PUID: __________ Name: __________

Industrial Engineering 546 — Spring 2019

Economic Decisions in Engineering

Exam 2 Solutions

Directions:

- You have 120 minutes to complete this exam. If you open the test before stated or do not turn it in on time you will automatically lose 20 points.
- You are allowed two $8\frac{1}{2} \times 11$ inch sheets, with notes front and back. Otherwise, this exam is closed book and notes.
- Your are allowed to use a calculator (graphing or simple). No other electronic devices are permitted.
- Show all your work. Your answers must be legible. Circle, underline, or leave sufficient white-space to distinguish your answers from intermediate work.
- Write your name and PUID on each sheet used.
- Do not write along the edge of the paper or on the back-side. The final pages are blank and you may use them for scratch work or overflow.

Grade:

1. [20] __________
2. [10] __________
3. [10] __________
4. [10] __________
5. [15] __________
6. [35] __________
7. [Bonus 10] __________

Total: __________

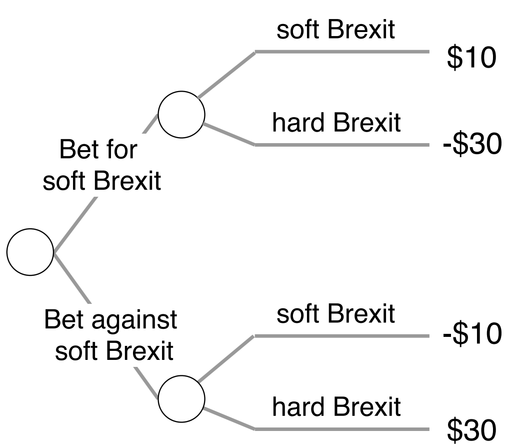
Problem 1. [20 points] Your friend Sue is thinking of expanding her retail business to Europe, but how lucrative the expansion will be depends on Britain's ultimate trade agreement. Namely, will it be a "soft" Brexit or a "hard" Brexit? Assume it will just be one of the two. Do not worry about utility for this problem (e.g. Sue is risk-neutral).

(A) Make a symmetric lottery to assess Sue's belief that there will be a soft Brexit. She tells you her initial gut feeling is that there is a $\frac{3}{4}$ chance it will be a soft Brexit. Set the initial payouts to correspond to that belief.

We want to pick payments so for $p = \frac{3}{4}$, the expected value of the two bets are zero. So pick x and y so

$$\begin{aligned} 0 &= xp - (1-p)y \\ &= \frac{3}{4}x - \frac{1}{4}y \\ \implies \quad y &= 3x \end{aligned}$$

So $x = \$10$ and $y = \$30$ works.

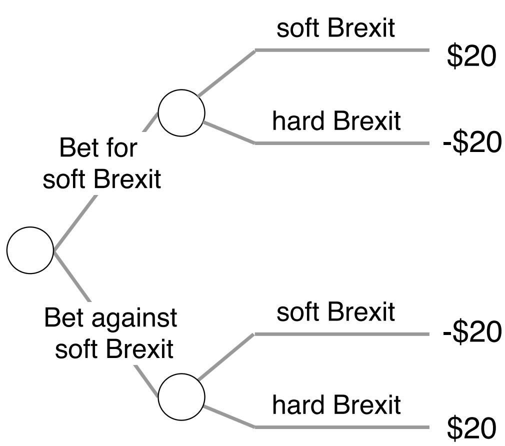


(B) Suppose after Sue thinks carefully about the lottery you set up, she decides she prefers the "bet against" branch. Does this mean her belief is more or less than $\frac{3}{4}$? Revise the symmetric lottery accordingly.

If she prefers the bet-against branch, then she thinks it has higher expected value, which for the symmetric lottery means it is positive,

$$\begin{aligned} -\$10p^* + \$30(1-p^*) &> 0 \\ -\$10p^* + \$30 - \$30p^* &> 0 \\ -\$40p^* &> \$30 \\ p^* &< \frac{3}{4} \end{aligned}$$

We want to pick payments so that for some $p < \frac{3}{4}$, such as $p = \frac{1}{2}$, the expected value of the two bets are zero. Let's use $x = 20$ and $y = 20$.



