

## Exam 3

Student Information (print neatly)

Name: \_\_\_\_\_

Purdue email: \_\_\_\_\_

### Directions:

- You have 120 minutes to complete this exam. If you open the test before stated or do not turn it in on time you will lose 20 points.
- This exam is closed book and notes. You will receive a zero for this exam for using books, notes, electronic devices (cell phone, ipad, calculator, laptop, etc.). You will also be reported and appropriate disciplinary action will be taken.
- Your answers must be legible. Circle, underline, or leave sufficient white-space to distinguish your answers from intermediate work. The last two pages are scratch paper and may be torn out.
- Show all your work.
- There are 115 points total. The exam will be graded out of 100, so there are 15 bonus points. However, the last two questions will have limited partial credit.

Grade:

1. [35] \_\_\_\_\_

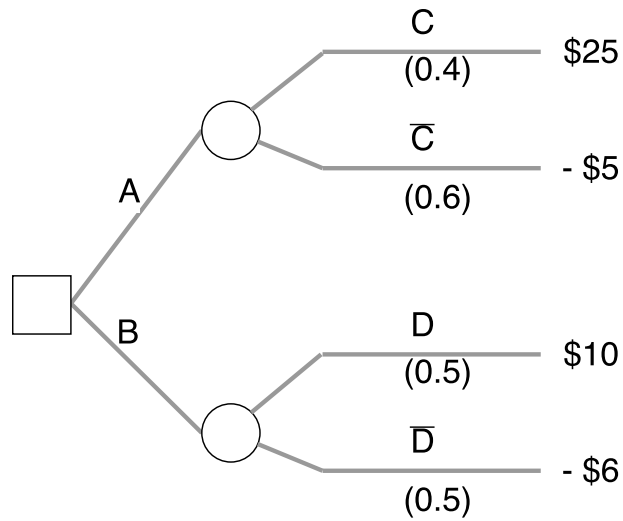
2. [35] \_\_\_\_\_

3. [25] \_\_\_\_\_

4. [20] \_\_\_\_\_

Total: \_\_\_\_\_

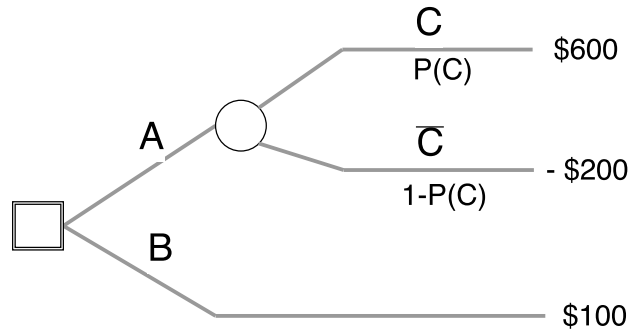
**Problem 1.** [35 points] This problem deals with the value of information. Do not calculate. Leave terms like “ $0.8 * 0.3$ ” as a product. For reference,  $E[\text{earnings A}] > E[\text{earnings B}]$ .



- A. Consider the decision tree above. Suppose you consult a perfect expert who will tell you the outcomes of both events. The events are independent. Draw the new decision tree where you use this information. Fully label the tree.

B. Write down the formula for value of information, plugging in numbers but do not calculate.

C. How is minimum expected regret related to the value of information? (brief answer)



D. Consider the decision tree above. Suppose you consult an imperfect expert who will tell you his prediction for the outcome of the uncertainty node. His accuracy is

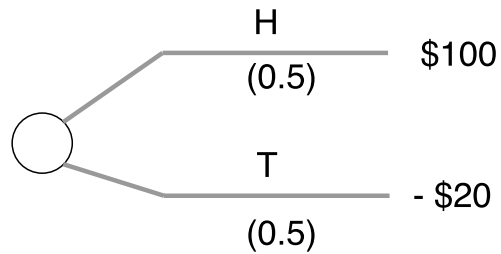
$$P(\text{ says } C \mid C)$$

$$P(\text{ says } \bar{C} \mid \bar{C})$$

Draw the new decision tree where you use this information. Fully label the tree.

E. For each probability you need in the new tree, write them in terms of probabilities  $(P(\cdot), P(\cdot|\cdot))$  you know.

**Problem 2.** [35 points] This problem deals with utility.



A. Consider the gamble above. Your friend Jill has the opportunity to play this. You want to take her place. Jill says the least she would sell her place for is \$10. What is Jill's risk premium for this gamble? (calculate the value)

B. What phrase describes her attitude toward risk?

C. Describe what a risk premium is in simple terms.

Your friend Jack is deciding between several job offers. He asks you to assess his utility curve  $U(x)$  for salary.

- D. You set the range to be from \$100k to \$200k. You will infer a normalized utility curve over this range. What utility values do the endpoints have?

