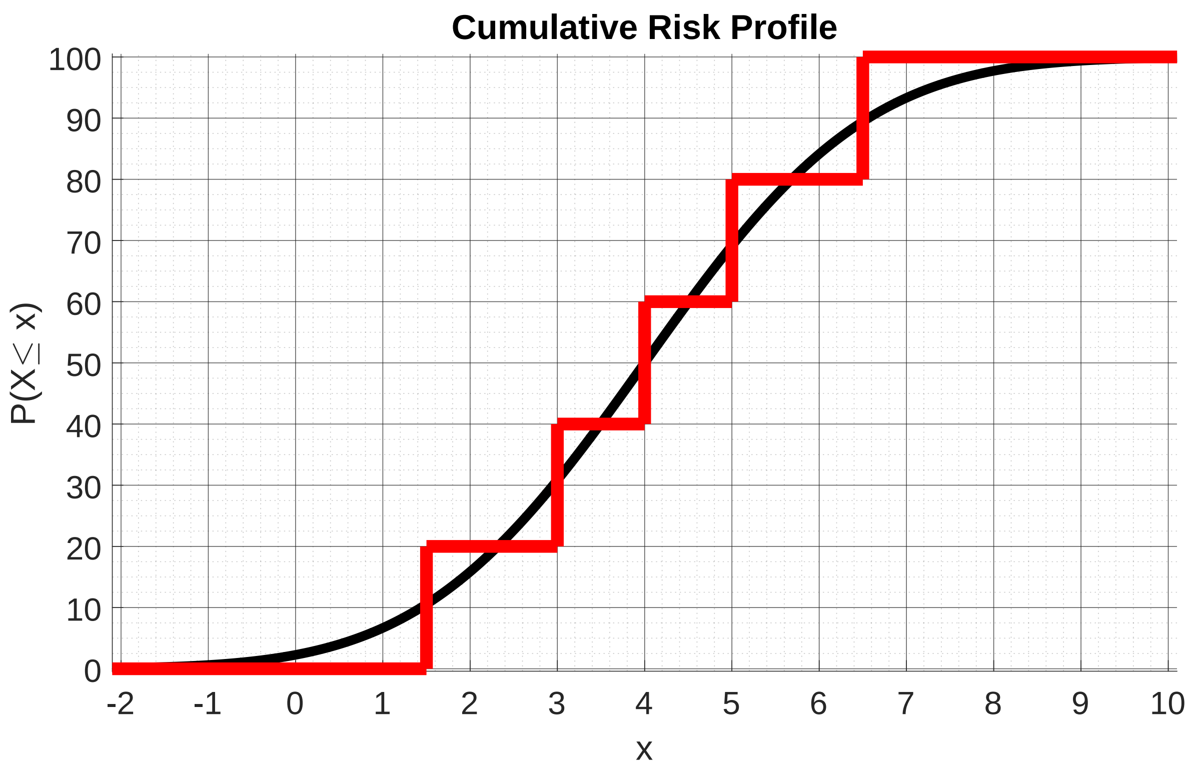
(the probabilities are shown on the last page)

Using the graph, first get the fractiles $x_{0.05}$, $x_{0.5}$, $x_{0.95}$.

$$x_{0.05} = 0.5 \qquad x_{0.5} = 4 \qquad x_{0.95} = 7.5$$

This results in the approximate, discrete distribution $Q(x)$, whose cumulative mass function ("cumulative risk profile") is plotted above.

$$Q(x) = \begin{cases} .185 & \textit{if } x = 0.5 \\ .63 & \textit{if } x = 4 \\ .185 & \textit{if } x = 7.5 \\ 0 & o/w \end{cases}$$



B. Find an approximation Q_2 using five equal-sized brackets. Write down the distribution. Also plot the corresponding cumulative risk profile of Q_2 on the graph above.

Since we will have five equal sized brackets, each will have width $100\%/5 = 20\%$*, so* $[0, 20]$, $[20, 40]$*, etc. For each we will use the median. Using the graph, first get the fractiles* $x_{0.10}$, $x_{0.30}$, $x_{0.50}$, $x_{0.70}$, $x_{0.90}$*. Then assign each a probability of* 0.2.

$$x_{0.10} = 1.5 \qquad x_{0.3} = 3 \qquad x_{0.5} = 4 \qquad x_{0.7} = 5 \qquad x_{0.9} = 6.5$$