Problem 2. [10 points] You want to assess the conditional likelihood for event A given event B in a decision. Use notation $\bar{A}$ and $\bar{B}$ to denote complements[1].

Two experts, "Q" and "R," are available and can provide any joint or conditional probability you ask, such as $Q(B|\bar{A})$ or $Q(A, \bar{B})$ or $R(A)$. You decide to average their beliefs. Let's refer to the group-averaged belief as "P". So for instance, if you want $P(\bar{A}, B)$, then you can use

$$P(\bar{A}, B) = \frac{1}{2}\left[Q(\bar{A}, B) + R(\bar{A}, B)\right].$$

Is the following the correct way to calculate the group averaged probability $P(A|B)$? If not, show how to calculate it correctly.

$$P(A|B) = \frac{1}{2}\left[Q(A|B) + R(A|B)\right].$$

No, that is incorrect. The correct way is to first compute

$$P(A, B) = \frac{1}{2}\left[Q(A, B) + R(A, B)\right]$$

and

$$P(\bar{A}, B) = \frac{1}{2}\left[Q(\bar{A}, B) + R(\bar{A}, B)\right]$$

Then compute

$$P(B) = P(A, B) + P(\bar{A}, B).$$

(or by averaging the marginals)

Lastly,

$$P(A|B) = \frac{P(A, B)}{P(B)}$$

[1] meaning the events do not occur

Problem 3. [10 points] The following are short answer questions.

1. For assessing beliefs from multiple experts, we discussed several biases that can occur if the experts all discuss together. Name or describe two such biases.

   conformity bias (wanting to agree), authority bias / deference (assuming others more authoritative), biases arising from agendas, egos, or adversarial interactions (e.g. several members competing for promotion), among others

2. Your friend Eric will pass by a penny, nickel, or a dime laying on the sidewalk, without the slightest care. But if he sees two dimes or a quarter on the sidewalk he will aggressively rush to pick them up and even fight for it[2]. How would you describe his preferences for small amounts of money (less than a dollar) in terms of risk?

   Risk seeking

3. Label the following examples with the name of the corresponding bias. There is at most one example of each bias: anchoring, gambler's fallacy, conjunctive bias, retrievability, ignoring priors (aka base-rate fallacy).

   1. You have been quite successful investing in the stock market over several years. However, a company you invested in just went bankrupt, causing you to lose over five thousand dollars. A friend asks about investing and you tell her there is a very high chance she will lose money.

      retrievability

   2. You are designing a robot which requires thousands of parts to function properly. Each part has a small probability of failure (one in a thousand) so you think there's a very high chance the robot will function properly.

      conjunctive bias

   3. Your boss asks you to estimate quarterly sales for a new product line. You start by using the quarterly sales of a current product, but decide to be conservative and only revise down slightly.

      anchoring

[2] remember that time with the opossum?

Problem 4. [10 points] Your friend Jenny is deciding about a possible investment in a startup. You help her assess that there is a 1/5 chance she will make \$100,000, and a 4/5 chance she will lose \$10,000. She concludes that her certainty equivalent for this lottery is \$5,000.

What is her risk premium? Explain how the sign of the risk premium tells you about whether she is risk-seeking/averse/neutral.

The expected value of the investment is

$$1/5 * \$100{,}000 - 4/5 * \$10{,}000 = \$20{,}000 - \$8{,}000 = \$12{,}000.$$

Her risk premium is

$$RP = ExpVal - CE = \$12{,}000 - \$5{,}000 = \$7{,}000.$$

It is positive, which means that she is willing to "pay" (in loss of potential money) to avoid the risk of the investment. She is risk-averse.

Problem 5. [15 points] You can assess utility functions for a single, continuous-valued attribute (like salary) using the non-parametric "certainty equivalent" method[3].

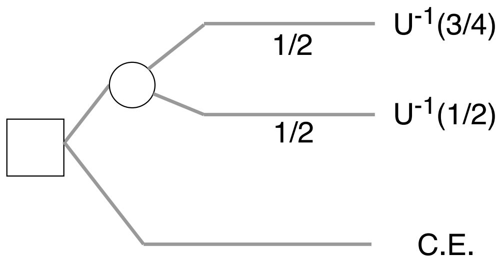
Briefly describe the sequence of points assessed. Draw the decision tree and show the calculations for obtaining $U^{-1}(\frac{5}{8})$.