You decide to infer his utility curve by partitioning the  $y$ -axis; that is, pick utility values and infer the corresponding salary.

- E. First set up a lottery to infer what salary  $x$  corresponds to  $U(x) = \frac{1}{2}$ . (draw the decision tree representing this lottery) Also show the formulas for why the lottery you designed yields  $U(x) = \frac{1}{2}$ .

- F. Suppose you conclude that for Jack,  $U(130) = \frac{1}{2}$ . Now set up two lotteries to infer what salary  $x$  corresponds to  $U(x) = \frac{1}{4}$  and what salary  $x$  corresponds to  $U(x) = \frac{3}{4}$ .

To check for consistency, you re-assess his utility curve by partitioning the  $x$ -axis; that is, picking salaries  $x$  and inferring the utility  $U(x)$ . (the range is still \$100k to \$200k)

G. Set up a lottery to infer Jack's utility for the salary \$120k. What will you ask Jack to contemplate for this lottery? (draw the corresponding decision tree)

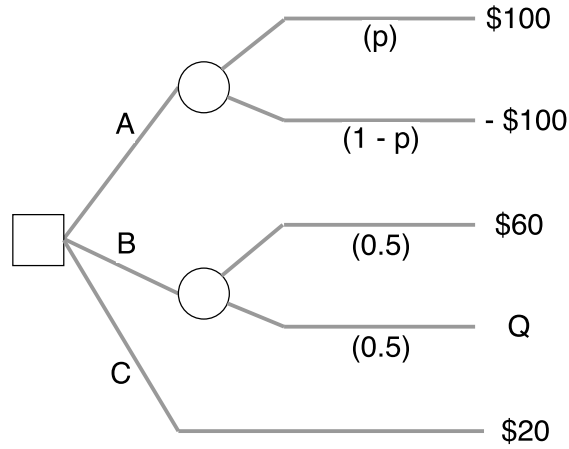
H. Show how to calculate Jack's utility based on his response to the lottery.

I. What are one pro and one con for using the method on this page (partition  $x$ -axis) compared to the method on the previous page (partition  $y$ -axis).





**Problem 3.** [25 points] This question is about sensitivity analysis. Consider the following decision tree.



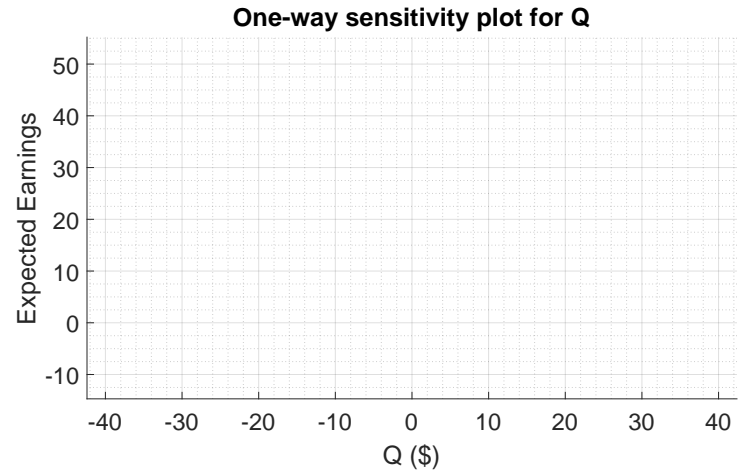
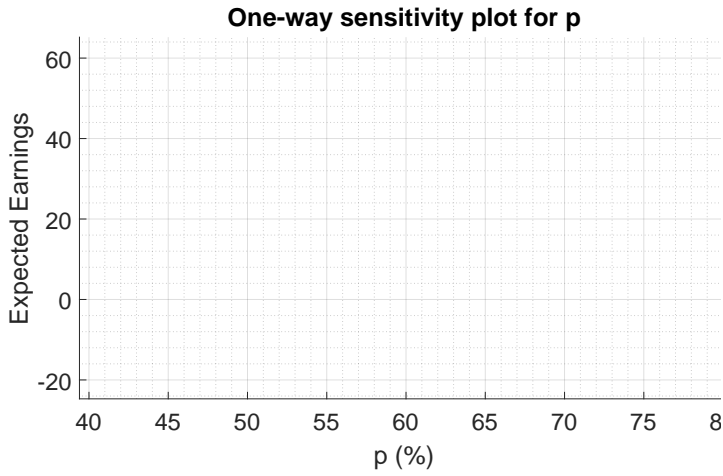
There are two parameters,  $p$  and  $Q$ , which are not known precisely.