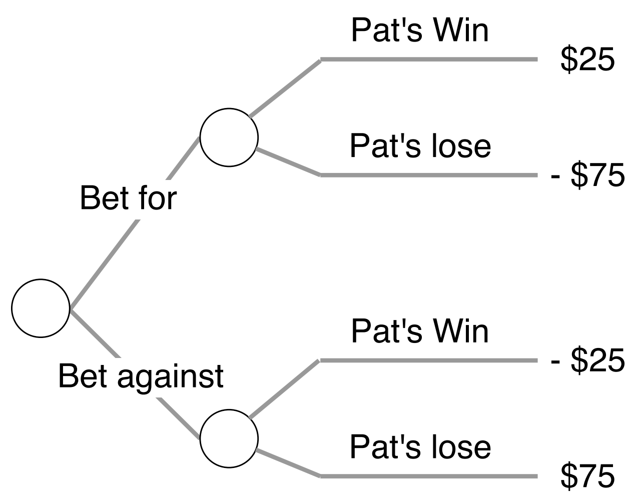
$$Q(x) = \begin{cases} .2 & \text{if } x = 1.5 \\ .2 & \text{if } x = 3 \\ .2 & \text{if } x = 4 \\ .2 & \text{if } x = 5 \\ .2 & \text{if } x = 6.5 \\ 0 & o/w \end{cases}$$

C. Visually, which approximation appears more accurate?

The median approximation.

Problem 3. [30 points] Prof. Quinn says he thinks the New England Patriots have a 75% chance of beating the Atlanta Falcons when they rematch in the 2017 season. Label the following trees appropriately.

A. You want to test if that is actually what he believes. Set up a symmetric lottery to assess his belief, starting with 75%. The magnitudes of the dollar amounts in each gamble should add to $100.



To pick a prize X, realize that the penalty is $-(\$100 - X)$ and we want to pick X such that if Prof. Quinn's belief is indeed 75%, then

$$0.75X + 0.25(-100 + X) = 0 \qquad \Longrightarrow \qquad X = \$25$$

B. Suppose you iterate through a few trees until Prof. Quinn says he is indifferent. Briefly explain the logic for how his indifference to the revised lottery lets you infer his belief. Let $\$X$ and $-\$Y$ denote values in the top revised gamble.

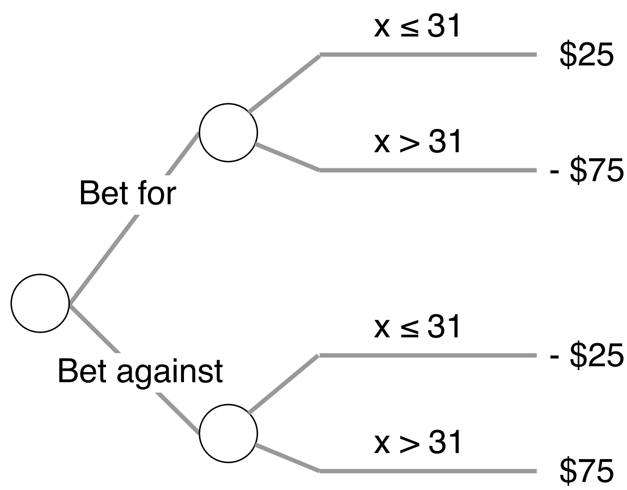
[this is more thorough than expected:]

By construction, for any probability p for "Pat's win," the expected value of the bet for alternative is the negative of the bet against. Indifference implies the expected values are equal, which only happens when both are zero. Since we know the expected value (zero), we can use the formula for expected value to solve for the unknown p. Let X denote the known, final prize and $-Y$ the known penalty,

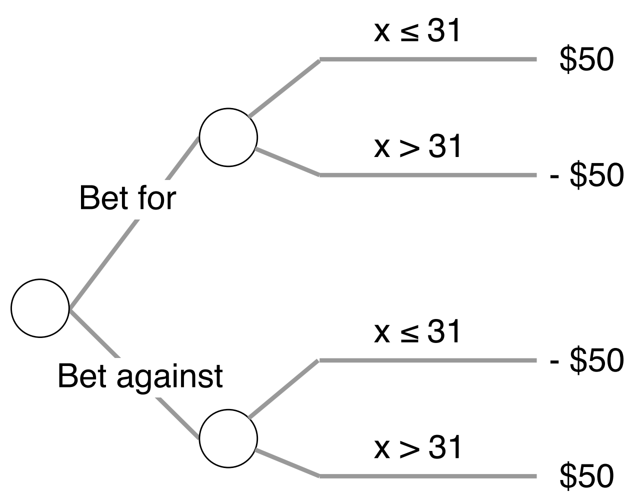
$$0 = p(X) + (1-p)(-Y) = p(X+Y) - Y \qquad \Longrightarrow \qquad p = \frac{Y}{X+Y}$$

Now you want to assess Prof. Quinn's belief for the the Patriot's final score. Treat the score as a continuous variable. Thus, you will try to infer his (implicit) continuous risk profile $P(X \leq x)$ for the score.