By normalization, we first set the best alternative as $U^{-1}(1)$ and the worst alternative as $U^{-1}(0)$.

Next we use a lottery (similar to that shown below) to obtain $U^{-1}(\frac{1}{2})$. Using that point and the first two, we can then get $U^{-1}(\frac{1}{4})$ and $U^{-1}(\frac{3}{4})$. Using all these, we can then get $U^{-1}(\frac{1}{8})$, $U^{-1}(\frac{3}{8})$, $U^{-1}(\frac{5}{8})$, and $U^{-1}(\frac{7}{8})$ and so on.

(It is fine to just list them in order)

To assess $U^{-1}(\frac{5}{8})$, we ask the decision maker to determine his/her certainty equivalent for the following lottery:



The CE is where he/she is indifferent, which implies equal expected utility,

$$\begin{aligned} U(CE) &= \frac{1}{2}U(U^{-1}(\frac{3}{4})) + \frac{1}{2}U(U^{-1}(\frac{1}{2})) \\ &= \frac{1}{2}\frac{3}{4} + \frac{1}{2}\frac{1}{2} \\ &= \frac{5}{8} \end{aligned}$$

Hence, the CE is $U^{-1}(\frac{5}{8})$.

It is fine to come up with example alternatives for $U^{-1}(\frac{3}{4})$ and $U^{-1}(\frac{1}{2})$.

[3] e.g. the method where we find what points on the x-axis correspond to specific utilities and which uses lotteries with outcome probabilities fixed at $\frac{1}{2}$ and $\frac{1}{2}$.

Problem 6. [35 points] Your neighbor Mr. Jones is thinking of buying a new car. You offer to help him by modeling his preferences. You determine that an additive model is appropriate.

Mr. Jones only cares about three attributes: price, fuel efficiency, and color. Prices range from \$20k to \$35k (lower is better). Fuel efficiencies range from 25mpg to 50mpg (higher is better). Colors include blue (best), yellow (worst), and black (half-way between).

Suppose you've already assessed his marginal utility functions $U_{\mathrm{p}}(x)$, $U_{\mathrm{e}}(x)$, $U_{\mathrm{c}}(x)$ for price, efficiency, and color respectively. Now help Mr. Jones assess the coefficients k_p, k_e, and k_c.

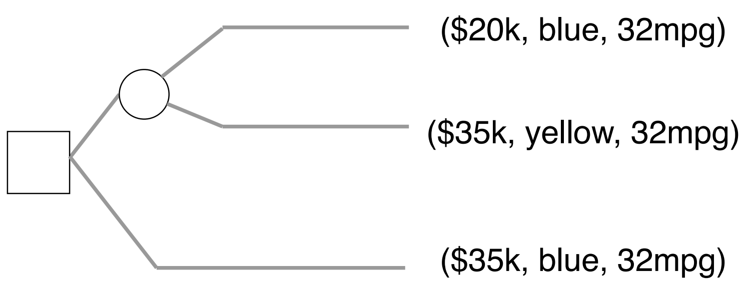
You may use "best" and "worst" as placeholders, such as "U(best, worst, best)" instead of "U(\$20k, yellow, 50mpg)."

(A) Set up a decision for Mr. Jones to think about in order to find the ratio between k_p and k_c. You can use either the "pricing out" method or the "lottery-based" method.

"pricing out" method. Pick two alternatives (fairly arbitrary), with price unspecified for alternative 2, values varying for the second column, and rest of attributes at the same value. Ask the decision-maker to think about at what price x he/she would be indifferent between the two alternatives.

	price	color	efficiency
Alternative 1	\$25k	black	40mpg
Alternative 2	x	yellow	40mpg

"lottery-based method" Construct three alternatives. They will have the same (arbitrary) attribute values for efficiency. For price and color, they will be best-in-both, worst-in-both, and worst-in-one (either worst in price or worst in color). The first two alternatives are in a lottery. The third is a guaranteed outcome. Mr. Jones should think about the lottery and determine for what probability p (or portion of shaded area for a wheel) he would be indifferent.