	low	base	high
$p$	40%	50%	80%
$Q$	-\$40	\$0	\$40

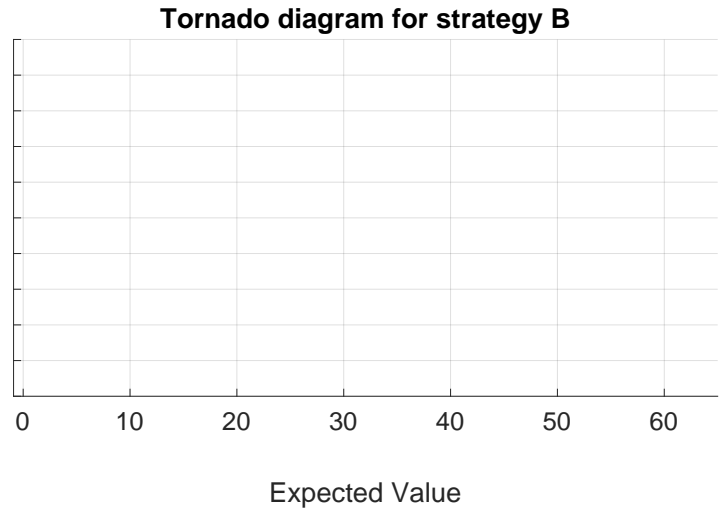
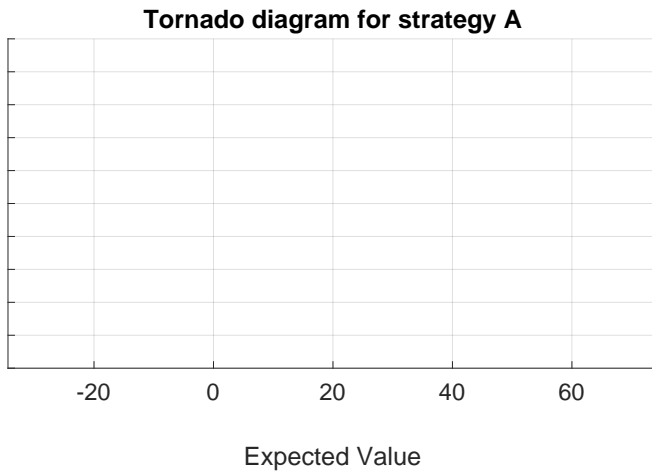
Conduct sensitivity analysis. First, determine the expected earnings as a function of the variables  $p$  and  $Q$ . Then calculate the expected earnings for low, base case, and high values of  $p$  and  $Q$ :

	formula (in terms of $p$ and $Q$ )	low	base	high
$E[\text{earnings of } A] =$				
$E[\text{earnings of } B] =$				
$E[\text{earnings of } C] =$				

Draw the one-way sensitivity plots for  $p$  and  $Q$ . Label the curves.

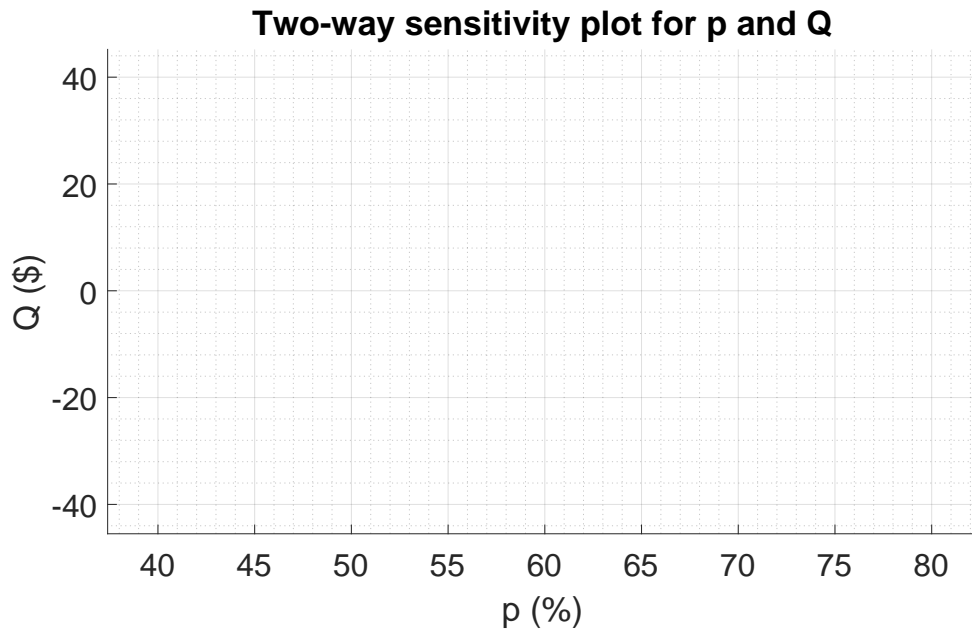


Draw the tornado diagrams for strategies  $A$  and  $B$ . Label the axes.