C. Consider the case that you partition the x-axis. Set up an initial symmetric lottery for the x-coordinate 31 with probability 0.75.

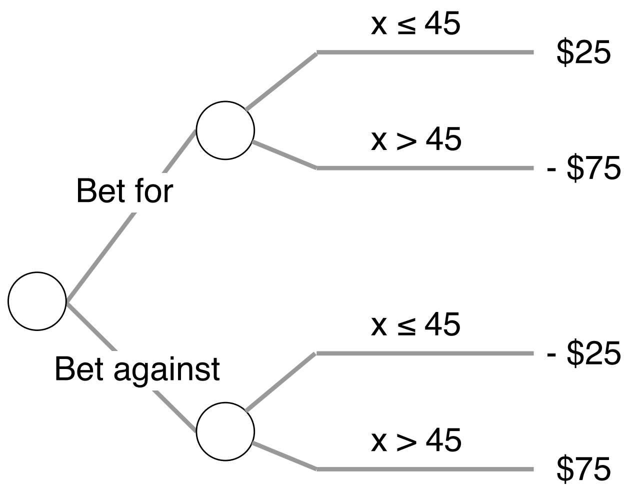


D. Suppose he says he prefers the bottom gamble. What does that mean about the probability 0.75 for x-value of 31? Construct a new lottery in response to his preference. (the specific values don't matter, just how they differ from the lottery above)



If he prefers the bottom gamble, he thinks it has a higher expected value, or equivalently the top gamble has a negative expected value. This means his belief $P(x \leq 31) < 0.75$. So we should change the prizes to correspond to a smaller probability.

E. Consider the case that you partition the y-axis. Suppose you use the same initial lottery as on the previous page (with probability 0.75 for x-coordinate 31), and Prof. Quinn says he prefers the top gamble. Construct a new lottery in response to his preference. (the specific values don't matter, just how they differ from the lottery above)



Following from part D, his belief $P(x \leq 31) < 0.75$. *Here, we want to keep that probability fixed (and thus prize money). Instead we change the event, increasing the value* 31 *to make the corresponding probability larger.*

Problem 4. [20 points]

A. Consider two events, A and B. Express the following relationships using probability formulas.

1. A and B are independent

$$P(A,B) = P(A)P(B) \qquad \text{or} \qquad P(B|A) = P(B)$$

2. A and B are mutually exclusive

$$P(A,B) = 0 \qquad \text{or} \qquad P(B|A) = 0$$

B. Your IE 546 prof tells you that a 1973 study of UC Berkeley graduate admissions found that men had a 44% acceptance rate while women had a 35% acceptance rate. Yet if you look at individual departments, women had the same or higher admissions in many of the departments. Is this possible? Briefly explain why.

Yes, and not necessarily due to any discrimination. We are looking at percentages, but if the underlying populations are not balanced, this can be misleading.

[This is a case of Simpson's paradox. Women tended to apply to departments with lower admission rates, with men tending to apply to departments with higher admission rates.]

C. Briefly explain the main pro and con of using decomposition to assess beliefs for complex events.

- *main pro: if an event A is hard to think about, assessments could be bad. With decomposition, you can assess probabilities that are easier to think about*
- *main con: more terms to assess*

D. Label which example corresponds to which of the following biases (one each): anchoring, gambler's fallacy, retrievability, prior blindness (aka base-rate fallacy).

1. "what are the odds of winning the lottery?" Having won last week, you reply "pretty good!"
   retrievability

2. You get an email with the name of your assigned teammate for an ECE senior design class. You haven't seen that name before and can't distinguish gender from it. You guess there's a 50/50 chance your teammate is male.
   ignoring priors / base-rate fallacy

3. You are at a roulette table and black has come up several times in a row. You decide to go all in for red.
   gambler's fallacy

4. To decide how much you should study for this exam, you first thought about how much you studied last exam and then decided to study some more. Now, with 5 min left in the test, you realize it wasn't enough.
   anchoring

Scratch

- Pearson-Tukey approximation

$$Q(x) = \begin{cases} .185 & \text{if } x = x_{0.05} \\ .63 & \text{if } x = x_{0.5} \\ .185 & \text{if } x = x_{0.95} \\ 0 & \text{o/w} \end{cases}$$