(B) Show how to calculate the ratio between k_p and k_c based on Mr. Jones's response. You may either use a variable for Mr. Jones's response or plug-in a reasonable number.

"pricing out method"

Indifference implies

$$\begin{aligned} U(\$25k, \text{black}, 40mpg) &= U(x, \text{yellow}, 40mpg) \\ k_pU_p(\$25k) + k_cU_c(\text{black}) + k_eU_e(40mpg) &= k_pU_p(x) + k_cU_c(\text{yellow}) + k_eU_e(40mpg) \\ k_c[U_c(\text{black}) - U_c(\text{yellow})] &= k_p[U_p(x) - U_p(\$25k)] \\ k_c &= k_p\frac{U_p(x) - U_p(\$25k)}{U_c(\text{black}) - U_c(\text{yellow})} \end{aligned}$$

"lottery-based method"

Indifference means equal expected utilities,

$$\begin{aligned} U(\$35k, \text{blue}, 32mpg) &= pU(\$20k, \text{blue}, 32mpg) + (1-p)U(\$35k, \text{yellow}, 32mpg) \\ k_pU_p(\$35k) + k_cU_c(\text{blue}) + k_eU_e(32mpg) &= p\left[k_pU_p(\$20k) + k_cU_c(\text{blue}) + k_eU_e(32mpg)\right] \\ &\quad + (1-p)\left[k_pU_p(\$35k) + k_cU_c(\text{yellow}) + k_eU_e(32mpg)\right] \\ k_p * 0 + k_c * 1 &= p\left[k_p * 1 + k_c * 1\right] + (1-p)\left[k_p * 0 + k_c * 0\right] \\ k_c(1-p) &= pk_p \\ k_c &= k_p\frac{p}{1-p} \end{aligned}$$

(C) We've assumed the additive model was appropriate. Describe how to check if even the multilinear can be used. Give an example question you would ask Mr. Jones to check this.

We need to check for preferential independence. For any two alternatives with some attributes at the same value, the preference should only depend on the attributes with different values. For example, we might ask the decision maker to consider the alternatives

$$(\$30k, \text{color}, 35mpg) \qquad vs. \qquad (\$40k, \text{color}, 25mpg)$$

If his preference depends on what color the car is, e.g. for yellow he prefers the first alternative and for blue he prefers the second, he does not exhibit preferential independence.

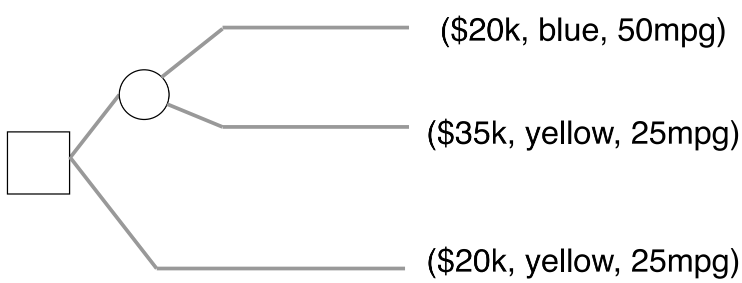
(D) Suppose the multilinear model is appropriate but additive is not.

1. Can you re-use the coefficients from the additive model? Why or why not?

No. Additive models have normalized coefficients and the pricing out and lottery methods enforce that. We should assess coefficients for the multilinear model with different methods.

2. If not, draw a diagram of the decision (with lottery) for assessing k_p, describe how to use it (what gets adjusted), and the formulas for calculating k_p.

Mr. Jones should consider the lottery below and decide for what probability p (or proportion of shaded area if using the wheel) he is indifferent between entering the lottery and taking the guaranteed outcome.