Draw the two-way sensitivity plot for  $p$  and  $Q$ .

- For each boundary denote which strategy is preferred on each side.
- For each region denote the preferred strategy.
- Show your work for solving for the boundary lines.



(Hint: for plotting the boundary line for A vs. B, solve for  $Q$  when  $p$  is 0.6 and 0.7)

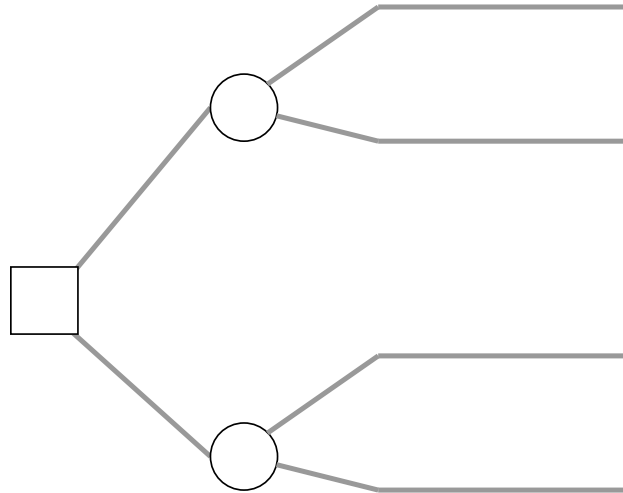


**Problem 4.** [20 points] Your friend Jake just finished a 72 hour ME 677 - Nonlinear Feedback Controller Design exam. Surprisingly, that was in-class, not a take-home.

He needs to decide whether he should sleep or study for his ME 680 - Bifurcations And Chaos exam, which starts at 10pm tonight.

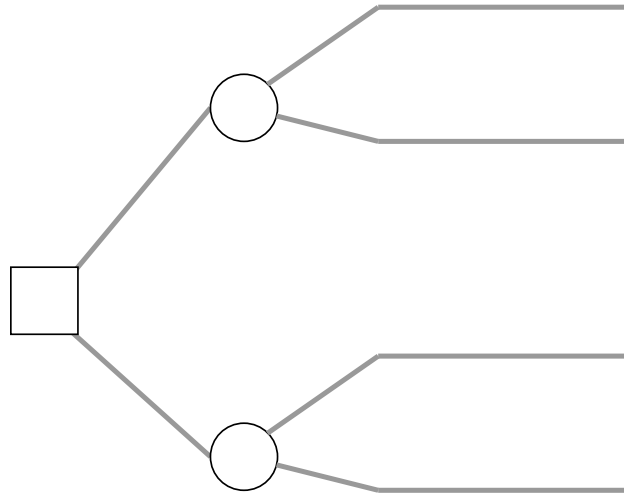
He asks you to use all your IE 546 wizardry to help him decide. You begin by assessing his belief about his ME 680 exam score were he to not study.

- A. He initially states that there's a 60% chance he won't get above an 85. Set up an initial symmetric lottery corresponding to this statement. Choose dollar amounts whose magnitudes add up to \$100. Label the tree.

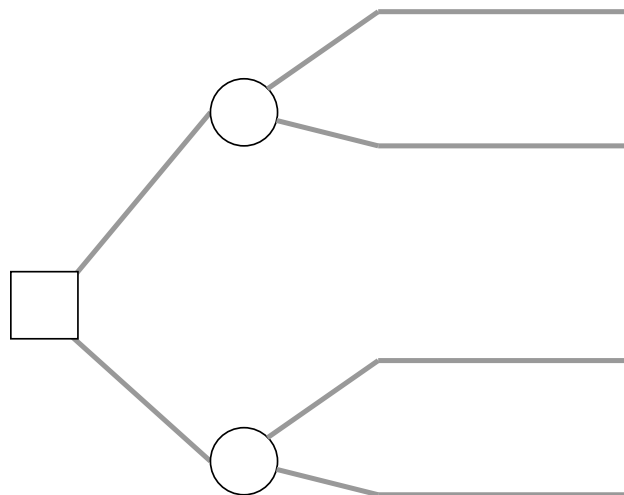


- B. Suppose he says he prefers the top gamble. What does that mean about the probability 0.6 for a grade of 85? [Don't think about utility for this problem.]

- C. Suppose you want to find what his true belief is about getting at most an 85. Construct a new lottery in response to his preference. Label the tree. (the specific values don't matter, just how they differ from the lottery above)



- D. Consider instead that you are trying to find the score that corresponds to a 60% belief (cumulative risk = 60%). Construct a new lottery in response to his preference. Label the tree. (the specific values don't matter, just how they differ from the original lottery)







# Scratch

Name (if detach but put work here): \_\_\_\_\_

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