Indifference will imply equal expected utility. The utility of the guaranteed outcome:

$$\begin{aligned} U(best, worst, worst) &= k_p U_p(best) + k_c U_c(worst) + k_e U_e(worst) \\ &\quad + k_{pc} U_p(best) U_c(worst) + K_{pe} U_p(best) U_e(worst) \\ &\quad + k_{ce} U_c(worst) U_e(worst) + k_{pce} U_p(best) U_c(worst) U_e(worst) \\ &= k_p \qquad \text{(all other products include a 0 term)} \end{aligned}$$

The utility of the lottery, for the p specified by Mr. Jones:

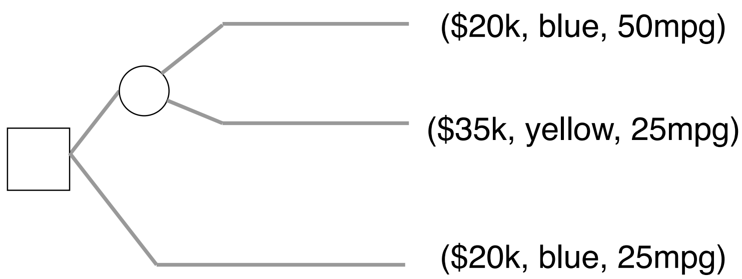
$$pU(best, best, best) + (1-p)U(worst, worst, worst) = p * 1 + (1-p) * 0 = p$$

Hence, the coefficient k_p is simply the probability p at which Mr. Jones is indifferent.

It is not necessary to write out all terms that end up having value of zero.

3. Do the same for k_{pc} Assume you already assessed k_p, k_c, and k_e.

Mr. Jones should consider the lottery below and decide for what probability p (or proportion of shaded area if using the wheel) he is indifferent between entering the lottery and taking the guaranteed outcome.



Indifference will imply equal expected utility. The utility of the guaranteed outcome:

$$\begin{aligned} U(best, best, worst) &= k_p U_p(best) + k_c U_c(worst) + k_e U_e(worst) \\ &\quad + k_{pc} U_p(best) U_c(worst) + K_{pe} U_p(best) U_e(worst) \\ &\quad + k_{ce} U_c(worst) U_e(worst) + k_{pce} U_p(best) U_c(worst) U_e(worst) \\ &= k_p + k_c + k_{pc} \qquad \text{(all other products include a 0 term)} \end{aligned}$$

The utility of the lottery, for the p specified by Mr. Jones:

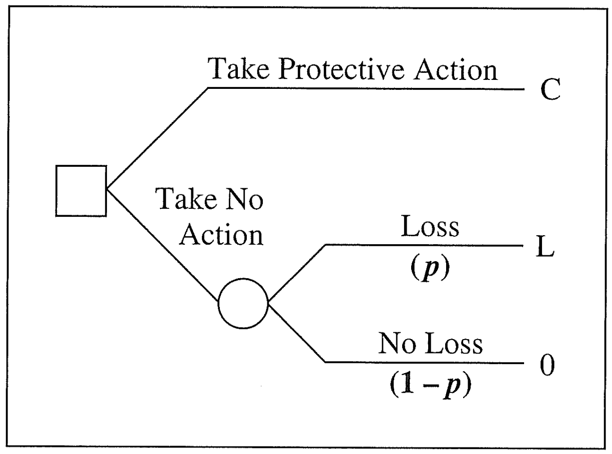
$$pU(best, best, best) + (1-p)U(worst, worst, worst) = p * 1 + (1-p) * 0 = p$$

Thus, indifference means expected utilities match and so

$$k_p + k_c + k_{pc} = p \qquad \Longrightarrow \qquad k_{pc} = p - k_p - k_c$$

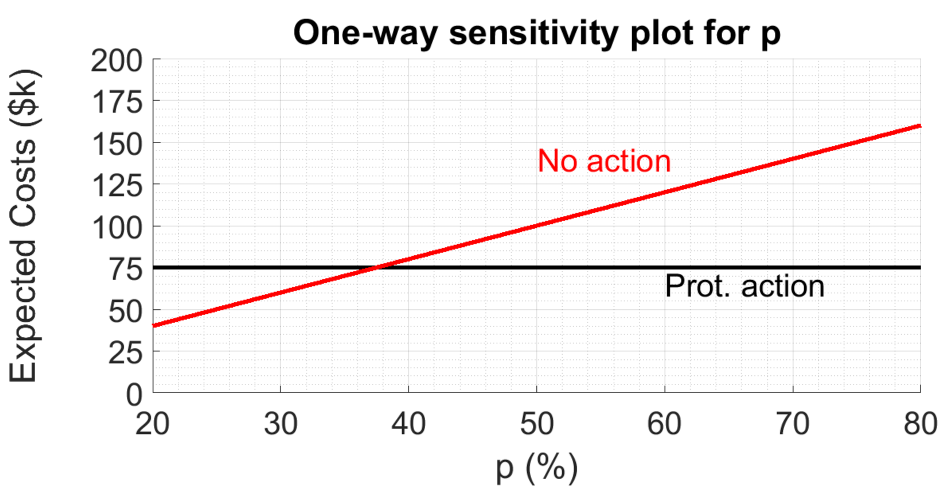
It is not necessary to write out all terms that end up having value of zero.

Problem 7. [Bonus 10 points] A farmer hears of potential bad weather next week. He can pay $\$C$ to take protective action to save his crop. If he does not take protective action, with probability p his crop will be severely damaged and he will incur a loss of $\$L$. The following decision tree models his decision.



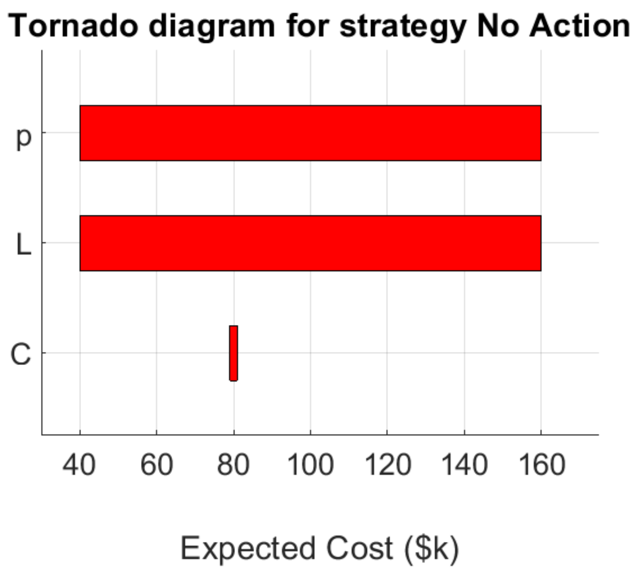
	low	base	high
p	20%	40%	80%
C	50k	75k	150k
L	100k	200k	400k

Draw the one-way sensitivity plot for p. Label the curves.

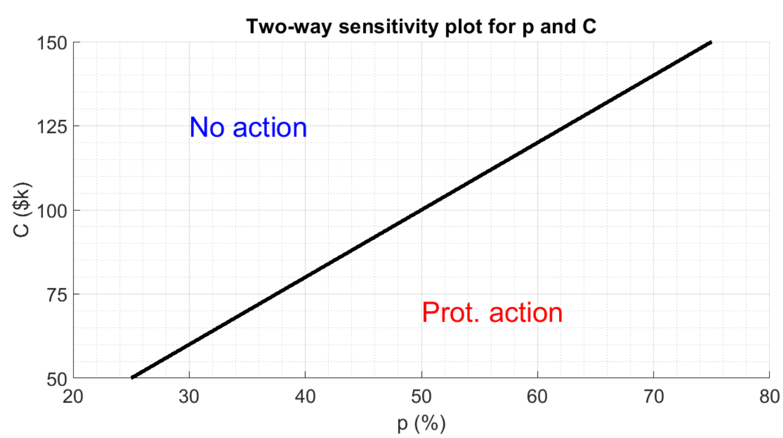


	low	base	high
p	20%	40%	80%
C	50k	75k	150k
L	100k	200k	400k

Draw the tornado diagram for strategy 'no action.' Label the axes.



Draw the two-way sensitivity plot for p and C. For each region, denote the preferred strategy. (Recall the values are *costs*, so smaller is better.)



What is the preferred alternative on the boundary itself?

Along the boundary, we are indifferent between the two alternatives.